

Monetary Policy under Imperfectly Observable
Commitment: The Return of the Time Inconsistency
Problem¹

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Abstract

This paper reconsiders monetary policy games with incomplete information concerning the central bank's ability to precommit. Unlike in the previous literature, commitment levels are taken to be imperfectly observable. This modification has drastic implications, even if the imperfection is arbitrarily small. In particular, reputation effects break down, and the power of precommitment becomes useless. This brings back the time inconsistency problem in full force. *Journal of Economic Literature* Classification Number: E52 Monetary Policy.

1 Introduction

This paper reconsiders the role of reputation in monetary policy games with incomplete information. Unlike in the previous literature, commitment levels are taken to be imperfectly observable.

In the literature on monetary policy games two interpretations of incomplete information have been used. In one branch, represented by Backus and Driffill [1985a,b], Barro [1986], and Cukierman and Liviatan [1991], incomplete information concerns *commitment*. The public does not know whether the central bank is truly committed to implement its policy target. Whereas in the other branch, represented by Vickers [1986] and Stein [1989], incomplete information concerns *preferences*.¹ The present paper is in the tradition of the first branch of the literature.

The commitment interpretation of incomplete information took off from Kydland and Prescott [1977] who pointed out the time inconsistency of optimal policies, and explained why the lack of credible commitment gives rise to an inflationary bias, that is however perfectly anticipated, and therefore has no benefits in terms of employment.

This pessimistic outlook was later challenged by Backus and Driffill [1985a,b], and Barro, [1986] who showed that the time inconsistency problem vanishes under incomplete information, because reputation mechanisms serve as a substitute for precommitment.² Their main result was that a weak central bank that has no power of precommitment can gain by mimicking the strong that has the power of precommitment. In order to maintain its reputation of being strong it chooses price stability, at least for some time.

¹Backus and Driffill [1985a,b] and Barro [1986] claim that their analysis allows both interpretations. As pointed out by Vickers [1986], results are however not robust if incomplete information concerns preferences. They then hold only in the extreme case when the strong central bank does not care for employment. Results change substantially, if the central bank is slightly concerned about employment. As shown by Vickers [1986], only a *separating* equilibrium is reasonable, unlike in Backus and Driffill [1985a,b] and Barro [1986] where the equilibrium is *pooling*.

²These contributions were inspired by Kreps and Wilson [1982] and Milgrom and Roberts [1982] who showed that the chain-store paradox may vanish under incomplete information.

The present paper casts doubt upon this resolution of the policy dilemma and brings back the time inconsistency of optimal plans in full force. Specifically we show that the public completely ignores the received signal of the commitment level. The public forms the same inflationary expectations no matter what signal it receives, if there is the slightest degree of imperfection in the transmission or perception of information. Consequently, the strong central bank loses its first mover advantage, and in equilibrium, both types of central banks behave exactly as if the public were sure that precommitment is not feasible.

The loss of the first mover advantage due to imperfectly observable commitment levels was recently observed by Bagwell [1995] in the context of duopoly games. However, Bagwell only considered the far simpler case of complete information.

In one regard our model is similar to Cukierman and Meltzer [1986] and Fudenberg and Levine [1992]. There, the public does not observe the central bank's action, which is the rate of monetary growth, but instead observes inflation, which is influenced by money growth but also by a stochastic and unobserved shock. Just as in our model, the link between the central bank's choice and the observed signal is weakened, and the central bank thus is subject to "moral hazard" in trying to maintain its reputation of being the "commitment type".

However, our results differ substantially. In Cukierman and Meltzer [1986] and Fudenberg and Levine [1992] reputation mechanisms still work and the central bank almost receives the "Stackelberg-payoff", which is the most she can obtain by a public commitment in the complete information context. Whereas, in the present paper, reputation effects break down and the central bank only receives the "Cournot-payoff", which is the most she can obtain in the absence of commitment. The key difference in the underlying assumptions is that the observed signal is payoff *relevant* in Cukierman and Meltzer [1986] and in Fudenberg and Levine [1992], whereas it is payoff *irrelevant* in the present paper.

There are two major motivations for the assumption of imperfectly observable commitment levels.

First, one cannot exclude misunderstanding or misinterpretation. In history,

there are many incidents where something went fatally wrong in the transmission of important information. Therefore, our approach may be seen as an attempt to account for the imperfections of real-life communication. For example, the Berlin wall finally came down when the press secretary of the *Politbureau* of the former GDR was misinterpreted in the news, and subsequently a tide of people rushed to border checkpoints. Of course, the transmission and perception of information may be almost perfect, meaning that the right message is received almost with certainty. But we believe that it is never perfect.

Second, the analysis of games with imperfectly transmitted messages provides an appealing alternative to common belief-based equilibrium refinements. It is an appealing alternative, because one can ensure that Bayes' rule is applicable everywhere.

