

A dynamic model of Bertrand competition with entry

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Abstract

This paper analyzes a simple, repeated game of simultaneous entry and pricing. We report a surprising property of the symmetric equilibrium solution: If the number of potential competitors is increased above two, the market breaks down with higher probability, and the competitive outcome becomes less likely. More potential competition lowers welfare – *another Bertrand paradox*. The model can also be applied to auctions to explore whether a revenue maximizing auctioneer should restrict the number of bidders if bidder participation is costly. © 1999 Elsevier Science B.V. All rights reserved.

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1. Introduction

Modern industrial organization has revealed that the decision of a potential entrant to enter into an industry will generally depend upon the nature of the oligopolistic game after entry. If there are multiple entrants, the entry problem is complex because an entrant must know the number of such entrants and whether they would enter sequentially or simultaneously.

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The analysis of the impact of varying the number of entrants and different interactions among them was initiated by Sherman and Willett (1967). They observed that, if a large number of entrants move simultaneously, it becomes possible that no one enters due to the losses that would be incurred if all entered.

Dasgupta and Stiglitz (1988) explored the entry problem and the role of the number of potential competitors in the context of a Bertrand model with positive set up costs and sequential entry. Their game has a unique subgame perfect equilibrium in which the first mover is an uncontested monopolist (see Dasgupta and Stiglitz (1988)); hence, potential competition is completely ineffective. Based on this finding, Dasgupta and Stiglitz (1988) advanced the general principle that “*the fiercer competition after entry, the less effective is potential competition*”.

Sharkey and Sibley (1993) modified that game by assuming simultaneous entry and pricing in the form of two-part tariffs. Their main result was that an increase in the number of potential competitors tends to shift probability mass towards higher prices, which in a particular sense confirmed the anticompetitive effect of more potential competition.

Assuming a Cournot framework, Dixit and Shapiro (1986) explored mixed entry strategies in a two-stage oligopoly model with simultaneous entry decisions at stage one and simultaneous quantity decisions at stage two. Nti (1989) showed that the effect of more potential competition is less disturbing in this model because it leads to higher welfare, at least if demand is linear.

In the present article we maintain the assumptions of Dixit and Shapiro, except that we replace quantity by price competition at the second stage of the constituent game. This modification drastically simplifies the analysis, allows us to derive closed form solutions, and to explicitly evaluate the welfare effect of potential competition without invoking particular assumptions concerning demand functions.

Like Sharkey and Sibley, we consider the entry problem in the framework of Bertrand competition. However, while Sharkey and Sibley consider a one-stage, one-shot game where firms simultaneously randomize entry and pricing, our model is a repeated two-stage game. And while Sharkey and Sibley only compare the equilibrium probability distribution of prices, our analysis admits a full scale and unambiguous welfare evaluation of potential competition.

The paper is organized as follows. The basic framework is set up in Section 2. Section 3 contains a welfare analysis of the one-shot game. Section 4 generalizes the results to the repeated game. Incomplete information is explored in Section 5 and Section 6 sketches an application to auctions and procurements.

2. The model

Consider the classical Bertrand oligopoly model with constant unit costs (normalized to zero), augmented by an entry game, in a two stage framework. The

players are $n \geq 2$ identical potential competitors. At stage one, they decide whether to enter without knowing their rivals' choice. Entry entails a sunk entry investment $I > 0$. At stage two, players observe their rivals' entry decisions and play a Bertrand competition game. Since firms cannot precommit to price strategies, subgame perfectness is invoked, and since firms are identical, symmetry of equilibrium is required.

The stage two subgame has a unique equilibrium outcome: If only one firm has entered, the market is served at the monopoly price; however, if more than one firm has entered, the price is equal to zero ("two is enough for competition").¹ Therefore the reduced form payoffs (before deducting entry costs) are equal to zero if more than one firm or none have entered, and equal to the monopoly profit π_m otherwise ($\pi(\tilde{n}) = 0$ if $\tilde{n} \geq 2$ or $\tilde{n} = 0$ and $\pi(1) = \pi_m$, where \tilde{n} denotes the number of firms that have entered). To rule out the trivial case when the market is never served, $\pi_m > I$ is assumed.

