Which Firm to Subsidize, the More or the Less Efficient?

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Abstract

Different from the previous studies in R&D subsidy literature, where firms are assumed to be symmetric and the question is whether to subsidize and how much to subsidize, this paper considers the behavior of asymmetric firms and addresses the question of which firm to subsidize. It is shown that the government subsidizes the firm with lower (higher) production cost, if R&D efficiency is relatively low (higher).

Keywords: innovation, licensing, R&D efficiency, R&D subsidy policy,

JEL: L13, O34, D45

1 Introduction

Governments provide substantial amount of R&D subsidies to private industries. For example, the annual average R&D subsidy received by private firms from their governments in EU countries amounts to 3.9 billion Euro over the period of 1995-1997. In Germany, more than 15 million Euro R&D investments are undertaken by the government every year since 1990; The governments of France, Denmark and the Netherlands spend even more money (relative to their GDP) on R&D subsidy than Germany.

One justification for R&D subsidies is that when R&D spillovers are high, the amount of R&D undertaken by private firms is insufficient from the social point of view. However, inventing new products or technologies is not sufficient for economic progress. Exploiting and diffusing the innovation through licensing and adoption is also critical for economic development. It is plausible that governments' R&D subsidy not only affects firms' incentive to innovate but also their incentive to license the innovation.

Casual empiricism shows that firms' incentive to transfer their technology is too low from the social point of view (Tirole, 1988). Thus, there may be room for the government to intervene in order to encourage firms to sell their superior technology to rival firms. This paper attempts to investigate whether and how R&D subsidies affect firms' incentives to license as well as to innovate. The focus of this paper is on how the government picks a winner of the R&D subsidy, and how the firms' incentives both to innovate and to license are strategically influenced by the government's R&D subsidy policy.

Our analysis is built on two branches of the literature. One is the literature on licensing, which is concerned with questions of optimal licensing((Jensen, 1992), (Kamien, Oren, and Tauman, 1992), (Wang, 1998) etc.), the strategic use of licensing ((Gallini, 1984), (Rockett, 1990), (Yi, 1999) etc.) as well as the effects of licensing on firms' incentives to innovate ((Katz and Shapiro, 1985), (Katz and Shapiro, 1987) etc.). The other is the R&D subsidy literature ((Bagwell and Staiger, 1994), (Qiu and Tao, 1998), (Spencer and Brander, 1983) etc.) which justifies governments' R&D subsidies as a way to increase the domestic firms' profit at the expense of foreign firms.

This paper considers two asymmetric firms that have different production costs and may engage in licensing either before or after innovation. The firms' R&D investments can be subsidized by the government, which has three options: Subsidize R&D investment under the condition that the subsidized firm undertakes a minor nondrastic innovation and afterwards engages in ex-post licensing, subsidize R&D conditional on a nondrastic innovation regardless of ex-post licensing, or provide no subsidy at all. Due to considerations of antitrust law, the government will not subsidize firms' drastic innovations. Taking the reaction of the firms into account, the government decides whether to subsidize firms' R&D investment and which firm to subsidize.

It is shown that the government has incentives to subsidize firms' R&D investments only when the firms can innovate sufficiently efficiently, and the subsidized firm will license its superior new technology to its competitor. The R&D efficiency, which measures the degree of the cost reduction through a given amount of R&D investment, also decides which firm the government will choose to subsidize. When R&D efficiency is relatively low, the more efficient firm will be subsidized because it is relatively cheap to induce the firms to engage in ex-post licensing. Whereas, if the R&D efficiency is relatively high, the less efficient firm's R&D will be subsidized, which reduces the market asymmetry, and increases industry efficiency substantially. With R&D subsidy from the government, the subsidized firm will only develop minor nondrastic innovations.

This study examines the relationship between R&D subsidy policy and licensing in a formal way for the first time. A rationale for R&D subsidy policy is found, that is, government's R&D subsidy may aim to stimulate firms' incentives to license. Another contribution of this paper is that it extends the R&D subsidy literature to asymmetric cases. In the R&D subsidy literature, firms are assumed to be symmetric and the question is whether to subsidize and how much to subsidize ((Qiu and Tao, 1998), (Spencer and Brander, 1983)). In contrast, we consider the behavior of asymmetric firms and address a quite different question, namely, which firm should the government subsidize, the more or the less efficient firm? We do this by allowing firms to have the possibility of licensing.

In the following section, the basic model under fixed-fee licensing is investigated. In Section 3, the model is analyzed under three R&D subsidy policies. Section 4 examines the equilibrium of the game. Section 5 provides the proof of the equilibrium. In Section 6 the results under different licensing contracts are compared to show that fixed-fee licensing can bring the highest net social welfare to the government. Section 7 contains concluding remarks.

2 The Model

There are three players in this game: a government and two asymmetric firms: firm 1 and firm 2. The marginal costs of the firms are constant and are represented by c_1 and c_2 respectively, $c_1 < c_2$, that is, firm 1 is more efficient than firm 2.