The idea of introducing imperfect signals for equilibrium selection purposes has recently been exploited by Carlsson and Dasgupta [1994]. They propose the following equilibrium selection procedure: Introduce noise, such that each possible signal is consistent with the sender's underlying equilibrium actions, and then let noise vanish. The equilibria that survive such small perturbations of the receiver's perception of a signal are referred to as "noise-proof" equilibria.³ The present paper can be viewed as an application of noise-proofness to monetary policy games.

The plan of the paper is as follows. In *Section 2*, we explain the policy game. In *Section 3* we start with the preliminary assumption that commitment levels are never observed, and derive the properties of the unique equilibrium. In addition we show that perfectly observable commitment levels give rise to a continuum of equilibrium outcomes, including the one of the game with no observability. In *Section 4* we introduce the notion of imperfectly observable commitment levels and show that

³Carlsson and Dasgupta [1994] consider a signaling game where the message is payoff relevant for all sender's types, and payoff irrelevant for the receiver, and where the receiver has only two pure strategies — for instance to enter or not to enter the market. As they show, noise-proof equilibria are never separating, and they are "insufficiently revealing" in the sense that there is always a positive probability of the receiver's choosing an action which would be suboptimal in a full information context.

this game has an unique equilibrium outcome which coincides with that under no observability. The paper closes with a summary in *Section 5*.

2 The Policy Game

Our model differs from the standard monetary policy game with incomplete information in two regards. First, we determine the commitment level endogenously. In this regard we follow Cukierman and Liviatan [1991], and modify the standard monetary policy game by allowing the central bank to choose a policy target. Second, and more important, we admit that commitment levels are imperfectly observable.

Apart from these major modifications, we generalize the usual analysis in some more technical detail. One concerns the central bank's payoff function, the other the way how to handle a nonstrategic player in a game.

Players, strategies The game has two players. The central bank that is either weak (W) or strong (S), and the nonstrategic public. The central bank first chooses a target inflation rate $\pi^a \in \mathbb{R}$, and then chooses actual inflation $\pi \in \mathbb{R}$. S is committed to the target, W is not.

As in all policy games, the public is not a strategic player. Its only role is to process information, to form and revise beliefs concerning the central bank's type, and to compute inflationary expectations $\pi^e \in \mathbb{R}$.

Sequence of moves

Nature selects S with probability α , and W with probability $1 - \alpha$.

Stage 1 The central bank chooses a target inflation rate π^a , of which all players observe the blurred signal x . Commitments to random selection rules are not permitted.

Stage 2 The public computes the expected inflation rate π^e ; at the same time, the central bank chooses the inflation rate π . S always implements π^a , whereas W can choose any inflation rate $\pi \in \mathbb{R}$. Mixed strategies are permitted.

This two-stage game is played T times. All players have perfect recall.

Payoffs The central bank's per period payoff $u(\pi, \ell)$ is a function of the inflation rate π and of employment ℓ . She likes high employment but dislikes price instability. Both W and S have the same payoff function. They differ only in their ability to precommit.

Let $h(\pi - \pi^e)$ be the expectational Phillips curve. Then, the central bank's payoff can be written as a function of π and π^e

$$v(\pi, \pi^e) := u(\pi, h(\pi - \pi^e)). \quad (1)$$

The following assumptions are made:

(A1) u and h are differentiable and strictly concave, with $u_2 > 0$, $h' > 0$ everywhere; u has a global maximum at $\pi = 0$, for each given ℓ .

(A2) v is continuously differentiable with $\lim_{\pi \rightarrow \infty} u_2(\pi, h(\ell)h'(\ell)) < -\lim_{\pi \rightarrow \infty} u_1(\pi, h(\ell))$ for all ℓ , and $0 \leq v_{12} < -v_{11}$ everywhere.

Of course, these assumptions cover the usually assumed, simple payoff function

$$v(\pi, \pi^e) = -\frac{1}{2}\pi^2 + c(\pi - \pi^e), \quad c > 0, \quad (2)$$

as a special case.

Modeling the nonstrategic public A peculiar feature of policy games is that they include a nonstrategic player. This makes it impossible to apply the definition of a perfect Bayesian equilibrium. In order to avoid this complication, we introduce the following per period payoff function of the public

$$g(\pi^e, \pi) = -(\pi^e - \pi)^2. \quad (3)$$

Given this payoff function, the public can be viewed as an ordinary player, and expectations π^e can be treated like strategies. In this perspective the public is a player that simply wants to imitate his opponents action. Its only goal is to make

the best prediction of the rate of inflation — just as is commonly assumed in policy games.

Note, if S is committed to choose price stability in each period our game coincides with that of Barro (1986). However, S may have chosen another commitment or no commitment at all.⁴

3 Benchmark Cases

As a point of reference, suppose, for the moment, that the central bank has no commitment power and that this is common knowledge. The game then has a unique equilibrium with the following properties.

Proposition 1 (No commitment) *Consider the associated game of complete information, without commitment ($\alpha = 0$). It has a unique subgame perfect equilibrium. In each period the central bank chooses the same positive inflation rate $\pi^* > 0$, and employment is at its natural rate, $\ell^* = h(0)$.*

Proof The Proof is in *Appendix A*. ■

Given that employment is at its natural rate, the central bank would be strictly better off if it chooses price stability. However, under *complete* information, price stability is attainable only if the central bank has commitment power, and if commitment levels are perfectly observable.⁵ These results are fairly obvious, and need not be proved here.

