SCALE ECONOMIES AND THE DYNAMICS OF RECURRING AUCTIONS

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We analyze the dynamics of a game of sequential bidding in the presence of stochastic scale effects, either economies or diseconomies of scale. We show that economies of scale give rise to declining expected equilibrium prices, whereas the converse is not generally true. Moreover, first- and second-price auctions are not always revenue equivalent. Economies of scale make second-price auctions more profitable for the seller, whereas revenue equivalence may be preserved in the case of diseconomies. (*JEL* D44)

I. INTRODUCTION

Many auction settings are characterized by examples along the following lines:

• An art collector or specialized art dealer participates in auctions for Expressionistic paintings from the early 1920s. As the collection grows, the "missing" pictures become more valuable—the whole is more valuable than the sum of its parts.

• A contract to develop and supply a new-generation fighter jet is auctioned recurrently. Experience gives a competitive advantage. Winning several auctions is more valuable than the sum of winning each auction alone.

• A license to supply catering services in a university cafeteria is awarded in a recurring auction. Success breeds failure. As the licensee becomes more established, organizational slack builds up. Winning several auctions is less valuable than the sum of winning each auction alone.

The essential characteristic of these examples is that several units or licenses are auctioned recurrently, after some lapse of time, and that bidders' valuations are stochastically dependent across auctions, due to learning, complementarities, or wear-out and organizational slack.

To gain insight into this common setting, this article analyzes recurring auctions in a simple framework where two prizes are auctioned in sequence to two bidders. Prior to each auction, bidders draw their private valuations for the item on the trading block. When the first prize is auctioned, random valuations are symmetric and independent. However, in the second auction, the probability distributions of valuations depend on the history of either winning or losing the first auction.

Two scenarios are considered: stochastic economies and diseconomies of scale. Economies of scale occur if the winner of the first prize has a higher random valuation for the second prize, due to complementarities or learning effects. Diseconomies occur if the winner's random valuation declines, for example due to organizational slack in the framework of licensing and procurement.

We analyze the equilibrium solution of first- and second-price auctions, provide necessary and sufficient conditions for declining and increasing equilibrium prices, and for the revenue ranking of first- versus second-price auction formats. In particular, we show

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that stochastic economies of scale give rise to declining expected prices, whereas the converse is not generally true. Moreover, standard auction formats are not revenue equivalent: the second-price format is generally more profitable in the case of economies of scale, yet revenue equivalence may be preserved otherwise.

There is a sizable literature on sequential auctions. This literature was stimulated by the observation of systematically declining prices in auctions of identical bottles of wine (Ashenfelter, 1989), and in real estate auctions for nearly identical condominium units (Ashenfelter and Genovese, 1992). Similar evidence for declining prices is available for commercial real estate auctions, transponder leases, and stamp auctions.

Several explanations were proposed to account for this anomaly. Black and de Meza (1992) show that declining prices may occur if bidders demand several units and have the option to purchase several goods. McAfee and Vincent (1993) explore the role of risk aversion as an explanation of declining prices for identical items. Bernhardt and Scoones (1994) and Engelbrecht-Wiggans (1994) show that prices may decline if items are stochastically equivalent and each bidder demands at most one unit, and von der Fehr (1994) explains declining prices as a result of reduced competition in the second auction due to costly participation. Gale and Hausch (1994) analyze auctions of stochastically equivalent items in which winners have the right to chose among the items. Milgrom and Weber (1982) suggest that declining prices may be due to the use of agents who are instructed to win an item at any price up to a specified limit.

The present article deviates from this literature in several regards. First of all, we do not consider the sale of identical objects that are auctioned immediately one after another, as in wine and real estate auctions. Instead, we are interested in understanding the dynamics of *recurring* auctions, when there is a sufficiently long lapse of time between subsequent rounds of auctions, and bidders do not yet know the value of the second item when they bid for the first item. Second, we consider situations in which a bidder is interested in winning all auctions, unlike the frequently used unit-demand assumption. Third, we are interested in modeling situations where bidders' demand is either stochastically increasing or decreasing due to complementarities, learning, or wear out and organizational slack.

Independently, von der Fehr and Riis (1998) study the properties of the equilibrium price sequence in the English auction, in a similar framework, allowing deterministic links for winners and/or losers in the first auction in addition to stochastic scale effects. Consequently, our results concerning the English format are mirrored in parts of their analysis. Similar results are also found in independent work by Branco (1997) and Menezes and Monteiro (1999).

Regarding the modeling of scale effects, it is also worth pointing out that Martin (1999), in a model of interjurisdictional competition for firm location, considers "agglomeration" or scale economies by shifting the supports of types. The main difference in that setting is that there are two sellers and the scale economies affect the sellers' sales in the sequential auctions, not the buyers' purchases.