In this game, the government is the first mover, it decides on its R&D subsidy policies–whether to subsidize firms' R&D investment and which firm to subsidize. In particular, the government has three possible R&D subsidy policies, and must decide which policy is applied in the first stage:

- Policy (1): No R&D subsidy
- Policy (2): Subsidy conditional on a *minor* nondrastic innovation (which leads to ex-post licensing).
- Policy (3): Subsidy conditional on a nondrastic innovation regardless of ex-post licensing.

For competitive reasons, it is assumed that drastic innovations will not be subsidized.

If the government provides no subsidy, the two asymmetric firms may engage in ex-ante licensing (case 1). In this case, a licensing agreement is struck for the usage of the more efficient existing technology. If R&D subsidies are provided, the subsidized firm undertakes R&D investment, and then decides whether to ex-post license (case 2). An ex-post licensing contract is struck after innovation for the use of the more efficient technology discovered to date.

After the licensing stage in both of the above cases, firms compete in the last stage in a Cournot market game. Later on, the first case (with no R&D subsidy) is referred to as subgame 1 and the second case (with R&D subsidy) as subgame 2.

In subgame 2, it is assumed that the government will only subsidize one of the firms instead of both of them. This is due to the following reasons: Subsidizing only one firm can avoid the excess R&D investment that arises when the firms are involved in R&D rivalry, while simultaneous subsidization to both firms will only lead to duplicated efforts that may result in redundant discoveries, and therefore is a waste of the government's subsidy cost.

For the time being, we consider fixed-fee licensing and assume that the licensor makes a take-it-or-leave-it offer.

The government's objective is to maximize social welfare U_G, defined as

$$U_G = CS + PS - S_i \tag{1}$$

where CS denotes consumer surplus, PS producer surplus, and S_i is the subsidy to firm i. For ease of calculation, it is assumed that the government's subsidy is equal to the entire R&D investment expenditure x_i, i.e. $S_i = x_i$. Also, it is assumed that firms do not engage in R&D if no subsidy is forthcoming.

Firm i's payoff function is

$$\pi_i := \Pi(q_i, q_j, x_i), \tag{2}$$

with

$$\Pi(q_i, q_j, x_i) = p(q_{i,}q_j)q_i - (c_i - \Delta(x_i))q_i \tag{3}$$

where $p(q_1, q_2) := a - q_1 - q_2$ is the linear inverse demand function. And $\Delta(x_i) := \Delta_i$ is the cost reduction of firm i as a function of x_i, that is,

$$\Delta(x_i) = r\sqrt{x_i} \tag{4}$$

here r is the efficiency of R&D investment.

Furthermore, let firm i be the innovating firm, firm j the noninnovating firm, we make the following assumptions:

- Assumption 1: $\Delta(x_i) \leq c_i$, $\Delta'(x_i) > 0$, $\Delta''(x_i) < 0$, the firms' marginal cost can not be negative and R&D investments are subject to positive but diminishing returns.

- Assumption 2: $a < 2c_j$, the market is not so big that the innovating firm may develop a drastic innovation.

- Assumption 3: $r^2 < \overline{r} := \frac{18(5a+6c_i-11c_j)}{43(a-c_j)}$, the efficiency of R&D investment is bounded from above.

3 Analysis of the game

3.1 Policy 1: No Subsidy

To study subgame perfect equilibrium, we begin with the final stage of the game–Cournot market game. If the government does not subsidize, the firms will not innovate but may engage in licensing due to their initially different technologies. Before licensing, the firms produce

$$q_i^{nex} = \frac{a - 2c_i + c_j}{3}, i, j = 1, 2 \tag{5}$$

with their own technologies. After licensing, since both firms will use the same technology of the more efficient firm, the firms become equally efficient, $c_2 = c_1$, and produce the same output:

$$q_1^{ex} = q_2^{ex} = \frac{a - c_1}{3}. \tag{6}$$

Fixed-fee licensing occurs if and only if firms' profits increase with licensing. That is, the incremental profit of the licensee from cost savings exceeds what the licensor loses from intensified competition due to licensing. Since the licensor (firm 1) has all the bargaining power, the fixed fee is equal to the licensee's (firm 2's) incremental profit, $F = \frac{(a-c_1)^2}{9} - \frac{(a-2c_2+c_1)^2}{9}$, which is the benefit from the licensing. Meanwhile, firm 1's profit loss due to more competition after licensing, $\frac{(a-2c_1+c_2)^2}{9} - \frac{(a-c_1)^2}{9}$, can be regarded as the cost of licensing. When the benefits of licensing are greater than its cost, licensing will occur.