Now introduce *incomplete* information. Then, perfectly observable commitment levels give rise to a continuum of equilibrium outcomes, including the one of complete information without commitment.⁶ In the following, π^* always denotes the

⁴Of course, S may commit itself to the time consistent action; in this sense, we do not exclude that S makes essentially no commitment.

⁵As pointed out by Schelling [1960], commitment has no value if the opponent receives no signal of the commitment level.

⁶This generalizes a result by Cukierman and Liviatan [1991], who consider the particular payoff function (2), and show that each $\pi^{\alpha*} \in [(1 - 2\alpha)c, c]$ is an equilibrium target of the one-shot game.

equilibrium inflation rate under *complete information*, in the absence of commitment.

Proposition 2 (Incomplete information with perfect observability) *Consider the one-shot game ($T = 1$) of incomplete information with perfectly observable commitment levels. This game has multiple perfect Bayesian equilibria where S and W choose the same target π^{a*} .*

- (i) *There exists an inflation rate $\tilde{\pi} < \pi^*$ such that each $\pi^{a*} \in [\tilde{\pi}, \pi^*]$ is an equilibrium target.*
- (ii) *The equilibrium with $\pi^{a*} = \pi^*$ has the same outcome as the associated complete information game without commitment.*

Proof The proof is in the *Appendix B*. ■

This multiplicity property obviously holds also in the repeated game. It crucially depends on the assumption of perfectly observable commitment levels. For suppose, commitment levels are not observable at all, but maintain the assumption of *incomplete* information. Then, S and W become identical players and the game of incomplete information degenerates to one of complete information, where the central bank and the public choose π and π^e simultaneously — just as under complete information without commitment. Therefore

Proposition 3 (Incomplete information with no observability) *Consider the game of incomplete information, and suppose commitment levels are not observable. This game has an unique perfect Bayesian equilibrium, and the equilibrium outcome is as under complete information without commitment, for all priors α .*

4 Imperfect Observability

We now return to imperfectly observable commitment levels, where the public observes only a blurred signal of S' 's commitment level π^a . Instead of directly observing π^a , the public observes only the realization x of a random variable X .

The conditional probability density function $f(x | \pi^a)$ satisfies the following requirements:

(A3) The likelihood ratio is strict monotone increasing

$$\hat{\pi}^a > \pi^a \Rightarrow \frac{f(\hat{x} | \hat{\pi}^a)}{f(\hat{x} | \pi^a)} > \frac{f(x | \hat{\pi}^a)}{f(x | \pi^a)} \quad \forall \hat{x} > x$$

(A4) $f(x | \pi^a) > 0$ for all $x \in \mathbb{R}$, and $\pi^a \in \mathbb{R}$

(A5) f is differentiable.

Assumption (A3) ensures a stochastic ordering such that higher targets become more likely when the observed signal x increases (see *Proposition 1* in Milgrom [1981]). Assumption (A4) ensures that Bayes's rule is applicable everywhere.

4.1 Equilibrium of the One-Shot Game

In a perfect Bayesian equilibrium the strategy chosen by the central bank, and the public's equilibrium expectations must be mutual best replies, given the public's beliefs about which node in the information set has been reached by the play of the game. These equilibrium conditions must hold not only for the overall game, but also for the continuation game that starts at *stage 2*. Further, beliefs must be consistent with Bayesian updating, and they must be confirmed in equilibrium. In the remaining text "equilibrium" means "perfect Bayesian equilibrium".

At *stage 2*, the central bank chooses π , and at the same time the public computes π^e . S sticks to the target π^a , and W 's equilibrium action is the best response to the public's equilibrium expectations π^{e*} (subscripts denote S 's and W 's action)

$$\pi_S^* = \pi^a, \quad \pi_W^* \in \operatorname{argmax}_{\pi} v(\pi, \pi^{e*}). \quad (4)$$

Since x is already observed at *stage 2*, conditions (4) must hold for each possible x .

The observed x is a signal of the central bank's type, and of the underlying target π^a . Let $\beta(x)$ denote the posterior of facing S , and let $\mu(y, x)$ denote the posterior

that π^a is lower than y , conditional on observing x and facing S

$$\beta(x) := \Pr\{\text{bank is } S \mid x \text{ is observed}\} \quad (5)$$

$$\mu(y, x) := \Pr\{\pi^a \leq y \mid x \text{ is observed and bank is } S\}. \quad (6)$$

Then, the publics' expected payoff is⁷

$$E[g(\pi^e, \Pi) \mid x] = -\beta(x) \int_{-\infty}^{\infty} (\pi^e - y)^2 d\mu(y, x) - (1 - \beta(x))(\pi^e - \pi_W^*)^2. \quad (7)$$

It has a unique maximum at⁸

$$\pi^{e*} = \beta(x) \int_{-\infty}^{\infty} y d\mu(y, x) + (1 - \beta(x)) \pi_W^*, \quad (8)$$

where $\int_{-\infty}^{\infty} y d\mu(y, x)$ is the publics' belief-based prediction of S 's choice of inflation that is formed after x is observed.