The plan of the article is as follows. In sections II and III we state and solve the model for both scale assumptions when bidders' valuations come from a two-point distribution. In section IV we analyze the resulting distributions of equilibrium bids and state necessary and sufficient conditions for declining and increasing equilibrium prices. In section V we analyze the implied revenue ranking of first- versus second-price auctions. In section VI we consider some extensions of the model and assess the robustness of our results concerning the equilibrium price sequence and the auctioneer's revenue. The article closes with a brief discussion in section VII.

II. THE MODEL

Consider a sequence of two auctions, either first- or second-price, in each of which a single prize is sold to two bidders. Prior to the first auction each bidder privately observes his valuation for the first prize, V, but not for the second. After conclusion of the first auction, the winner is announced and, after some lapse of time, bidders privately observe their valuation for the second prize, V_H. Due to scale effects, the random variable V_H depends on the history H of winning or losing the first auction.

The valuations of the first prize V are independent random variables drawn from a support normalized to $\{0, v\}$, $v > 0$, with $0 < \rho := Pr\{V = v\} < 1$.

The valuation of the second prize V_H depends on bidders' histories. We refer to the winner of the first auction as "incumbent" (I), and the loser as "contestant" (C). Their valuations, V_I, V_C, are stochastically independent and drawn from the same support $\{0, v\}$, but the probability of the event $V_H = v$, ($H \in \{I, C\}$) is not the same for incumbent and contestant.

Because the contestant has not gained experience, his valuation for the second prize is drawn from the same distribution. Whereas the incumbent's probability of drawing the high valuation, $\sigma := Pr\{V_I = v\}$, differs from ρ.

Stochastic "economies" of scale occur if the incumbent has a higher probability of drawing the high valuation, that is, $0 < \rho < \sigma < 1$, whereas "diseconomies" occur if $0 < \sigma < \rho < 1$. Economies lead to an increasing and diseconomies to a decreasing expected value of the prize.

Bidders are risk-neutral and, for simplicity, maximize the sum of the payoffs from each auction by placing any real-valued bid (no discounting).

The payoffs in the second auction are described by $U_H(V_H) := Pr\{\text{win 2nd auction}\}(V_H - P_2)$, $H \in \{I, C\}$, where P_2 denotes the price paid in that auction. The overall payoff, evaluated at the time of the first auction, can thus be represented by $U(V) := Pr\{\text{win 1st auction}\}(V - P_1 + E[U_I(V_I)]) + (1 - Pr\{\text{win 1st auction}\})E[U_C(V_C)]$.

Letting Δ denote the (ex ante) value of incumbency, we define

$$\Delta := E[U_I(V_I)] - E[U_C(V_C)]. \tag{1}$$

Hence, the overall payoff U can be written in the convenient form $U(V) := Pr\{\text{win 1st auction}\}(V - P_1 + \Delta) + E[U_C(V_C)]$.

Ties are broken in the first auction by the flip of a fair coin, whereas, for convenience, in the second auction the incumbent is favored in the case of economies and the contestant in the case of diseconomies.

III. EQUILIBRIUM STRATEGIES

In this section we solve the equilibrium strategies for both auction formats. We start with the simpler English auction and then analyze the Dutch auction. We analyze the case of economies of scale in detail and then sketch how results change in the case of diseconomies.

As a benchmark we frequently refer to the symmetric, one-shot auction, where both bidders draw the valuation $V = v + \Delta$ with probability ρ and the valuation $V = \Delta$ with probability $1 - \rho$. Therefore, at the outset, we state the equilibrium solutions of the static benchmark auctions for arbitrary Δ. (The proofs are standard and can be found in many game theory texts, for example, Fudenberg and Tirole [1991]).

PROPOSITION 1 (Benchmark Auction). *In the unique equilibrium of the static Dutch (first-price) benchmark auction bidders with valuation $V = \Delta$ bid $b^*(\Delta) = \Delta$, and bidders with $V = v + \Delta$ play the mixed strategy F^*: $[\Delta, \rho v + \Delta] \to [0, 1]$:*

$$F^*(b) = [(1 - \rho)(b - \Delta)]/[\rho(v - b + \Delta)]. \tag{2}$$

In the static English (second-price) benchmark auction the unique equilibrium surviving the iterated elimination of weakly dominated strategies is $b^*(V) = V$.

English (Second-Price) Auctions

Consider the strategically simpler English or second-price auction format. The following proposition gives the equilibrium strategies in the sequence of auctions; they apply to economies and diseconomies alike.

PROPOSITION 2 (English Auctions). *The English auction has an equilibrium in weakly dominant pure strategies*

$$b_1(V) = V + \Delta_E \quad \text{(first auction)} \tag{3}$$

$$b_2(V_H) = V_H, \quad H \in \{I, C\} \quad \text{(second auction)} \tag{4}$$

$$\Delta_E = (\sigma - \rho)v. \tag{5}$$

Proof. As in the benchmark, iterated elimination of weakly dominated strategies in the second auction yields (4). On the basis of this Δ_E can be computed and then further elimination of weakly dominated strategies yields (3) for the first auction.