The overall game has several subgame perfect Nash equilibria: exactly n asymmetric equilibria in pure and a unique symmetric equilibrium which involves mixed strategies.² In each asymmetric pure strategy equilibrium only one firm enters and all others stay out. In the unique symmetric mixed strategy equilibrium each firm enters with the same positive probability less than one.³ Like Dixit and Shapiro, we analyze the unique symmetric equilibrium which is the most plausible (see also Section 4).

3. The one-shot game

Consider the entry decision evaluated by the reduced form payoff function. A mixed strategy equilibrium where each firm randomizes its entry decision is necessarily a symmetric equilibrium.⁴ There, each firm enters with the same probability $\mu \in (0,1)$.

Each firm has $n - 1$ rivals. Entry is profitable if and only if they all stay out. In a symmetric mixed strategy equilibrium this event occurs with probability $(1 - \mu)^{n-1}$. Hence, the payoff from entry is $(1 - \mu)^{n-1} \pi_m - I$. Of course, the payoff

¹We mention, if more than two firms have entered, every nonnegative price vector with at least two components equal to zero is an equilibrium strategy vector. This suggests an equilibrium selection problem. Invoking symmetry gives uniqueness.

²And, of course, many combinations of pure and mixed strategies where, for example, $k < n - 1$ firms stay out and the remaining $n - k$ firms randomize.

³One may wonder whether communication could lead to self-enforcing coordination as suggested by Farrell (1987). However, this works only if each firm would rather see rivals serve the market than see it not served at all, as in the "battle-of-the-sexes" game.

⁴Proof: The conditions of indifference for entrants j and k are: $\prod_{i \neq j} (1 - \mu_i) \pi_m = I$, $\prod_{i \neq k} (1 - \mu_i) \pi_m = I$. Therefore, $(1 - \mu_k)/(1 - \mu_j) = 1$, and hence $\mu_j = \mu_k$.

from non-entry is zero. The strategy μ is an equilibrium if it is a mutual best response. This occurs if and only if each firm is indifferent between entry and non-entry

$$(1 - \mu)^{n-1} \pi_m - I = 0, \quad (1)$$

which gives the unique solution

$$\mu(n, t) = 1 - {}^{n-1}\sqrt{t}, \text{ where } t = \frac{I}{\pi_m} < 1. \quad (2)$$

In equilibrium there are three possible market outcomes: market *breakdown* (b), if no firm has entered, *monopoly* (m), if only one entered, and *competition* (c), if more than one entered.⁵ They occur with the following probabilities

$$\rho_b(n, t) = (1 - \mu)^n = {}^{n-1}\sqrt{t^n} \quad (3)$$

$$\rho_m(n, t) = n\mu(1 - \mu)^{n-1} = nt(1 - {}^{n-1}\sqrt{t}) \quad (4)$$

$$\rho_c(n, t) = 1 - \rho_b - \rho_m = 1 - {}^{n-1}\sqrt{t^n} - nt(1 - {}^{n-1}\sqrt{t}). \quad (5)$$

We want to compare these probabilities as we go from $n \geq 2$ to $n+1$ potential competitors.

Proposition 1. *Suppose there are at least two potential competitors. Adding another firm makes market breakdown more and competition less likely.*

Proof. 1) By Eq. (3) it follows immediately that, for each $t \in (0, 1)$, $\rho_b(n, t)$ is strict monotone increasing in n .

2) Consider the probability increment

$$\Delta_c(n, t) = \rho_c(n, t) - \rho_c(n+1, t) = t((n-1)^{n-1}\sqrt{t} - n - t + 2). \quad (6)$$

Differentiating with respect to n , one obtains

$$\frac{\partial \Delta_c(n, t)}{\partial n} = -t \frac{{}^{n-1}\sqrt{t}(\ln t - n + 1)}{n-1} - t \quad (7)$$

$$< 0 \Leftrightarrow \ln t > (n-1)(1 - t^{-1/(n-1)}). \quad (8)$$

⁵Evidently, the market can be monopolized, but the monopoly profit is completely offset by expected losses due to the probability of overcrowding. Therefore, this variation of the Bertrand model nicely exemplifies Posner's (Posner, 1975) proposed reinterpretation of the social loss of monopoly which includes the monopoly profit in addition to the usual deadweight loss of monopoly.