In turn, the central bank's equilibrium target (π_S^{a*}, π_W^{a*}) is the best response to its subsequent equilibrium action (4), and to the publics' equilibrium expectations (8).

$$\pi_S^{a*} \in \operatorname{argmax}_{\pi} \int_{-\infty}^{\infty} v(\pi, \pi^{e*}) f(x \mid \pi) dx \quad \text{s.t. (4) and (8),} \quad (9)$$

$$\pi_W^{a*} \in \operatorname{argmax}_{\pi} \int_{-\infty}^{\infty} v(\pi_W^*, \pi^{e*}) f(x \mid \pi) dx \quad \text{s.t. (4) and (8).} \quad (10)$$

Beliefs must be consistent with Bayes's rule, and they must be confirmed on the equilibrium path. Therefore, $\beta(x)$ is determined by Bayesian updating together with S 's and W 's equilibrium targets as follows

$$\beta(x) = \frac{\alpha f(x \mid \pi_S^{a*})}{\alpha f(x \mid \pi_S^{a*}) + (1 - \alpha) f(x \mid \pi_W^{a*})}. \quad (11)$$

⁷ We use the same notation for continuous and discrete random variables. For instance, if the public applies probability q to $\hat{\pi}^a$ and probability $1 - q$ to $\tilde{\pi}^a$, then

$$\int_{-\infty}^{\infty} (\pi^e - y)^2 d\mu(y, x) = q(\pi^e - \hat{\pi}^a)^2 + (1 - q)(\pi^e - \tilde{\pi}^a)^2.$$

⁸It may seem a little strange to treat expectations like strategies. Note however that we have defined the publics' payoff in such a way that this is equivalent to making the best prediction of π .

Also $\mu(y, x)$ must be confirmed on the equilibrium path.⁹

Does the belief $\mu(y, x)$ concerning S 's underlying target depend on the observed signal x ? In other words, is the public's prediction of S 's choice of inflation $\int_{-\infty}^{\infty} y d\mu(y, x)$ contingent on x ?

Proposition 4 (Irrelevance of signals I) *Consider the one-shot game ($T = 1$). In equilibrium, the public's prediction of S 's choice of inflation is independent of the observed signal x . The signal is completely uninformative concerning S 's underlying policy target.*

Proof Since commitments to random selection rules are not permitted, S chooses π_S^{a*} with certainty in equilibrium. Given the particular target π_S^{a*} , the public may observe *each* possible signal x from the support of X , by (A4). Hence, μ confirms on the equilibrium path if and only if the public applies probability one to the event that S has chosen π_S^{a*} , no matter which signal x is observed. Therefore

$$\int_{-\infty}^{\infty} y d\mu(y, x) = \pi_S^{a*} \quad \text{for all } x. \quad (12)$$

■

Next we show, that the received signal x is also completely uninformative concerning the central bank's type. For this purpose we first prove the following *Lemma*.

Lemma 1 *Consider the one-shot game ($T = 1$).*

- (i) *Suppose $\pi_S^{a*} = \pi^*$. Then W chooses the same inflation rate as S no matter which signal x is observed.*
- (ii) *Suppose $\pi_S^{a*} < \pi^*$. Then W chooses $\pi_W^* > \pi_S^{a*}$ no matter which signal x is observed.*
- (iii) *Suppose $\pi_S^{a*} > \pi^*$. Then W chooses $\pi_W^* < \pi_S^{a*}$ no matter which signal x is observed.*

⁹Obviously, each prediction of S 's choice of inflation $\int_{-\infty}^{\infty} y d\mu(y, x)$ that is based on beliefs $\mu(y, x)$ that fail to confirm on the equilibrium path, is self-destroying. Because, for each such prediction one can always find a better one, namely S 's best response to this very prediction.

Proof As we have already shown in the proof of *Proposition 1*, W 's best response function $R(\pi^e)$ is monotone increasing with slope less than one everywhere.

By the mean value theorem there exists a $\hat{\pi}^e$ such that

$$R(\pi^{e*}) - R(\pi^*) = (\pi^{e*} - \pi^*)R'(\hat{\pi}^e). \quad (13)$$

Recall that $\pi_W^* = R(\pi^{e*})$, $\pi^* = R(\pi^*)$, and $\pi^{e*} = \beta(x)\pi_S^{a*} + (1 - \beta(x))\pi_W^*$ by (8) and (12). Substitute these expressions into (13), this gives

$$\pi_W^* - \pi^* = [\beta(x)\pi_S^{a*} + (1 - \beta(x))\pi_W^* - \pi^*]R'(\hat{\pi}^e). \quad (14)$$

(i) Suppose $\pi_S^{a*} = \pi^*$. By (14) one then obtains:

$$\pi_W^* - \pi^* = (1 - \beta(x))(\pi_W^* - \pi^*)R'(\hat{\pi}^e). \quad (15)$$

Since $R'(\hat{\pi}^e) \in [0, 1)$ and $\beta(x) \in (0, 1)$ for all x , this implies that $\pi_W^* = \pi^*$ for all x .