To compute Δ_E, notice that in the second auction a bidder's payoff is equal to zero unless two conditions are met: the bidder has valuation $V_H = v$, and his rival's valuation is $V_H = 0$. Therefore, $U_I(v) = (1-\rho)v$ and $U_C(v) = (1-\sigma)v$. Hence, $\Delta_E = \sigma U_I(v) - \rho U_C(v) = (\sigma - \rho)v$. □

In the first auction bidders account for the fact that winning awards not only the first prize but also the position of incumbency. If incumbency is valuable, which occurs in the case of economies, the first auction equilibrium bids are higher and vice versa. Also notice that the equilibrium strategy b_2 is equivalent to that of the static, symmetric benchmark English auction with $\Delta = 0$; similarly, b_1 is equivalent to the equilibrium strategy of the static benchmark auction with $\Delta = \Delta_E$.

Dutch (First-Price) Auctions

Now consider the Dutch auction format. Suppose there are economies of scale. Then, the winner of the first auction has a higher likelihood of drawing the high valuation. Consequently, the second auction is asymmetric. Its unique equilibrium strategies are as follows.

PROPOSITION 3 (Second Auction). *Suppose* $\sigma > \rho$ (*economies*). *In equilibrium bidders with valuation* $V_H = 0$ *bid their value and bidders with* $V_H = v$ *play mixed strategies* $F_H : [0, \rho v] \to [0, 1]$. *Specifically,*

(6) $b_H(0) = 0, \quad H \in \{I, C\}$ (bidder with $V = 0$);

(7) $F_I(b) = 1 - ([\rho v - b]/[\sigma(v - b)])$ (incumbent with $V = v$);

(8) $F_C(b) = [(1-\rho)b]/[\rho(v - b)]$ (contestant with $V = v$).

Bidders' equilibrium payoffs are

(9) $U_H(0) = 0, \quad U_H(v) = (1-\rho)v, \quad H \in \{I, C\}.$

Proof. It is straightforward to confirm that the candidate equilibrium strategies are mutual best replies. It is equally straightforward to compute the equilibrium payoffs. Note, however, that F_I has a mass point at $b = 0$. Uniqueness is established similarly to the case of symmetric auctions. □

The intuition for the uniqueness of the equilibrium is given following Corollary 1.

To compute the equilibrium value of incumbency, Δ_D, notice that in the second auction a bidder has a positive expected payoff only if he has valuation $V_H = v$. Hence, after conclusion of the first auction, but before valuations of the second prize are drawn, the incumbent's payoff is $E[U_I(V_I)] = \sigma U_I(v) = \sigma(1-\rho)v$, whereas the contestant's payoff is $E[U_C(V_C)] = \rho U_C(v) = \rho(1-\rho)v$. Therefore, by definition (1),

(10) $$\Delta_D = (\sigma - \rho)(1-\rho)v.$$

Just as in the English auction format, bidders account for the fact that winning the first auction not only awards the first prize but also the position of incumbency.

PROPOSITION 4 (First Auction). *Suppose* $\sigma > \rho$ (*economies*). *In equilibrium bidders with valuation* $V = 0$ *bid the value of incumbency, bidders with* $V = v$ *play a mixed strategy* F: $[\Delta_D, \rho v + \Delta_D] \to [0, 1]$. *That is,*

(11) $$b(0) = \Delta_D$$

(12) $$F(b) = [(1-\rho)(b - \Delta_D)] / [\rho(v - b + \Delta_D)],$$

and both continue in the second auction as summarized in Proposition 3.

Proof. Follows from (10) and Proposition 1. □

Consider now the case of diseconomies ($\sigma < \rho$). The second auction is nearly identical to that in the case of economies. Indeed, the strategies and payoffs are as in Proposition 3, except that σ and ρ, as well as I and C, exchange places. Thus, bidders with valuation $V_H = 0$ bid 0, and bidders with $V_H = v$ play the mixed strategies F_H: $[0, \sigma v] \to [0, 1]$:

(7′) $$F_I(b) = [(1-\sigma)b]/[\sigma(v - b)]$$

(8′) $$F_C(b) = 1 - [\sigma v - b]/[\rho(v - b)]$$

(9′)
$$U_H(0)=0, \quad U_H(v)=(1-\sigma)v, \quad H\in\{I,C\}.$$

The equilibrium strategies in the first auction are given by equation (13); however, the value of incumbency for the case of diseconomies is