The condition for licensing to occur is that the net gain,

$$NG_1 = \frac{(c_2 - c_1)(2a + 3c_1 - 5c_2)}{9},$$

be positive. That is,

$$c_2 \leq \frac{2a + 3c_1}{5}. \tag{7}$$

With no subsidy, the government's payoffs are either U_G^{ex} if ex-ante licensing occurs or U_G^0 if no licensing occurs, where

$$U_G^{ex} = \frac{4(a - c_1)^2}{9} \tag{8}$$

$$U_G^0 = \frac{(2a - c_1 - c_2)^2}{18} + \frac{(a - 2c_1 + c_2)^2}{9} + \frac{(a - 2c_2 + c_1)^2}{9} \tag{9}$$

It can be shown that $U_G^{ex} > U_G^0$.

Lemma 1 *Assume firms do not engage in R&D, then licensing with a fixed-fee increases social welfare.*

Fixed-fee licensing reduces the less efficient firm's production cost and increases industry output, from $\frac{2a-c_1-c_2}{3}$ to $\frac{2(a-c_1)}{3}$. It therefore leads to a lower price and a higher consumer surplus.

Lemma 2 *The equilibrium of subgame 1 is:*

- *Suppose the cost difference between the firms is small, i.e.* $c_2 \leq \frac{2a+3c_1}{5}$*, then the firms engage in licensing. Firm 1's and firm 2's equilibrium payoffs are* $2(q_1^{ex})^2 - (q_2^{nex})^2$ *and* $(q_2^{nex})^2$ *respectively while the government's payoff is* U_G^{ex}*;*
- *Suppose the cost difference between the firms is big, i.e.* $c_2 > \frac{2a+3c_1}{5}$*, then there is no licensing. The firms' equilibrium profits are* $(q_1^{nex})^2$ *and* $(q_2^{nex})^2$ *respectively while the government's payoff is* U_G^0*.*

3.2 Policy 2: Subsidy conditional on a minor innovation

Policy 2 means that R&D subsidy is conditional on innovation size[1]. The government provides R&D subsidies if the innovating firm develops a small innovation so that ex-post licensing with a fixed fee will occur, otherwise it does not subsidize.

Let firm i be the innovating firm and firm j the non-innovating firm, $i, j = 1, 2$. The firms' output levels after innovation but before ex-post licensing are, respectively,

$$q_i = \frac{a - 2(c_i - \Delta_i) + c_j}{3} \quad (10)$$

$$q_j = \frac{a - 2c_j + (c_i - \Delta_i)}{3} \quad (11)$$

If firm i shares its innovation with firm j, then both firms have the same production cost after licensing and produce:

$$q_i^{ep} = q_j^{ep} = \frac{a - (c_i - \Delta_i)}{3}, i = 1, 2 \quad (12)$$

Through licensing, firm j's (the licensee's) profit increases, due to cost saving, from $(q_j)^2$ to $(q_j^{ep})^2$. Since the licensor has all the bargaining power, the fixed fee the licensee pays equals its incremental profit. Meanwhile, licensing decreases firm i's (the licensor's) profit, due to more competition, from $(q_i)^2$ to $(q_i^{ep})^2$. If the fixed fee the licensor gets from the licensee is greater than what it loses, licensing will occur. That is, licensing is desirable only if the net gain from licensing,

$$NG_2 = 2(q_i^{ep})^2 - (q_i)^2 - (q_j)^2,$$

[1]Under this conditional policy, the government does not have to inspect whether a licensing contract is signed or not. Since the net gain from licensing a minor nondrastic innovation is greater than zero, the firms will be willing to engage in ex-post licensing as long as the innovation is minor.

is positive, which gives rise to the condition $\Delta_i \leq \Delta_i^L$, where

$$\Delta_i^L = \frac{2a + 3c_i - 5c_j}{3}. \tag{13}$$

The innovation is minor, if $\Delta_i \leq \Delta_i^L$, under which ex-post licensing will occur. If $\Delta_i > \Delta_i^L$, licensing is undesirable. Note that Δ_i^L is also the biggest innovation the government is willing to subsidize under policy 2. Therefore, the government's subsidy policy can be summarized as

$$S(\Delta_i) = \begin{cases} x_i, \text{if } \Delta_i \leq \Delta_i^L \\ 0, \text{if } \Delta_i > \Delta_i^L \end{cases}$$

Moreover, by Assumption 2, $\Delta_i^L < c_i$.

Furthermore, since firms differ, the cost reduction that leads to ex-post licensing differs as well. If firm 1 is subsidized, then $\Delta_1^L = \frac{2a+3c_1-5c_2}{3}$ while if firm 2 is subsidized, then $\Delta_2^L = \frac{2a+3c_2-5c_1}{3}$. Because $\Delta_1^L < \Delta_2^L$, it is cheaper to subsidize firm 1 and induce it to engage in ex-post licensing. Nevertheless, we will show later on that the government will not necessarily provide R&D subsidy to the more efficient firm.