(ii) Let $\pi_S^{a*} < \pi^*$. Assume, *per absurdum*, that there exists a signal x where $\pi_W^* \leq \pi_S^{a*}$. By (14) one then obtains:

$$\pi_W^* - \pi^* \geq (\pi_W^* - \pi^*)R'(\hat{\pi}^e). \quad (16)$$

Since $R'(\hat{\pi}^e) \in [0, 1)$ this implies that $\pi_W^* \geq \pi^*$.

However, $\pi_S^{a*} < \pi^*$ and $\pi_W^* \leq \pi_S^{a*}$ imply that $\pi_W^* < \pi^*$, which is a contradiction.

(iii) The proof is similar to (ii) and hence omitted. ■

Proposition 5 (Irrelevance of signals II) *Consider the one-shot game ($T = 1$). Suppose $\pi_S^{a*} \neq \pi^*$. Then, W chooses the same target as S , which makes the observed signal x completely uninformative concerning the central bank's type, $\beta(x) = \alpha$ for all x .*

Proof By (8) and (12) one has $\pi^{e*} = \beta(x)\pi_S^{a*} + (1 - \beta(x))\pi_W^*$. Substitute this expression into W 's payoff and define

$$v(\pi_W^*, \beta(x)\pi_S^{a*} + (1 - \beta(x))\pi_W^*) =: V(x). \quad (17)$$

Then, the best response requirement (10) can be written in the form

$$\int_{-\infty}^{\infty} V(x)[f(x | \pi_W^{a*}) - f(x | \pi^a)]dx \geq 0, \quad \forall \pi^a. \quad (18)$$

As shown in *Appendix C* requirements (18) are met if and only if W chooses the same target as S . $\pi_W^{a*} = \pi_S^{a*}$ implies that $\beta(x) = \alpha$ for all x , by (11). ■

Utilizing *Lemma 1 (i)* and *Propositions 4-5* we now show that the public completely ignores the observed signal x . It forms the same inflationary expectations no matter which signal x it receives. The equilibrium outcome of the game of imperfectly observable targets thus corresponds to the equilibrium outcome of the game where policy targets are not observable at all.

Proposition 6 (Futility of commitment) *Consider the one-shot game ($T = 1$). The game has an unique equilibrium outcome, which is the same as that under complete information without commitment, for each prior α .*

Proof First, suppose $\pi_S^{a*} = \pi^*$. Then, $\pi_W^* = \pi^*$, by *Lemma 1*. Therefore, $\pi^{e*} = \pi^*$, which is independent of x .

Next suppose that $\pi_S^{a*} \neq \pi^*$. By *Propositions 4-5* one then obtains $\pi^{e*} = \alpha\pi_S^{a*} + (1 - \alpha)\pi_W^*$. The prior α and S 's equilibrium target π_S^{a*} are independent of x (π_S^{a*} is chosen before x is observed). As shown in *Appendix D*, π_W^* is also independent of x .

Hence, π^{e*} is independent of x , for each possible π_S^{a*} . The set of equilibrium outcomes of the game with *imperfectly* observable commitment levels thus corresponds to the set of equilibrium outcomes of the game with *no* observability. From here the assertion follows immediately by *Propositions 3* and *1*. ■

The game might have multiple equilibria, if we permitted commitment to random selection rules.¹⁰ However, most equilibrium selection rules favor strict equilibria.

¹⁰In the framework of a Stackelberg game with complete information Bagwell [1995] and van Damme and Hurkens [1994] have shown that imperfectly observable commitment give rise to multiple equilibria. One pure strategy equilibrium — there the first mover advantage gets lost — and several mixed strategy equilibria. One mixed strategy equilibrium supports the Cournot outcome,

The unique pure strategy equilibrium of the present game is strict, whereas each mixed strategy equilibrium is weak. Therefore, the adoption of standard equilibrium selection rules always leads to the equilibrium stated in *Proposition 6*.

4.2 Equilibrium of the Repeated Game

In the repeated game past inflation rates are additional signals of the central bank's type. But the addition of these signals cannot give rise to reputation effects to emerge. By *Proposition 6*, the equilibrium outcome of the last period is unique, and independent of the public's beliefs concerning the central bank's type. Hence, the equilibrium outcome of the last period is independent of the actual history of the game. By backwards induction, this extends to each period. Thus, there is no link between different periods, and in each period players behave exactly as in the one-shot game.

Proposition 7 (Repeated game) *Consider the finitely repeated game of incomplete information with imperfect signals. The game has an unique equilibrium outcome, which is the same as under complete information without commitment, stated in Proposition 1.*

Proof Using *Proposition 6*, the assertion follows immediately by backward induction. ■

Note, that our results hold for all probability distributions of the signal X that satisfy assumptions (A3)–(A5). Therefore, it also applies if one puts almost all probability mass on a arbitrarily small neighborhood of $X = \pi^a$. Therefore, our result holds also in the limit, when the signal imperfection tends to vanish, and the public observes something very close to the true policy target almost with certainty. yet another converges in probability to the Stackelberg equilibrium outcome if imperfections tend to vanish.