(10′) $$\Delta_D = (\sigma-\rho)(1-\sigma)v.$$

Having solved the game, we turn to a comparison of equilibrium strategies and the associated probability distribution of bids. In the following we denote the bids of I and C, as seen from the seller's perspective, by the random variables B_I and B_C. Moreover, we will say that a random variable X is stochastically "larger" than Y, $X > Y$, if X first-order stochastically dominates Y.[1]

For purposes of comparison note that in the static benchmark auction with $\Delta = 0$ a bidder's equilibrium bids, denoted by the random variable B^*, has distribution

(13) $$G^*(b) := Pr\{B^* \le b\} = 1-\rho+\rho F^*(b) = [(1-\rho)v]/(v-b).$$

COROLLARY 1. *From the seller's perspective, the equilibrium bids by incumbent and contestant, B_I, B_C, are stochastically the same: $B_I = B_C =: B_2$. In particular,*

(14) $$Pr\{B_2 \le b\} = [(1-\min\{\sigma,\rho\})v]/(v-b) =: G_2(b),$$

which is, incidentally, also the distribution of bids in a symmetric one-shot auction where $V = v$ is drawn with probability $Pr\{V_H = v\} = \min\{\sigma,\rho\}$.

Proof. By (7), (8), (7'), and (8') one has $G_I(b) := Pr\{B_I \le b\} = 1-\sigma+\sigma F_I(b) = G_2(b)$ and similarly $G_C(b) := Pr\{B_C \le b\} = 1-\rho+\rho F_C(b) = G_2(b)$. □

To understand this result note first that the auction is efficient. That is, a bidder with valuation $V = v$ will not lose to a bidder whose valuation is $V = 0$ in equilibrium. This implies the expected payoffs to the bidders and consequently the supports of the strategies used by bidders with valuation $V = v$. In turn, these imply that the incumbent's and contestant's random bids are identical to the those placed in the benchmark auction when $Pr\{V_H = v\} = \min\{\sigma,\rho\}$.

Now consider the first auction. Comparing equilibrium bids in the first auction, denoted by B_1, with equilibrium bids of the benchmark auction, B^*, one obtains a stochastic order relationship analogous to the first auction of the English format.

COROLLARY 2. *Compare the first auction with the static benchmark auction for $\Delta = 0$. From the seller's perspective, bidders' equilibrium bids in the first auction are stochastically larger: $B_1 > B^*$, if $\sigma > \rho$, and smaller: $B_1 < B^*$, if $\sigma < \rho$.*

Proof. Because $\Delta_D \gtrless 0 \iff \sigma \gtrless \rho$ and

(15) $$G_1(b) := Pr\{B_1 \le b\} = (1-\rho)+\rho F_1(b) = [(1-\rho)v]/[v-b+\Delta_D],$$

one has $G_1(b) \lesseqgtr G^*(b) \iff \sigma \gtrless \rho$. □

IV. EQUILIBRIUM PRICE SEQUENCE

Now we examine how economies and diseconomies of scale affect the sequence of equilibrium prices. Declining and increasing prices are characterized either by a first-order stochastic dominance ranking of prices or a comparison of expected prices.

English (Second-Price) Auctions

Let P_1^E and P_2^E denote the random equilibrium prices in the first and second auctions. These random variables cannot be ranked by the stochastic larger relationship; however, the ranking of expected equilibrium prices is unambiguously governed by the scale assumption.

THEOREM 1. *In the English auction, the expected equilibrium prices are declining if there are economies and increasing if there are diseconomies:*

$$E[P_1^E] \gtrless E[P_2^E] \iff \sigma \gtrless \rho.$$

1. Definition: Let F_X, F_Y be the c.d.f.'s of X and Y. Then, $X > Y$ if for all z, $F_X(z) \le F_Y(z)$, with strict inequality somewhere. Of course, if the supports differ, F_X, F_Y must be extended to cover the union of the two supports.

Proof. Recall from Proposition 2 that P_2^E is equal to v iff $V_I = V_C = v$, and equal to 0 otherwise; similarly, P_1^E is equal to $v + \Delta_E$ iff both bidders draw the valuation $V = v$ and equal to Δ_E otherwise. Therefore,

$$(16)\quad E\left[P_1^E\right] = (1-\rho^2)\Delta_E + \rho^2(v + \Delta_E) = \rho^2 v + (\sigma - \rho)v,$$

$$(17)\quad E\left[P_2^E\right] = \rho\,\sigma\,v,$$

and the assertion follows immediately. □

Interestingly, in the case of economies the expected value of the prize increases, whereas the expected price declines and vice versa. This is due to the fact that in the first round of bidding, bidders account for the value of incumbency.

Dutch (First-Price) Auctions

Let P_1^D, P_2^D, P^* denote the random equilibrium prices in the first, second, and benchmark auctions, respectively. In the case of economies, as in the English auction, the expected equilibrium prices are declining, that is, $E[P_1^D] > E[P_2^D]$. However, we obtain an even stronger result because the random prices in the consecutive Dutch auctions can be ranked by the stochastic larger relationship.