The latter condition is indeed satisfied, since⁶

$$\ln t \equiv -(n-1) \ln t^{-1/(n-1)} > (n-1)(1-t^{-1/(n-1)}). \quad (9)$$

3) Finally, observe that

$$\rho_c(n+1, t) - \rho_c(n, t) = \Delta_c(n+1, t) - \Delta_c(n, t). \quad (10)$$

■

Proposition 2. *Suppose there are at least two potential competitors. More potential competition lowers welfare.*

Proof. 1) Let S_m denote consumer surplus under monopoly and S_c that under competition (both are independent of n). Then, expected consumer surplus is

$$\begin{aligned} S(n, t) &= (1 - \rho_c(n, t) - \rho_b(n, t))S_m + \rho_c(n, t)S_c \\ &= S_m + \rho_c(n, t)(S_c - S_m) - \rho_b(n, t)S_m. \end{aligned} \quad (11)$$

Since $S_c > S_m$, Proposition 1 entails immediately that S is strict monotone decreasing in n .

2) Each firm's equilibrium expected value of profit is equal to zero, by the condition of indifference between entry and non-entry Eq. (1). Hence, increasing n does not affect expected producer surplus.

3) Total welfare – the sum of expected consumer and producer surplus – is strict monotone decreasing in n by 1) and 2). ■

Example. In order to gain a feel for the quantitative effects involved take a look at Fig. 1, where we plot the probability increments $\Delta_b := \rho_b(n, t) - \rho_b(2, t)$, $\Delta_m := \rho_m(n, t) - \rho_m(2, t)$, and $\Delta_c := \rho_c(n, t) - \rho_c(2, t)$ against the parameter t , if the number of firms is raised from 2 to 3. Clearly, breakdown becomes more and competition less likely. If $t < 1/4$, some probability mass also shifts from the competitive outcome towards the monopoly outcome.

But as t is increased beyond $1/4$, probability mass shifts toward market breakdown not only from the competitive but also from the monopoly outcome. In either case, welfare is unambiguously reduced. The chance of breakdown increases by up to 10.5% (at $t = 9/16$) and that of the competitive outcome goes down by up to 6.3% (at $t = 1/4$). These are not negligible quantities.

Altogether, the driving force behind the adverse effect of potential competition is that each firm reduces its probability of entry when the number of potential competitors is increased. In and by itself this effect is not surprising. The

⁶The inequality follows from the well-known property of the \ln function: $\ln x \leq x - 1$, with strict inequality for all $x \neq 1$.

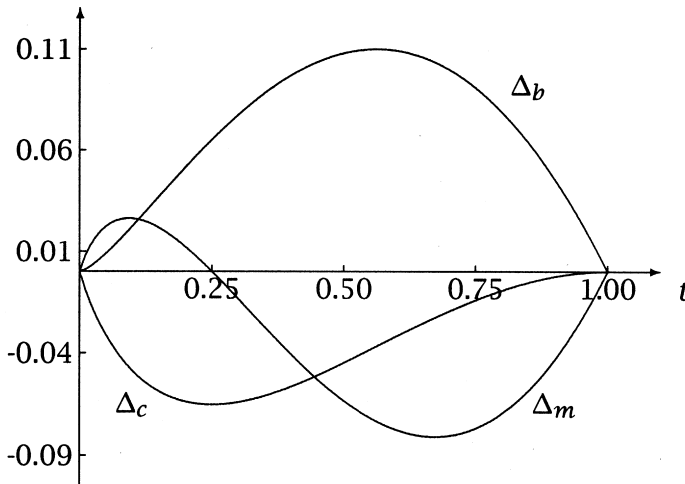


Fig. 1. Switching from 2 to 3 competitors.

remarkable part is that the probability of entry goes down to such an extent that market breakdown becomes more and competition less likely, and welfare is unambiguously reduced.