5 Conclusions

This paper reconsidered the standard monetary policy game with incomplete information concerning the central bank's ability to precommit, in the tradition of Backus and Driffill [1985a,b], Barro [1986] and Cukierman and Liviatan [1991]. There, the public is uncertain whether the central bank is committed to its policy target.

Our model differs from the standard monetary policy game, because we admit that commitment levels are imperfectly observable. This modification was shown to have drastic implications. Altogether, reputation effects break down, and the power of precommitment becomes useless.

As in Barro [1986] and Cukierman and Liviatan [1991] the weak central bank that has no power of precommitment always pursues the same monetary policy as the strong that has the power of precommitment. However, while in Barro [1986] and Cukierman and Liviatan [1991] the weak central bank mimics the strong, and leans toward price stability, in our model the strong "imitates" the weak, because imperfect signals destroy the strategic benefit of precommitment.

Appendix

A Proof of Proposition 1

In the associated game of complete information without commitment ($\alpha = 0$), the central bank and the public choose π and π^e simultaneously. First consider the one-shot game ($T = 1$). There, the equilibrium inflation rate π^* and the equilibrium expectations π^{e*} are

$$\pi^{e*} \in \operatorname{argmax}_{\pi^e} g(\pi^e, \pi^*) \quad \pi^* \in \operatorname{argmax}_{\pi} v(\pi, \pi^{e*}). \quad (19)$$

We first show that $\pi^* > 0$ for each given π^e . Notice that $v_1 = u_1 + u_2 h' > 0$ if $\pi = 0$, by (A1), and that $v_1 < 0$ if $\pi \rightarrow \infty$ by (A2) — for each π^e . Hence, for each given π^e there exists a $\pi^* > 0$ where $v_1 = 0$, by the continuity of v_1 . Further, W' 's best respond against π^e is *unique* since v is strictly concave, by the strict concavity of u and h , and by $u_2 > 0$.

Next notice that $\pi^{e*} = \pi^*$ by (19) and (3). Hence, employment is at its natural rate, $l^* = h(0)$ and $\pi^* = R(\pi^*)$. There, R denotes W 's best response function, that is implicitly defined by the first order conditions $v_1(R(\pi^e), \pi^e) = 0$ for all π^e .

It remains to be shown that the fixed point $\pi^* = R(\pi^*)$ exists and that it is unique. R is monotone increasing with slope less than one everywhere, since $0 \leq -v_{12}/v_{11} < 1$ by (A2). Hence, R is a contraction and thus has a unique fixed point.

Up to this point we have shown that the one-shot game has an *unique* Nash equilibrium with the asserted properties. Therefore, the finitely repeated game has an unique subgame perfect equilibrium, where the equilibrium of the one-shot game is played in each period.

B Proof of Proposition 2

Let β denote the public's posterior of facing central bank S , and suppose

$$\beta = \begin{cases} \alpha & \text{if } \pi^a = \pi^{a*} \\ 0 & \text{otherwise.} \end{cases} \quad (20)$$

Then, the public's expectations are given by

$$\pi^e = \begin{cases} \alpha\pi^{a*} + (1-\alpha)\pi_W^* & \text{if } \pi^a = \pi^{a*} \\ \pi^* & \text{otherwise.} \end{cases} \quad (21)$$

There $\pi_W^* = R(\alpha\pi^{a*} + (1-\alpha)\pi_W^*)$. Note that the fixed point $\pi_W^* = R(\alpha\pi^{a*} + (1-\alpha)\pi_W^*)$ exists for each given π^{a*} , by Assumption (A2).¹¹

Given these expectations, π^{a*} is S 's and W 's equilibrium target if

$$v(\pi^{a*}, \alpha\pi^{a*} + (1-\alpha)\pi_W^*) \geq v(\pi^*, \pi^*), \quad (22)$$

$$v(\pi_W^*, \alpha\pi^{a*} + (1-\alpha)\pi_W^*) \geq v(\pi^*, \pi^*). \quad (23)$$

(i) Utilizing these results we now show that there exists a $\tilde{\pi} < \pi^*$ such that each target $\pi^{a*} \in [\tilde{\pi}, \pi^*]$ is an equilibrium target. (23) holds whenever (22) is satisfied,

¹¹ Define $k(\pi_W^*) := R(\alpha\pi^{a*} + (1-\alpha)\pi_W^*)$. Then, $k' = -(1-\alpha)\frac{v_{12}}{v_{11}}$, and in conjunction with (A2) one obtains that $0 \leq k' < 1$, for each given π^{a*} . Hence k is a contraction and thus has a (unique) fixed point.

since $v(\pi_W^*, \pi^{e*}) \geq v(\pi^{a*}, \pi^{e*})$ by (4). Hence, we need only prove that there exists a $\tilde{\pi} < \pi^*$ such that requirement (22) is met for all $\pi^{a*} \in [\tilde{\pi}, \pi^*]$.