THEOREM 2 (Economies). *Suppose* $\sigma > \rho$. *Then, the random equilibrium price is declining*: $P_1^D \succ P_2^D$.

Proof. Using Corollaries 1 and 2 one has $Pr\{P_2^D \le p\} = G_2(p)^2 = G^*(p)^2$, and $Pr\{P_1^D \le p\} = G_1(p)^2 < G^*(p)^2$. Therefore, $P_1^D \succ P^* = P_2^D$, as asserted. □

As prices decline in the case of economies, one might conjecture that diseconomies should give rise to increasing equilibrium prices. After all, diseconomies make incumbency undesirable, which lowers bidding in the first auction. However, this conjecture is not generally correct.

Employing an argument similar to the proof of Theorem 2 one sees immediately that the introduction of diseconomies lowers both P_1^D and P_2^D, relative to the equilibrium price of the benchmark auction P^*. Specifically, due to diseconomies the value of incumbency is negative, which shifts the support of P_1^D below that of P^*; also, because bidding in the second auction is characterized by $\sigma\,(<\rho)$, bids are lower in the second auction than in the benchmark, that is, $P_2^D \prec P^*$. This suggests that there is no unambiguous stochastic ranking of equilibrium prices in the diseconomies case.

Indeed, equilibrium prices cannot be ranked by the stochastic larger relationship in the case of diseconomies. This follows from the fact that $Pr\{P_2^D \le p\}$ is below $Pr\{P_1^D \le p\}$ for all $p < 0$, but jumps above it at $p = 0$. This evidently violates first-order stochastic dominance.

Moreover, in the diseconomies case one cannot even unambiguously rank expected equilibrium prices. Instead, expected equilibrium prices decline for some combinations of σ and ρ and increase for others. In fact, regardless of the value of $\sigma \in (0, \rho)$ there is always a sufficiently large value of ρ so that prices are expected to decline. More to the point, increasing prices are only expected for relatively few combinations of ρ and σ—illustrated by the shaded area of Figure 1.

THEOREM 3 (Diseconomies). *Suppose* $\sigma < \rho$. *Then, expected equilibrium prices decline if* $\rho > 1 - 2\sigma$, *and increase if* $\rho < 1 - 2\sigma$,

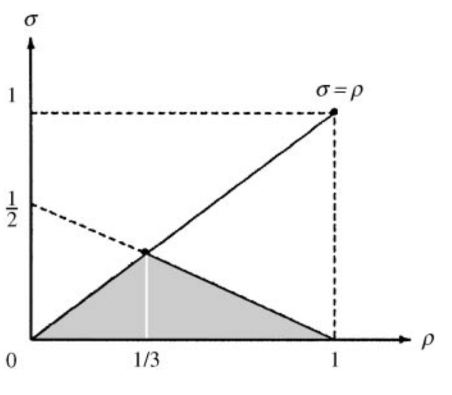
$$E\left[P_1^D\right] \gtrless E\left[P_2^D\right] \iff \rho \gtrless 1 - 2\sigma.$$

Proof. Using (14) and (15) one obtains

$$Pr\{P_2^D \le p\} = G_2(p)^2 = ([(1-\sigma)v]/[v-p])^2, \quad p \in [0, \sigma v]$$

FIGURE 1

Declining and Increasing Expected Prices



$$Pr\{P_1^D \leq p\}$$
$$= G_1(p)^2 = ([(1-\rho)v]/[v-p+\Delta_D])^2,$$
$$p \in [\Delta_D, \rho v + \Delta_D].$$

Hence, using (10') and defining $\phi(\sigma) := E[P_1^D] - E[P_2^D]$, gives[2]

$$(18) \qquad E\left[P_2^D\right] = \sigma^2 v$$

$$(19) \qquad E\left[P_1^D\right] = \rho^2 v + \Delta_D$$
$$= \rho^2 v + (\sigma - \rho)(1-\sigma)v$$
$$\phi(\sigma) = (\sigma - \rho)(1 - \rho - 2\sigma)v.$$

Since $\sigma < \rho, \phi(\sigma) \gtreqless 0$ iff $\rho \gtreqless 1 - 2\sigma$, as asserted. □

The reason for declining expected equilibrium prices for large ρ is the way in which the equilibrium prices in the two consecutive auctions are affected by ρ and σ, respectively. When increasing ρ while leaving σ unchanged, the second auction expected price, $E[P_2^D]$, remains the same (see Corollary 1). However, the first auction expected price, $E[P_1^D]$, is affected in two ways. First, it leads to a higher probability of bidders having valuation $V = v$ for the first prize. Second, it decreases the value of incumbency. From Equation (19), one sees that the marginal impact of the former effect is to raise $E[P_1^D]$ by $2\rho v$, whereas the latter effect leads to a marginal decrease in $E[P_1^D]$ by $(1-\sigma)v$. Because the latter is independent of ρ, the greater ρ is, the smaller the impact of the second effect is relative to the first, so that the first effect dominates. This results in declining expected equilibrium prices for sufficiently large ρ.