4. An incomplete information perspective

Many economists feel uneasy about mixed strategies, especially when the game also admits pure strategy equilibria. However, the mixed strategy equilibrium can be viewed as a reflection of uncertainty about rivals' actions due to incomplete information concerning rivals' payoffs. We close with a brief defense of the mixed strategy equilibrium in the spirit of Harsanyi's (Harsanyi, 1973) celebrated "purification theorem".⁷

Suppose there is incomplete information about entry costs in the sense that each firm knows its own entry cost, but not that of others. Then each firm views rivals' entry cost as a random variable. For simplicity, assume each rival's entry cost is $I + \tilde{\theta}_i$, where the $\tilde{\theta}_i$'s are *iid* and uniformly distributed on the support $[0, \varepsilon]$, $\varepsilon > 0$.

We explore the pure strategy Bayesian Nash equilibrium, where each firm's entry strategy is contingent upon its own private entry cost parameter $\theta_i \in [0, \varepsilon]$. We will show that, in this solution, each rival enters exactly with that probability

⁷For empirical evidence on mixed strategies see the coordination game experiments by Ochs (1991) and the "unstable pricing" observed in natural monopoly experiments by Millner et al. (1990).

which solves the symmetric mixed strategy equilibrium of the above complete information game, if we make the payoff distortion vanish by letting $\varepsilon \rightarrow 0$.

Each firm plays the pure entry strategy

$$e(\tilde{\theta}) = \begin{cases} \text{entry} & \text{if } \tilde{\theta} \leq c(\varepsilon) \\ \text{non-entry} & \text{otherwise} \end{cases}, \quad (12)$$

where c denotes the ‘‘cutoff’’ point. In order to solve the equilibrium cutoff $c(\varepsilon)$, notice that at $\tilde{\theta} = c(\varepsilon)$, entry and non-entry must be payoff equivalent, if all rivals play that equilibrium strategy. Therefore, $c(\varepsilon)$ is implicitly determined by payoff equivalence, as applied to firm i (*rhs*: payoff from entry, *lhs*: payoff from non-entry)

$$0 = Pr\{\tilde{\theta}_j > c(\varepsilon), \forall j \neq i\} \pi_m - I - c(\varepsilon) = (1 - F(c(\varepsilon)))^{n-1} \pi_m - I - c(\varepsilon), \quad (13)$$

where F is the *c.d.f.* of $\tilde{\theta}$. Invoking the uniform distribution assumption, and substituting $I = t\pi_m$, this condition simplifies to (index i omitted)

$$\left(\frac{\varepsilon - c(\varepsilon)}{\varepsilon}\right)^{n-1} \pi_m = t\pi_m + c(\varepsilon). \quad (14)$$

In equilibrium, each rival enters with probability

$$Pr\{\text{entry}\} = F(c(\varepsilon)) = \frac{c(\varepsilon)}{\varepsilon} = :d(\varepsilon). \quad (15)$$

We want to solve $d(\varepsilon)$ for $\varepsilon \rightarrow 0$.

Substitute c by $d(\varepsilon)\varepsilon$ in Eq. (14). This gives $(1 - d(\varepsilon))^{n-1} \pi_m = t\pi_m + d(\varepsilon)\varepsilon$. And finally, compute the limit⁸

$$\lim_{\varepsilon \rightarrow 0} d(\varepsilon) = 1 - {}^{n-1}\sqrt{t} = \mu(n,t). \quad (16)$$

Hence, all firms play the same pure strategy; but, due to arbitrarily small payoff uncertainty, rivals’ entry is random, and each rival enters with the very same probability $\mu(n,t)$ that solved the symmetric mixed strategy equilibrium of the complete information game.