Notice that

$$v(\pi^{a*}, \alpha\pi^{a*} + (1-\alpha)\pi_W^*) \geq v(\pi^{a*}, \alpha\pi^{a*} + (1-\alpha)\pi^*) \quad \forall \pi^{a*} \leq \pi^* \quad (24)$$

This inequality holds since $v_2 = -u_2 h' < 0$, and since $\pi_W^* \leq \pi^*$ for each given $\pi^{a*} \leq \pi^*$.¹²

Define $w(\pi^{a*}) := v(\pi^{a*}, \alpha\pi^{a*} + (1-\alpha)\pi^*)$. Note that $w'(\pi^*) < 0$, since $v_1(\pi^*, \pi^*) = 0$ by definition of π^* , and $v_2(\pi^*, \pi^*) < 0$ by (A1). Hence, there exists an $\tilde{\pi} < \pi^*$ with $w(\pi^{a*}) > w(\pi^*)$ for all $\pi^{a*} \in [\tilde{\pi}, \pi^*)$. By definition of $w(\pi^{a*})$ one thus has

$$v(\pi^{a*}, \alpha\pi^{a*} + (1-\alpha)\pi^*) \geq v(\pi^*, \pi^*), \quad \forall \pi^{a*} \in [\tilde{\pi}, \pi^*], \quad (25)$$

which together with (24) shows that (22) is met for all $\pi^{a*} \in [\tilde{\pi}, \pi^*]$.

(ii) Suppose, $\pi_S^{a*} = \pi^*$. Then $\pi_W^* = \pi^*$ (see the proof of *Lemma 1 (i)*). Therefore, $\pi^{e*} = \pi^*$ and the equilibrium outcome is the same as under complete information without commitment.

C Proof of Proposition 5

Here we complete the proof of Proposition 5 and show that requirements (18) are met if and only if $\pi_W^{a*} = \pi_S^{a*}$ — given that $\pi_S^{a*} \neq \pi^*$.

Integration by parts shows that (18) is equivalent to

$$\int_{-\infty}^{\infty} V'(x)[F(x | \pi_W^{a*}) - F(x | \pi^a)]dx \leq 0, \quad \forall \pi^a, \quad (26)$$

where $F(x | \cdot) := \int_{-\infty}^x f(y | \cdot)dy$, and where $V(x)$ is defined by (17).

Notice, that $v_1(\pi_W^*, \beta(x)\pi_S^{a*} + (1-\beta(x))\pi_W^*) = 0$ for all x since $\pi_W^* = R(\pi^{e*})$. This in turn implies that $\frac{d\pi_W^*}{dx} = -\frac{v_{12}(\pi_S^{a*} - \pi_W^*)}{v_{11} + (1-\beta)v_{12}}\beta'$. Therefore,

$$V'(x) = \frac{v_2 v_{11}(\pi_S^{a*} - \pi_W^*)}{v_{11} + (1-\beta)v_{12}}\beta'(x). \quad (27)$$

¹²In order to show that $\pi^{a*} \leq \pi^* \Rightarrow \pi_W^* \leq \pi^*$, one needs an argument similar to the proof of *Lemma 1(ii)*. Let $\pi^{a*} \leq \pi^*$, (13) then yields $\pi_W^* - \pi^* \leq (1-\beta)(\pi_W^* - \pi^*)R'(\tilde{\pi}^e)$. This in turn implies that $\pi_W^* \leq \pi^*$, since $\beta \in \{0, \alpha\}$ by (20), and since $R'(\tilde{\pi}^e) \in [0, 1)$.

Also notice that

$$\frac{v_2 v_{11}}{v_{11} + (1 - \beta)v_{12}} < 0 \quad \text{everywhere,} \quad (28)$$

$$\text{sign}(\beta') = \text{sign}(\pi_S^{a*} - \pi_W^{a*}) \quad \forall x, \quad (29)$$

$$\text{sign}[F(x | \pi_W^{a*}) - F(x | \pi^a)] = \text{sign}[\pi^a - \pi_W^{a*}] \quad \forall x. \quad (30)$$

(28) holds since $v_2 = -u_2 h' < 0$, $v_{11} < 0$, and $v_{11} + (1 - \beta)v_{12} < 0$ by assumptions (A1)–(A2). (29) holds since $\beta'(x) = 0$ for all x if $\pi_W^{a*} = \pi_S^{a*}$, by (11), and since $\beta(x)$ is strictly increasing (decreasing) if $\pi_S^{a*} > \pi_W^{a*}$ ($\pi_S^{a*} < \pi_W^{a*}$) by the monotone likelihood ratio property (A3). (30) holds by *Proposition 1* in Milgrom [1981].

Utilizing these results we now show that conditions (26) hold if $\pi_W^{a*} = \pi_S^{a*}$. Suppose $\pi_W^{a*} = \pi_S^{a*}$, then $\beta'(x) = 0$ for all x , by (29). Hence, $V'(x) = 0$ for all x , by (27), and requirements (26) are met with equality.