Notice also that the marginal impact of increases in ρ on $E[P_D^1]$ though increasing $Pr\{V = v\}$, is independent of σ, whereas the marginal decrease in $E[P_D^1]$ brought about by changes in the value of incumbency due to changes in ρ is independent of ρ but decreasing in σ. Consequently, the larger σ is, the larger is the range of possible values of ρ that lead to decreasing expected prices.

The underlying economic reason for the asymmetry between the Dutch and the English formats in general and the case of economies and diseconomies in the Dutch format in particular stem from the impact of Corollary 1.

The intuition of the price paths in the English auctions are fairly straightforward. In the first auction bidders bid for the first auction prize, as well as the value of incumbency. If incumbency is valuable (economies), this affects both bidders' first auction strategies by shifting the entire support of bids, which more than offsets the higher average bid placed by the incumbent in the second auction on the lower support. Consequently, one observes declining expected prices. The case of diseconomies is the direct converse of this argument.

The intuition for the Dutch auction in the case of economies is along the same lines, but even stronger. Though a positive value of incumbency increases bidding in the first auction by shifting the support of bids, there is no increase in the distribution of bids by the incumbent in the second auction, as demonstrated by Corollary 1.

Now consider the Dutch auction for the case of diseconomies. Although incumbency is viewed unfavorably and hence lowers bidding in the first auction, in the second auction—unlike the case of economies in the Dutch auction—the distribution of bids are not unaffected. Indeed, not only are one bidders' distribution of bids lower, as was the case in the English format, but *both* bidders' distribution of bids are lower in the second auction. In addition to lowering bids in the second auction, the diminished likelihood of high valuations in the second auction yield a "lower-stakes" game when compared to the economies case, so that the impact of the second auction on first auction bidding is diminished.

In the following section we consider the implications of these results for the revenue of the auctioneer.

V. REVENUE RANKING

Which auction format—Dutch or English–gives the seller the higher expected revenue? We first consider the ranking of expected revenues in each of the two auctions and then proceed to rank the seller's overall expected revenues.

2. To compute $E[P_1^D]$, be careful and note that $Pr\{P_1^D \leq p\}$ has a mass point at $p = \Delta_D$.

PROPOSITION 5 (Second Auction Revenue). *For either scale assumption, the seller's equilibrium expected revenue in the second auction is higher in the English than in the Dutch auction, that is,* $E[P_2^D] < E[P_2^E]$.

Proof. From (18), $E[P_2^D] = \sigma^2 v$ if $\sigma < \rho$; similarly, $E[P_2^D] = \rho^2 v$ if $\sigma > \rho$. Finally, by (17), $E[P_2^E] = \sigma\rho v$. Hence, $E[P_2^D] = (\min\{\sigma,\rho\})^2 v < \sigma\rho v = E[P_2^E]$. □

The key to understanding the superiority of the English auction in the second auction is Corollary 1. There it is shown that in the Dutch auction the distribution of bids, and hence the seller's expected revenue, is the same as in a symmetric one-shot auction in which both incumbent and contestant draw the high valuation $V_H = v$ with the smaller probability, $Pr\{V_H = v\} = \min\{\sigma,\rho\}$. Of course, in a symmetric one-shot auction, Dutch and English formats yield the same expected revenues. However, since either $\sigma > \rho$ (economies) or $\sigma < \rho$ (diseconomies), one of the bidders has a higher chance of drawing $V_H = v$, whereas strategies in the English format are unaffected by variations in σ or ρ (see [4]). Therefore, compared to the symmetric one-shot English auction, one bidder's average bid is higher, which raises the seller's expected revenue in the English format.

However, this unconditional superiority of the English auction does not extend to the first round of bidding.

PROPOSITION 6 (First Auction Revenue). *The seller's equilibrium expected revenue in the first auction is higher in the Dutch auction if there are diseconomies, and higher in the English auction if there are economies, that is,* $E[P_1^D] \gtrless E[P_1^E] \iff \sigma \lessgtr \rho$.