5. The finitely repeated game

Surprisingly, the above results generalize to finitely repeated oligopoly games where those who have not yet entered may reconsider entry. However, the repetition itself contributes to increase the entry probability and thus tends to increase welfare, simply because if a supplier becomes a monopolist, he stays a

⁸ Notice, $d \in (0,1)$, since d is a probability. Therefore, $\lim_{\varepsilon \rightarrow 0} \varepsilon d(\varepsilon) = 0$.

monopolist forever. Therefore, even though more potential competition is harmful also in the repeated game, the welfare level in the repeated game is higher to begin with.

In order to prove the asserted generalization of our previous result, reinterpret the above game as a stage game that is repeated T times. For simplicity, take T to be large, $T \rightarrow \infty$.⁹

At the outset, notice that entry is permanent. Entry costs are sunk; therefore exit is not an issue. This allows us to focus entirely on the entry decision of those who have not yet entered (“outsiders”).

Of course, no outsider will ever contemplate entry when the market is already served by one or more “incumbent” firms. Therefore, entry only occurs as long as everybody is still an outsider. As soon as one firm has entered, everybody – incumbent and outsider alike – passively sits out the remaining game. Monopoly and competition are absorbing states; breakdown is not.

Suppose everybody is still an outsider. Let all rivals enter with the same probability μ . Then, firms’ lifetime expected profit from entry is

$$V(n,t) = Pr\{\text{no rival enters}\} \Pi_m - I = (1 - \mu(n,t))^{n-1} \Pi_m - I, \quad (17)$$

where $\Pi_m := \pi_m / (1 - \delta)$ denotes the present value of the permanent annuity π_m and $\delta \in (0,1)$ the common discount factor. Similarly, firms’ lifetime expected profit from non-entry is

$$\begin{aligned} V(n,t) &= Pr\{\text{no rival enters}\} \delta V(n,t) = (1 - \mu(n,t))^{n-1} \delta V(n,t) \\ &= 0, \text{ since } (1 - \mu(n,t))_{n-1} \delta < 1. \end{aligned} \quad (18)$$

In equilibrium, outsiders must be indifferent between entry and non-entry if rivals enter with the equilibrium probability μ . Therefore, the equilibrium probability of entry μ is just like in the one-shot case

$$\mu(n,t) = 1 - {}^{n-1}\sqrt{t}, \text{ where } t := \frac{I}{\Pi_m}, \Pi_m := \frac{\pi_m}{1 - \delta}, \quad (19)$$

provided the parameter t is reinterpreted to involve Π_m instead of π_m .

The dynamics of the market game can now be summarized by the matrix of transition probabilities, where ρ_{ij} denotes the transition probability that the market moves from state i to state j ($i, j \in \{b, c, m\}$)

⁹If the number of repetitions is infinite, the game admits many solutions with radically different properties, as is well-known from the “folk-theorem” of repeated games. Of course, this discontinuity suggests that the limit of the solution for $T \rightarrow \infty$ is more interesting than the solution at $T = \infty$.

$$\begin{bmatrix} \rho_{mm} & \rho_{mc} & \rho_{mb} \\ \rho_{cm} & \rho_{cc} & \rho_{cb} \\ \rho_{bm} & \rho_{bc} & \rho_{bb} \end{bmatrix} := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \rho_m & \rho_c & \rho_b \end{bmatrix}. \tag{20}$$

Thereby, ρ_m , ρ_c , and ρ_b are just like in the one-shot game (see Eqs. (3)–(5)), with the by now familiar proviso concerning the interpretation of t . Therefore, Proposition 1 applies, and we can turn directly to the welfare assessment of potential competition.