When firm 2 gets the R&D subsidy, there are two possibilities that licensing may occur. The first possibility is that firm 2 becomes more efficient than firm 1 after innovation, i.e. $\Delta_2^L > c_2 - c_1$. In this case, firm 2 will license to firm 1. Another possibility is that the new marginal cost of firm 2 may still be greater than firm 1's, i.e. $\Delta_2^L \leq c_2 - c_1$. Under this circumstance, both firms will use the more efficient existing technology of firm 1 after licensing. Although the small subsidy to firm 2 may make licensing more likely to occur, it would only be a waste for the government to subsidize firm 2 but still keep it less efficient than firm 1. The reason is that a small subsidy to firm 2 will not change social welfare but only the transfer amount between the firms (fixed fee) and increase subsidy cost. For this reason, we will only consider the first case, where firm 2 gets the subsidy.

Knowing that the government provides R&D subsidy if it develops a minor nondrastic innovation and engages in licensing, how much does the subsidized firm innovate? Using the envelope theorem the derivative of firm i's equilibrium profit,

$$\pi_i^*(x_i) := \Pi(q_i(x_i), q_j(x_i), x_i),$$

with respect to x_i is:

$$\frac{d\pi_i^*}{dx_i} = \frac{\partial \pi_i^*}{\partial x_i} + \frac{\partial \pi_i^*}{\partial q_j}\partial q_j^{*'}(x_i) = \frac{4}{3}\Delta_i' q_i, \tag{14}$$

which is positive. Therefore, firm i will invest as much as possible in R&D. However, to get the subsidy, the firm's innovation cannot be greater than Δ_i^L. Thus, under policy 2, the largest amount firm i can choose is x_i^L, where

$$x_i^L = \frac{(\Delta_i^L)^2}{r^2} = \frac{1}{9r^2}(2a + 3c_i - 5c_j)^2. \tag{15}$$

When the subsidized firm undertakes a minor nondrastic innovation and then engages in ex-post licensing the government's payoff is U_G^{epi},

$$\begin{aligned} U_G^{epi} &= \frac{4}{9}\left[a - (c_i - \Delta_i^L)\right]^2 - x_i(\Delta_i^L) \\ &= \frac{100}{81}(a - c_j)^2 - \frac{1}{9r^2}(2a + 3c_i - 5c_j)^2 \end{aligned} \tag{16}$$

Lemma 3 *Under Policy 2, the subsidized firm (firm i) invests* $x_i^* = \frac{(2a+3c_i-5c_j)^2}{9r^2}$*, and then licenses its new technology to the non-innovating firm. The equilibrium payoff of firm i is* $\frac{49(a-c_j)^2}{81}$*, that of firm j is* $\frac{(a-c_j)^2}{81}$*. The government's payoff is* U_G^{epi}.

3.3 Policy 3: Subsidy conditional on a nondrastic innovation.

A drastic innovation is one where the innovating firm becomes a monopoly while the firm using the less efficient technology is driven out of market[2]. Here the subsidized firm's innovation is drastic if its cost-reduction is greater than Δ_i^D,

$$\Delta_i^D = a - 2c_j + c_i \tag{17}$$

Note that $0 \leq \Delta_i^L \leq \Delta_i^D$ and $\Delta_i^D < c_i$ by Assumption 2.

With drastic innovation, firm i will produce the monopolistic output q_i^m,

$$q_i^m = \frac{a - (c_i - \Delta_i^D)}{2} = a - c_j. \tag{18}$$

If the innovation induces a cost reduction of Δ_i and $\Delta_i \in (0, \Delta_i^D)$, then the innovation is nondrastic. In particular, if $\Delta_i \in \left(0, \Delta_i^L\right]$, then it is a minor nondrastic innovation and ex-post licensing occurs. If $\Delta_i \in (\Delta_i^L, \Delta_i^D)$, then the innovation is a major nondrastic one and no ex-post licensing will occur.

Under policy 3 a firm knows that the government will subsidize its R&D investment only if it develops a nondrastic innovation. The firm will develop a major nondrastic innovation with a cost reduction that lies in the range of[3] $\Delta_i = \Delta_i^N \in (\Delta_i^L, \Delta_i^D)$, thus, no ex-post licensing will occur.

Moreover, since the firms will benefit from their R&D investment subsidized by the government, i.e. $\frac{d\pi_i}{dx_i} > 0$, Δ_i^N will be very close to Δ_i^D, i.e. $\Delta_i^{N*} = \Delta_i^D - \varepsilon$, where $\varepsilon > 0$ is an arbitrary small value.

[2]In other words, an innovation is drastic, if the less efficient firm's constant marginal cost is greater than the monopoly price of the innovating firm, i.e. $c_j \geq \frac{a-(c_i-\Delta_i)}{2}$.

[3]Here Δ_i^N denotes the cost reductions under which no ex-post licensing occurs.