Proof. If $\sigma < \rho$, using (19) and (16), and if $\sigma > \rho$, using $Pr\{P_1^D \le p\}$ from the proof of Theorem 2, we conclude

$$\begin{aligned} E\left[P_1^D\right] &= (\rho^2 + (\sigma-\rho)(1-\min\{\sigma,\rho\}))v \\ &\gtrless [\rho^2 + (\sigma-\rho)]v \iff \sigma \lessgtr \rho \\ &= E\left[P_1^E\right]. \end{aligned}$$

□

To understand this result, note first that the auctions are equivalent to a static, symmetric benchmark auction, in which bidders draw either the valuation Δ or $v + \Delta$. If the values of incumbency, Δ, were the same, the seller's expected revenues would also be the same, by the well-known revenue equivalence theorem. However, from equations (10), (10'), and (5) the values of incumbency differ as follows

$$(20) \qquad \Delta_E \gtrless \Delta_D \iff \sigma \gtrless \rho.$$

Of course, the seller's expected revenue is increased if bidders' valuations are uniformly increased. Hence, the auction with the higher Δ also yields the higher expected revenue.

The reason for (20) is that in the second auction of the Dutch format both the incumbent and the contestant benefit from the asymmetry of the auction (cf. Corollary 1), whereas in the English format it is only the incumbent in the case of economies and the contestant in the case of diseconomies who benefits. A straightforward implication is that the value of incumbency is greater in absolute value in the English auction. Consequently, scale effects have a greater impact in the English auction format than in the Dutch format.

We now turn to the overall expected revenues for the two formats. To this end, define $\Pi := E[P_1] + E[P_2]$, which is the seller's overall expected revenue.

THEOREM 4 (Revenue Ranking). *The seller's overall equilibrium expected revenue is higher in the English than in the Dutch format iff there are economies; otherwise the auction formats are revenue equivalent, that is,* $\Pi^E > \Pi^D \iff \sigma > \rho$, $\Pi^E = \Pi^D \iff \sigma < \rho$.

Proof. The superiority of the English auction in the case $\sigma > \rho$ (economies) follows from the fact that the English auction gives rise to higher revenues in both auctions, as shown in Propositions 5 and 6.

In turn, if $\sigma < \rho$ (diseconomies), one obtains:

$$\begin{aligned} \Pi^D - \Pi^E &= \left(E\left[P_1^D\right] - E\left[P_1^E\right]\right) \\ &\quad + \left(E\left[P_2^D\right] - E\left[P_2^E\right]\right) \\ &= (\sigma-\rho)v[(1-\sigma)-1] + \sigma(\sigma-\rho)v \\ &= 0, \end{aligned}$$

as asserted. □

What is critical for the superiority of the English auction is only the format of the second auction. Note that the second auction determines the equilibrium value of incumbency, Δ. For each given Δ the first auction is equivalent to a symmetric, one-shot auction. In a symmetric, one-shot auction, revenue equivalence holds. Therefore, the seller's revenue in the first auction is independent of the auction format and depends only on the value of incumbency, Δ, determined by the equilibrium of the second auction. Hence, if the game ends with either an English or a Dutch auction, it does not matter which format is used in the first auction. Thus, Theorem 4 can be extended as follows.

COROLLARY 3. *Consider all possible combinations of English and Dutch auctions, such as EE: English followed by English, DE: Dutch followed by English, etc. The seller ranks these combinations as follows (with strict inequality iff* $\sigma > \rho$): $\Pi^{EE} = \Pi^{DE} \geq \Pi^{DD} = \Pi^{ED}$.

VI. ROBUSTNESS

The present simple model should be viewed as a starting point for a more general analysis of recurring auctions. Of course, one cannot expect that all our unusually strong and clear-cut results generalize without qualification. Some of the main results can be extended under meaningful restrictions; others raise intriguing new issues for further research.

An obvious extension is to allow for three or more bidders. This affects the equilibrium price sequence in an intuitively predictable way. For simplicity consider the case of three bidders and economies. The increased competition lowers the gap between expected equilibrium prices. Nevertheless, our results concerning the price sequence in the Dutch auction and the revenue ranking of auction forms generalize without qualification. However, in the English auction the order of expected equilibrium prices can be reversed, but only if more competition has a "drastic" effect on equilibrium prices.

Another extension concerns the introduction of a strategic reserve price, above the seller's own valuation. Suppose that in the second auction the auctioneer imposes a minimum bid, $b_{\min}$, equal to the high valuation, v.[3] In this case bidders' expected surplus is completely extracted in the second auction, so that the second auction revenue is the same in both formats. Moreover, the expected payoff of both the incumbent and contestant is equal to 0, regardless of the auction format. Consequently, the value of incumbency is 0 as well, and—following the logic of Corollary 3—both English and Dutch formats yield the same overall expected revenue in the sequence of sales. Similarly, the auctioneer may choose to auction both prizes simultaneously, rather than sequentially, with the winner possibly having the option of resale of the second prize, should a value of zero be realized. Again, the auction becomes revenue equivalent across formats (yielding higher revenue than the sequential sale because all expected surplus of the second prize is auctioned off). We mention, however, that simultaneous sale may often not be possible, and, generally, the use of reserve prices may be undesirable if one takes into account how they affect participation in auctions, which may explain why strategically motivated reserve prices are rare in practical applications.[4]