It remains to be shown that conditions (26) hold only if $\pi_W^{a*} = \pi_S^{a*}$ — given that $\pi_S^{a*} \neq \pi^*$. Recall that $\pi_S^{a*} \neq \pi^*$ implies that either $\pi_W^* > \pi_S^{a*}$ for all x or $\pi_W^* < \pi_S^{a*}$ for all x , by *Lemma 1(ii)–(iii)*.

(i) Let $\pi_W^* > \pi_S^{a*}$ for all x . By (27)–(28) one then obtains

$$\text{sign}V'(x) = \text{sign}\beta'(x) \quad \forall x. \quad (31)$$

Suppose that $\pi_W^{a*} \neq \pi_S^{a*}$. Choose $\pi^a = \pi_S^{a*}$, (29)–(30) then yield

$$\text{sign}(\beta') = \text{sign}[F(x | \pi_W^{a*}) - F(x | \pi_S^{a*})] \neq 0 \quad \forall x. \quad (32)$$

By (31)–(32) one obtains

$$\text{sign}(V'(x)) = \text{sign}[F(x | \pi_W^{a*}) - F(x | \pi_S^{a*})] \neq 0 \quad \forall x. \quad (33)$$

Hence, $V'(x)[F(x | \pi_W^{a*}) - F(x | \pi_S^{a*})] > 0$ for all x , and (26) is violated.

(ii) Let $\pi_W^* < \pi_S^{a*}$ for all x . By (27)–(28) one then obtains

$$\text{sign}V'(x) = -\text{sign}\beta'(x) \quad \forall x. \quad (34)$$

Suppose that $\pi_W^{a*} \neq \pi_S^{a*}$. Choose $\pi^a > \pi_W^{a*}$ if $\pi_S^{a*} < \pi_W^{a*}$, and $\pi^a < \pi_W^{a*}$ if $\pi_S^{a*} > \pi_W^{a*}$. The rest of the proof is similar to (i) and hence omitted.

D Proof of Proposition 6

Here we complete the proof of *Proposition 6* and show that π_W^* is independent of x .

Since π_W^* is chosen after x is observed, the first order condition $v_1(\pi_W^*, \alpha\pi_S^{a*} + (1 - \alpha)\pi_W^*) = 0$ must hold for all x . This in turn implies that (α and π_S^{a*} are independent of x)

$$[v_{11} + (1 - \alpha)v_{12}] \frac{d\pi_W^*}{dx} = 0 \quad \forall x. \quad (35)$$

Condition (35) holds if and only if $\frac{d\pi_W^*}{dx} = 0$ for all x , since $v_{11} + (1 - \alpha)v_{12} < 0$ everywhere by (A2).

References

- [1] D. Backus and J. Driffill. Inflation and reputation. *American Economic Review*, 75:530–538, 1985.
- [2] D. Backus and J. Driffill. Rational expectations and policy credibility following a change in regime. *Review of Economic Studies*, 52:211–221, 1985.
- [3] K. Bagwell. Commitment and observability in games. *Games and Economic Behavior*, 8:271–280, 1995.
- [4] R.J. Barro. Reputation in a model of monetary policy with incomplete information. *Journal of Monetary Economics*, 17:3–20, 1986.
- [5] H. Carlsson and S. Dasgupta. Noise-proof equilibria in signaling games. Technical report, CentEr for Economic Research, and University of Southern California, 1994.
- [6] A. Cukierman and N. Liviatan. Optimal accommodation by strong policymakers under incomplete information. *Journal of Monetary Economics*, 27:99–127, 1991.
- [7] A. Cukierman and A. Meltzer. A theory of ambiguity, credibility and inflation under discretion and asymmetric information. *Econometrica*, 54:1099–128, 1986.
- [8] D. Fudenberg and D.K. Levine. Maintaining a reputation when strategies are imperfectly observed. *Review of Economic Studies*, 59:561–579, 1992.
- [9] D. Kreps and R. Wilson. Reputation and imperfect information. *Journal of Economic Theory*, 27:253–279, 1982.
- [10] F.E. Kydland and E.C. Prescott. Rules rather than discretion: The inconsistency of optimal plans. *Journal of Political Economy*, 85:473–591, 1977.
- [11] P. Milgrom. Good news and bad news, representation theorems and applications. *Bell Journal of Economics*, 12:380–391, 1981.

- [12] P. Milgrom and J. Roberts. Predation, reputation, and entry deterrence. *Journal of Economic Theory*, 27:280–312, 1982.
- [13] T. Schelling. *The Strategy of Conflict*. Harvard University Press, 1960.
- [14] J. C. Stein. Cheap talk and the Fed: A theory of imprecise policy announcements. *American Economic Review*, 79:32–43, 1989.
- [15] E. van Damme and S. Hurkens. Games with imperfectly observable commitment. Technical report, Center for Economic Research, Tilburg University, The Netherlands, 1994.
- [16] J. Vickers. Signalling in a model of monetary policy with incomplete information. *Oxford Economic Papers*, 38:433–455, 1986.