Proposition 3. *More potential competition also lowers welfare in the repeated market game.*

Proof. 1) The present value of a \$1 permanent annuity is $1/(1-\delta)$. To compute the equilibrium present value of expected consumer surplus, \bar{S} , recall that c and m are absorbing states, unlike b. Therefore,

$$\bar{S}(n,t) = \rho_c \frac{S_c}{1-\delta} + \rho_m \frac{S_m}{1-\delta} + \rho_b \delta \left(\rho_c \frac{S_c}{1-\delta} + \rho_m \frac{S_m}{1-\delta} + \rho_b \delta (\dots) \right).$$

Inserting $S := \rho_c S_c + \rho_m S_m$, which is the one-shot expected consumer surplus from Eq. (11), one obtains

$$\bar{S}(n,t) = \frac{S}{1-\delta} \sum_{i=0}^{\infty} (\rho_b \delta)^i = \frac{S}{(1-\delta\rho_b)} \frac{1}{(1-\delta)}. \tag{21}$$

It takes a few steps (elaborated in the Appendix) to show that $S/(1-\delta\rho_b)$ is strict monotone decreasing in n , from which it follows immediately that \bar{S} is strict monotone decreasing in n as well.

2) Firms' equilibrium expected present value of profits is zero, by the required indifference between entry and non-entry. Therefore, the welfare effect of increasing potential competition is fully determined by 1). ■

However, it should not be overlooked that the entry probability is higher in the repeated game because the benefit from monopoly is permanent and thus much increased.¹⁰ Therefore, even though more potential competition is harmful also in the repeated game, the numbers involved are much lower to begin with.

In order to gain a feel for the quantitative effects, we close with an example. Suppose $S_c = 2$, $S_m = 1$, and $t = \frac{1}{2}$. Then, increased potential competition gives rise to the percentage welfare changes (here equivalent to the change in consumer

¹⁰This reflects in the parameter $t := t/\Pi_m$ where $\Pi_m = \pi_m/(1-\delta)$ is the present value of the annuity π_m .

Table 1
Per cent change of welfare if n is increased from $n=2$ to:

	$n=5$	$n=10$	$n \rightarrow \infty$
$\delta=0$	-23.9	-29.7	-33.5
$\delta=0.5$	-7.8	-12.2	-14.9
$\delta=0.9$	-2.3	-3.0	-3.8

surplus) summarized in Table 1 for a sample of discount factors. There, $\delta=0$ corresponds to the one-shot game.

6. Discussion

We have analyzed the symmetric subgame perfect equilibrium of a repeated oligopoly model with sequential entry and pricing. Due to the sunk cost of entry, firms randomize entry. This contributes to coordination failures in the form of either insufficient or excessive entry, with positive probability. Interestingly, we showed that the expected loss due to such coordination failures tends to increase when the market has more potential suppliers. Therefore, we arrived at the double Bertrand paradox that more *potential* competition is harmful, while *actual* competition, if it occurs, is exceedingly effective.

All of the above applies readily to auctions if bidding entails fixed cost of participation. In this regard, our analysis indicates that in auctions it is optimal to restrict the number of potential bidders.

Consider a procurement or contractor game with many potential competitors for a specified task.¹¹ The procurer invites firms to participate and to quote prices, with the understanding that the task is awarded to the lowest price bidder (first-price auction).

Given these rules, bidders first decide whether to participate in the bidding (entry decision). Entry entails a one-time sunk cost. Then, the number of active bidders becomes known,¹² bids are placed, and the winner is selected. All bidders have (almost) identical costs (in the sense of Section 4).

Evidently, this procurement problem is a special case of the above model. The only specific feature is that the procurer takes the place of consumers. This leads

¹¹The solution of a procurement problem is easily transformed into the solution of a regular auction, and vice versa. On this and other features of procurement see the recent survey on auctions by Wolfstetter (1996).

¹²In written auctions, this may not always be appropriate. However, the procurer could always ask for a series of bids, contingent on the number of active bidders.

immediately to the conclusion that the procurer’s profit reaches a maximum if the number of potential bidders is equal to two.¹³

One limitation of our analysis is that we assumed that entry costs are completely sunk. This entails that firms never contemplate exit once they have entered. However, if some of the entry cost can be salvaged upon exit, firms consider exit if more than one firm has entered. An interesting issue for future research is to explore a more complex model that includes an exit game after each round of entry and pricing. It remains to be seen whether the adverse welfare effect of potential competition extends to such a more general model of entry, pricing, and exit.