The government's payoff when the subsidized firm develops a nondrastic innovation but no ex-post licensing occurs is U_G^{nepi},

$$U_G^{nepi} = \frac{1}{18}(2a - c_i^{'} - c_j)^2 + \frac{1}{9}(a - 2c_i^{'} + c_j)^2 + \frac{1}{9}(a - 2c_j + c_i^{'})^2 - x(\Delta_i^{N*}), \tag{19}$$

here $c_i^{'} = c_i - \Delta_i^{N*}$ is firm i's marginal cost after innovation.

Lemma 4 *Under policy 3, the subsidized firm invests $x_i(\Delta_i^{N*})$, where $\Delta_i^{N*} = \Delta_i^D - \varepsilon$, and earns the equilibrium profit of $\frac{(a-2(c_i-\Delta_i^{N*})+c_j)^2}{9}$, while the noninnovating firm's equilibrium profit is $\frac{(a-2c_j+c_i-\Delta_i^{N*})^2}{9}$. The government's payoff is U_G^{nepi}.*

In addition, if the government would subsidize a drastic innovation, its payoff would be U_G^{Mi},

$$U_G^{Mi} = \frac{3}{2}(a - c_j)^2 - \frac{(a - 2c_j + c_i)^2}{r^2} \tag{20}$$

Comparing U_G^{epi}, as shown in (16) with U_G^{Mi}, as given above, and by Assumption 3, we have

Lemma 5 *The government prefers ex-post licensing with a fixed fee to drastic innovation. That is, $U_G^{epi} > U_G^{Mi}$.*

Proof: See Appendix

4 Equilibrium

In the previous section, the options open to the government and firms were set out formally. The circumstances determining which policy leads to the most preferred outcome from the point of view of the government are now examined. One question that can be resolved immediately is whether the government would prefer policy 2 over policy 3. On comparing U_G^{epi} with U_G^{nepi}, as shown in (19), we obtain the equilibrium of subgame 2, which is given in the following lemma.

Lemma 6 *The government prefers policy (2) to policy (3).*
That is, $U_G^{epi} > U_G^{nepi}$.

Proof: see Appendix.

Here the intuition is similar to that behind Lemma 1, that is, ex-post licensing increases industry output and thus lowers the market price. The social welfare under ex-post licensing is higher than when there is no ex-post licensing but the cost of subsidizing ex-post licensing is lower than subsidizing non ex-post licensing. This lemma implies that the government encourages ex-post licensing. If the subsidized firm does not engage in ex-post licensing, the government will give no subsidy.

To prepare the main Theorems, two further concepts are introduced. We implicitly define r_f^{ex} as the critical value of the efficient parameter r, at which the government is indifferent between subsidizing R&D conditional on ex-post licensing and no R&D subsidy combined with ex-ante licensing.

Similarly, r_f^{nex} is the critical value of r at which the government is indifferent between subsidizing R&D conditional on ex-post licensing and no R&D subsidy combined with no ex-ante licensing:

$$U_G^{ep1}(r_f^{ex}) = U_G^{ex} \tag{21}$$

$$U_G^{ep1}(r_f^{nex}) = U_G^{0} \tag{22}$$

Evidently, $U_G^{ep1}(r) > U_G^{ex}$ iff $r^2 > r_f^{ex}$ and $U_G^{ep1}(r) > U_G^0$ iff $r^2 > r_f^{nex}$. Because high R&D efficiency means low subsidy cost, if the R&D efficiency is sufficiently high (if $r^2 > r_f^{ex}$ when the firms are initially similar or $r^2 > r_f^{nex}$ when the firms are very asymmetric), then it is worthwhile for the government to subsidize R&D so as to enhance consumer surplus as well as producer surplus substantially. The R&D efficiency is defined to be low if the above inequalities of r are reversed, i.e. if $r^2 \leq r_f^{ex}$ or $r^2 \leq r_f^{nex}$. Moreover, it will be shown that $r_f^{nex} < r_f^{ex} < 1.44$.

The subgame perfect equilibrium of the game is described in the following Theorem.

Suppose the efficiency of R&D investment is low, as defined above. Then, the government provides no R&D subsidy, and firms engage in licensing if and only if the gap between their marginal costs is "small", i.e. $c_2 - c_1 \leq \frac{2(a-c_2)}{3}$.

In turn, suppose the efficiency is high, the government subsides R&D, conditional on ex-post licensing. Specifically, the subsidy goes to the more efficient firm (firm 1), iff $r < 1.2$, and to the less efficient firm (firm 2) iff $r \geq 1.2$.

If R&D efficiency is low, then the subsidy cost is high, so it is not worthwhile for the government to subsidize firms' R&D activities. But if the R&D efficiency is sufficiently high, R&D subsidy reduces both firms' production cost through ex-post licensing, causes consumer and producer surplus to increase sufficiently and hence the government will choose to subsidize.

When R&D subsidy is provided, the government will subsidize firm 1, if the R&D efficiency is relatively low, because it is relatively cheaper to subsidize the more efficient firm and this will also induce the firms to engage in ex-post licensing.