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Appendix 1

Here we show that $S(n)/(1 - \delta\rho_b(n))$ is strict monotone decreasing in n , as we claimed in the proof of Proposition 3. Obviously, the assertion is equivalent to $(d/dn) \ln(S) < (d/dn) \ln(1 - \delta\rho_b)$, or

$$\frac{S'}{S} < -\frac{\delta\rho'_b}{1 - \delta\rho_b} \text{ for all } n \tag{22}$$

Using Eq. (11), the following inequalities are all equivalent to Eq. (22)

$$(\rho'_c(S_c - S_m) - \rho'_b S_m)(1 - \delta\rho_b) + \delta\rho'_b S < 0 \tag{23}$$

$$(\rho'_c k - \rho'_b)(1 - \delta\rho_b) + \delta\rho'_b(1 + \rho_c k - \rho_b) < 0 \tag{24}$$

¹³ Compare this to other contributions on entry in auctions such as McAfee and McMillan (1987) and Engelbrecht-Wiggans (1993). There it is assumed that potential bidders learn about their valuation only after entry costs in the form of costs of inspection have been sunk. The main result of this literature is that the seller should not impose a minimum price above the seller’s own valuation, contrary to a celebrated recommendation of the optimal auction literature that had ignored entry problems. We also mention that Lang and Rosenthal (1991) explored an auction problem with simultaneous entry and bidding that corresponds to the already mentioned oligopoly problem by Sharkey and Sibley (1993).

$$\rho'_c k - \delta k(\rho'_c \rho_b - \rho'_b \rho_c) + (\delta - 1)\rho'_b < 0 \quad (25)$$

$$k(\rho'_c(1 - \delta\rho_b) + \rho'_b \delta\rho_c) + \rho'_b(\delta - 1) < 0. \quad (26)$$

The second term of the latter inequality is negative by Proposition 1 and $\delta < 1$. Therefore, it is sufficient to show that

$$\delta(\rho'_b \rho_c - \rho'_c \rho_b) < -\rho'_c. \quad (27)$$

Since $\delta \in (0,1)$ and since the term in parenthesis is positive by Proposition 1, it is sufficient to show that

$$\rho'_b \rho_c - \rho'_c \rho_b < -\rho'_c. \quad (28)$$

Define

$$z(n) := \frac{n\mu(n)}{1 - \mu(n)}. \quad (29)$$

Then,

$$\rho_m = z\rho_b \quad (30)$$

$$\rho_c = 1 - \rho_b \quad (31)$$

$$\rho'_c = -\rho'_b(1 + z) - \rho_b z' \quad (32)$$

$$\rho'_b = -\mu'n(1 - \mu)^{n-1} - \frac{\mu'n}{1 - \mu}\rho_b. \quad (33)$$

Substitute, and it follows that all of the inequalities below are equivalent to Eq. (28)

$$-(\rho'_b(1 + z) + \rho_b z')(1 - \rho_b) + \rho'_b(1 - \rho_b(1 + z)) < 0 \quad (34)$$

$$-\rho'_b z - \rho_b z' + \rho_b^2 z' < 0 \quad (35)$$

$$\frac{\mu'n}{1 - \mu} \rho_b z - \rho_b z' + \rho_b^2 z' < 0 \quad (36)$$

$$\frac{\mu'n}{1 - \mu} z - z' + \rho_b z' < 0 \quad (37)$$

$$n^2 \mu \mu' - (1 - \rho_b)(\mu'n + \mu(1 - \mu)) < 0 \quad (38)$$

$$-n\mu'(1 - \rho_b - n\mu) < (1 - \rho_b)\mu(1 - \mu). \quad (39)$$

The right-hand side of Eq. (39) is positive and $\mu' < 0$ (see Eq. (2)). Therefore, Eq. (39) holds if

$$1 - \rho_b - n\mu < 0, \quad (40)$$

which is equivalent to

$$f(t) := t^{1/(n-1)}(n-t) < n-1. \quad (41)$$

As one can easily confirm, $f(0)=0$, $f(1)=n-1$, and $f(t)$ is strict monotone increasing in $t \in (0,1)$, for each n . Therefore, Eq. (41) holds and the proof is complete.

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