However, if the R&D efficiency is relatively high, the less efficient firm will be subsidized. With R&D subsidy, the less efficient firm will develop a bigger innovation than the more efficient firm, whereby the condition for ex-post licensing still holds. Because the new marginal cost of the less efficient firm ($c_2' = \frac{5c_1-2a}{3}$) is lower than that of the more efficient firm($c_1' = \frac{5c_2-2a}{3}$), both industry efficiency and consumer surplus increases substantially. If R&D efficiency is relatively high, the incremental social welfare will sufficiently offset the subsidy cost. Therefore, with relatively high R&D efficiency, it is optimal for the government to subsidize the less efficient firm.

5 Proof of the Theorem

The above theorems state the conditions for the government to decide whether to subsidize the firms' R&D investment and which firm to subsidize. To see whether the government has incentive to subsidize R&D, we need to compare the government's payoff under ex-post licensing U_G^{epi} with its payoff when there is no R&D subsidy (U_G^{ex} or U_G^0 , given in (8) and (9)). Here U_G^{epi} can be explicitly expressed by U_G^{ep1} (or U_G^{ep2}), denoting, respectively, the government's payoff when firm 1 (or firm 2) is subsidized, where

$$U_G^{ep1} = \frac{100}{81}(a-c_2)^2 - \frac{(2a+3c_1-5c_2)^2}{9r^2} \tag{23}$$

$$U_G^{ep2} = \frac{100}{81}(a-c_1)^2 - \frac{(2a+3c_2-5c_1)^2}{9r^2} \tag{24}$$

It can be shown that

(i) $U_G^{ep1} > U_G^{ex}$ if $r^2 > r_1^2 =: r_f^{ex}$

(ii) $U_G^{ep2} > U_G^{ex}$, if $r^2 > r_2^2$

(iii) $U_G^{ep1} > U_G^0$, if $r^2 > r_3^2 =: r_f^{nex}$

(iv) $U_G^{ep2} > U_G^0$, if $r^2 > r_4^2$

Note that r_1^2, r_2^2, r_3^2 and r_4^2 are all smaller than 1. [4]

In addition, it is readily proved that $U_G^{ep1} > U_G^{ep2}$ iff $r^2 < 1.44$. This means that, supposing R&D subsidy is provided, firm 1 will be chosen if $r < 1.2$, otherwise, firm 2 will be chosen. Thus, with relatively low R&D efficiency, we only need to know whether the government has incentive to subsidize firm 1. In other words, among the above cases, only case (i) and (iii) are relevant. By Lemma 1, the government's reservation payoff (with no R&D subsidy) when the firms are asymmetric is smaller than when the firms are similar, thus, we get $r_f^{nex} < r_f^{ex}$. The government is more likely to provide R&D subsidy when there is no licensing.

The government's decisions on R&D subsidies can be summarized as follows:

1. If licensing without innovation does not occur, then firm 1 is subsidized if $r_f^{nex} < r^2 < 1.44$.

2. If licensing without innovation also occurs, then firm 1 is subsidized if $r_f^{ex} < r^2 < 1.44$.

3. Firm 2 is subsidized if $1.44 \leq r^2 < \overline{r}$.

The range of r^2 is demonstrated more clearly in figure 1:

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$r_1^2 = \frac{9(2a+3c_1-5c_2)}{4(8a-3c_1-5c_2)}$, $r_2^2 = \frac{9(2a+3c_2-5c_1)^2}{64(a-c_1)^2}$,
$r_3^2 = \frac{18(2a+3c_1-5c_2)^2}{128a^2+72ac_1-328ac_2+126c_1c_2-99c_1^2+101c_2^2}$,
$r_4^2 := \frac{18(2a+3c_2-5c_1)^2}{128a^2-328ac_1+72ac_2+126c_1c_2+101c_1^2-99c_2^2}$.

No subsidy	*Firm 1 is subsidized*		*Firm 2 is subsidized*	*No subsidy*
	$If\ c_2 > \frac{2a+3c_1}{5}$	$If\ c_2 \leq \frac{2a+3c_1}{5}$		

r_f^{nex} $\quad r_f^x \quad 1.44 \quad \overline{r}$

6 Discussion: other licensing contracts

In this model, the form of licensing contracts with a fixed-fee is investigated. This is due to the following considerations: The first argument is that, although licensing with royalty per unit provides the innovating firm more profit than fixed fee licensing when the firms are competing in a homogenous goods market, the government which takes into account social welfare instead of profit alone will give preference to fixed-fee licensing. This is because the maximum payoff the government can achieve under fixed-fee licensing is greater than under royalty licensing.

Unlike fixed-fee licensing, whose occurrence depends on the size of the innovation, royalty licensing is always profitable. Since a net gain at the amount of rq_j is generated through licensing with royalty per unit, the firms are willing to engage in royalty licensing as long as the innovation is nondrastic. For this reason, policy 2 and 3 are equivalent under licensing with royalty per unit.

When the subsidized firm knows that its R&D costs will be covered by the government, it's R&D investment level x_i^* will be very close to x_i^D. Correspondingly, the cost reduction of the subsidized firm is $\Delta_i^* = \Delta_i^D - \varepsilon = \Delta_i^{N*}$.

The government's payoff, if its R&D subsidy policy is conditional on ex-post royalty licensing, is U_G^{si}, [5]

$$U_G^{si} = \frac{1}{18}(2a - c_i^{'} - c_j)^2 + \frac{1}{9}(a - 2c_i^{'} + c_j)^2 + \frac{1}{9}(a - 2c_j + c_i^{'})^2 + (c_j - c_i^{'})\frac{(a - 2c_j + c_i^{'})}{3} - \frac{(\Delta_i^*)^2}{r^2} \tag{25}$$

where $c_i^{'} = c_i - \Delta_i^*$ is the subsidized firm's marginal cost after innovation. With no R&D subsidy, the firms will always have incentive to engage in royalty licensing, so the government's payoff without giving a subsidy is

$$U_G^{NS} = \frac{(2a - c_1 - c_2)^2}{18} + \frac{(a - 2c_1 + c_2)^2}{9} + \frac{(a - 2c_2 + c_1)^2}{9} + (c_2 - c_1)\frac{(a - 2c_2 + c_1)}{3}. \tag{26}$$

Comparing U_G^{si} with U_G^{NS}, it is easily shown that the government will decide to subsidize iff R&D efficiency is sufficiently high. Furthermore, it can be proved that the government's payoff if the firms engage in ex-post royalty licensing U_G^{si} is smaller than its payoff if drastic innovation would be subsidized U_G^{Mi}. This result is recorded in lemma 7.

Lemma 7 $U_G^{si} < U_G^{Mi}$.

Proof: See Appendix

The intuition is that a drastic innovation reduces the cost of the innovating firm and the market price substantially and thus increases social welfare. Meanwhile, the subsidy cost of supporting a drastic innovation is only marginally greater than supporting a nondrastic innovation, taking into concern that the subsidized firm would undertake an innovation very close to a drastic innovation with R&D subsidy.

[5] The superscript "s" stands for "subsidy".

Summarily, with an intermediate level of R&D efficiency, the government's highest payoff from R&D subsidy under fixed-fee licensing is greater than its payoff under royalty licensing, according to Lemma 5 and 7. This supports our claim that the government prefers fixed-fee licensing to licensing with royalty per unit.

Several empirical works (Rostoker, 1984, Taylor and Silberston, 1973) found that most licensing contracts contain royalty payments. The motivation for the use of royalty licensing is that royalty licensing constitutes a form of risk sharing and can partially solve the problem of asymmetric information (Katz and Shapiro, 1986, Gallini and Wright, 1990). It is common knowledge that the licensor has better information about the value of the innovation than the licensee does. Since royalty payments vary with the licensee's output level, the licensee does not need to pay much when the innovation is not very useful. These reasons for employing royalty payments do not arise in this model where neither uncertainty nor asymmetric information is considered.

Lastly, in this model a licensing contract with a two-part tariff is not allowed. This is due to the consideration that licensing with a two-part tariff may facilitate collusive behavior, in the way that the licensor would raise the licensee's effective cost and then compensate it with a side payment. This is especially the case when the firms are producing the same product.

7 Conclusion

This paper examines the rationality of the government's R&D subsidy from a new perspective, namely, the government's R&D subsidy increases not only firms' incentives to innovate but also their incentives to engage in ex-post licensing. With an intermediate range of R&D

efficiency, the government will subsidize firms' R&D investment to encourage ex-post licensing, especially when fixed-fee licensing is employed. Besides, this paper also answers the question of which firm to subsidize when the firms are asymmetric. It is found that which firm is chosen depends crucially on the efficiency of R&D investment. When the efficiency of R&D investment is relatively high, the less efficient firm will be subsidized. In this case, the asymmetry between firms is reduced and competition is strengthened. On the other hand, when the efficiency of R&D investment is low, the more efficient firm will get the subsidy.

Appendix

Proof of Lemma 5: Compare the government's payoff under ex-post licensing U_G^{epi} (16) with that under drastic innovation (20). For convenience, they are copied here:

$$\begin{aligned} U_G^{epi} &= \frac{100}{81}(a-c_j)^2 - \frac{1}{9r^2}(2a+3c_i-5c_j)^2, \\ U_G^{Mi} &= \frac{3}{2}(a-c_j)^2 - \frac{(a-2c_j+c_i)^2}{r^2}. \end{aligned}$$

After simple calculation we obtain

$$U_G^{epi} - U_G^{Mi} = \frac{(a-c_j)}{9}\left[\frac{(5a+6c_i-11c_j)}{r^2} - \frac{43(a-c_j)}{18}\right]$$

Hence,

$$U_G^{epi} \geq U_G^{Mi} \text{ if } r^2 \leq \frac{18(5a+6c_i-11c_j)}{43(a-c_j)}$$

By assumption 3,

$$r^2 \leq \overline{r} := \frac{18(5a+6c_i-11c_j)}{43(a-c_j)},$$

so we have $U_G^{epi} \geq U_G^{Mi}$. ■

Proof of Lemma 6: The government's payoff when the subsidized firm undertakes a nondrastic innovation and then engages in ex-post licensing is U_G^{epi} and its payoff when the subsidized firm undertakes a nondrastic innovation but no ex-post licensing occurs is U_G^{nepi}, $U_G^{epi} = \frac{100}{81}(a-c_j)^2 - \frac{1}{9r^2}(2a+3c_i-5c_j)^2$,
$U_G^{nepi} = \frac{1}{18}(2a-c_i^{'}-c_j)^2 + \frac{1}{9}(a-2c_i^{'}+c_j)^2 + \frac{1}{9}(a-2c_j+c_i^{'})^2 - x(\Delta_i^{N*})$, where $\Delta_i^{N*} = \Delta_i^D - \varepsilon$ is the equilibrium amount of the subsidized firm's (firm i's) cost reduction under subsidy policy 3 and $c_i^{'} = c_i - \Delta_i^{N*}$ is firm i's new marginal cost.

First, we compare the social welfare (gross of subsidy cost) in the above cases, which are, for the sake of calculation, decomposed into two parts:

$$\begin{aligned}
&(1).\ \frac{2}{9}\left[a-(c_i-\Delta_i^L)\right]^2 - \frac{1}{18}(2a-c_i^{'}-c_j)^2 \\
&= \frac{50}{81}(a-c_2)^2 - \frac{1}{18}(2a-c_i^{'}-c_j)^2 \\
&= \frac{1}{18}\left(26a-23c_j-3c_i^{'}\right)\left(14a-17c_i^{'}+3c_j\right) > 0 \\
&(2).\ \frac{2}{9}\left[a-(c_i-\Delta_i^L)\right]^2 - \frac{1}{9}(a-2c_i^{'}+c_j)^2 - \frac{1}{9}(a-2c_j+c_i^{'})^2 \\
&= \frac{50}{81}(a-c_j)^2 - \frac{1}{9}(a-2c_i^{'}+c_j)^2 - \frac{1}{9}(a-2c_j+c_i^{'})^2 \\
&= \frac{1}{9}\left(2a-c_i^{'}-c_j\right)\left[16(a-c_j)-3(c_j-c_i)\right]
\end{aligned}$$

Under nondrastic innovation, we have $c_j - c_i \leq a - c_j$, therefore,

$$\frac{1}{9}(2a-c_i^{'}-c_j)\left[16(a-c_j)-3(c_j-c_i^{'})\right] > 0.$$

From (1) and (2), the gross social welfare generated from ex-post licensing is greater than in the case of non ex-post licensing.

Second, we compare the government's subsidy cost in the two cases. Since $\Delta_i^{N*} > \Delta_i^L$, we get $x_i(\Delta_i^L) < x(\Delta_i^N)$.

Therefore, $U_G^{epi} \geq U_G^{nepi}$. ■

Proof of Lemma 7: Comparing the government's payoff when the subsidized firm undertakes nondrastic innovation and then sells its new innovation to the less efficient firm by the means of royalty per-unit licensing U_G^{si} (25) to its payoff when the subsidized firm develops a drastic innovation U_G^{mi} (20), the following is obtained:

Since the subsidy costs with ex-post royalty licensing and with drastic innovation are nearly the same, we only need to compare the social welfare under the two cases.

First, compare U_G^{s1} with U_G^{M1}:

$$\begin{aligned}
U_G^{s1} - U_G^{M1} &= \frac{1}{18}(2a - c_1^{'} - c_2)^2 + \frac{1}{9}(a - 2c_1^{'} + c_2)^2 \\
&+ \frac{1}{9}(a - 2c_2 + c_1^{'})^2 + (c_2 - c_1^{'})\frac{(a - 2c_2 + c_1^{'})}{3} - \frac{3}{2}(a - c_2)^2 \\
&= \frac{(a - 2c_2 + c_1')}{9}\frac{-19a + 11c_1' + 8c_2}{2} + (c_2 - c_1^{'})\frac{(a - 2c_2 + c_1^{'})}{3} \\
&= \frac{(a - 2c_2 + c_1^{'})}{3}\left[\frac{-19a + 5c_1' + 14c_2}{6}\right] < 0 \\
&\Rightarrow U_G^{s1} < U_G^{M1}
\end{aligned}$$

Similarly, we get $U_G^{s2} < U_G^{M2}$. ■

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