Possibly the most insightful extension is to allow for more than two and possibly a continuum of valuations. Though the extension to discrete distributions with more than two valuations is straightforward but messy, the extension to a continuum of valuations is generally untractable. However, there are numerical solutions for special classes of distributions. Thus, assuming uniform distributions of valuations, one can model economies as increases in the upper end of the support and diseconomies as reductions in the upper end of the support. Our main results also extend to this specification of the model.[5]

The statements regarding the English auction format follow without qualification. Specifically, as the expected value of the prizes increase from the first to the second auction due to economies, the expected prices obtained in the auctions are declining and vice versa, so that Theorem 1 holds. Similarly, for the case of economies, the expected prices decline in the Dutch format, so a weaker version of Theorem 2 holds.

3. Recall, however, that the low valuation of $V = 0$, is simply a normalization. Hence imposing a minimum bid of v may be against the auctioneer's interest if his or her own valuation is substantially below the low valuation of the bidders.

4. See Levin and Smith (1994) and Engelbrecht-Wiggans (1993).

5. The detailed analysis of this case is in the appendix.

However, the expected prices in the case of diseconomies in the Dutch auction now unambiguously increase. Formally,

$$E\left[P_1^T\right] \gtrless E\left[P_2^T\right] \iff \Delta_T \gtrless 0, \quad T \in \{E, D\},$$

where $\Delta > 0$ comes about in the case of economies, and $\Delta < 0$ otherwise.

Regarding revenue, the English format generates more revenue than the Dutch format in the case of economies, so the first part of Theorem 4 holds without qualification. However, coinciding with the unambiguous ranking of the price sequence in the Dutch auction for the case of diseconomies, the Dutch auction generates higher revenue than the English auction under diseconomies, so that instead of the second part of Theorem 4, one has

$$\Pi^E \gtrless \Pi^D \iff \Delta_E, \Delta_D \gtrless 0.$$

Altogether, the analysis of the continuous distribution case indicates that our main results are robust, and the ambiguity of the equilibrium price sequence in the diseconomies case vanishes. In particular, economies lead to higher revenue of the English auction and to declining expected prices, regardless of the auction format and vice versa. The reason for this difference with the model studied in detail is that the asymmetric Dutch auction is no longer efficient. Consequently, the analogue of Corollary 1 does not hold.

The result that the English format generally raises more revenue in the case of economies, and that the Dutch format weakly dominates in the case of diseconomies is particularly strong. It is well known that in asymmetric auctions revenue rankings across auction formats can be reversed when changing distributional assumptions.[6] This is indeed the case here, so that in the continuous types case the opposite revenue ranking from the main model obtains. That is, the opposite of Proposition 5 holds. Nevertheless, under either specification of the auction format the value of incumbency is greater in absolute value in the English auction, and this drives the overall revenue ranking, due to the impact of the value of incumbency on the first auction bidding.

6. See, e.g., Maskin and Riley (2000).

Finally, one would also like to consider sequences of more than two auctions. However, this task is beyond the scope of the present article, already because it is not obvious how one should model the dynamics of the learning process in an economically convincing fashion.

VII. CONCLUSION

Many auctions are recurring events where bidders may have reasonable expectations to meet current opponents again at a future auction under terms that are affected by the outcome of previous auctions. In many such instances bidders who have won a previous auction experience either an increase or a decrease of their random valuation in subsequent auctions, due to complementarities, learning effects, or wear-out and organizational slack. The theme of this article is to gain insight into how this may affect bidders' strategies, equilibrium prices, and equilibrium revenues in the two most common auction formats.

We showed that stochastic economies of scale intensify competition in the first auction and result in declining expected prices. The converse, however, need not hold, even though diseconomies decrease bidding in the first auction. Indeed, whether expected prices increase or decline depends not only on specific parameter values but also on broader distributional assumptions. Finally, for all considered probability distributions of valuations, the second-price auction yields a higher expected revenue in the case of economies, whereas the converse need not hold, and revenue equivalence may be preserved in the case of diseconomies.

APPENDIX

In this appendix we derive the results that are reported in section VI. Specifically, we derive the price sequence and revenue implications of a sequential auction in which valuations are drawn from specific uniform distributions. As we have already mentioned, sequential auctions that involve an asymmetric auction in some subgame are not generally tractable. However, in the setup used here, closed-form solutions are available from Plum (1992).

The Model

Regardless of whether there are diseconomies, or economies, in the second auction one bidder's valuation is a draw from the uniform distribution on the unit interval, whereas the other bidder's valuation is an independent draw from the uniform distribution on the interval $[0,2]$.

In the case of economies the first auction valuations are independent draws from the uniform distribution on the unit interval, so that in the second auction it is the contestant's value that comes from the unit interval. In the case of diseconomies the valuations in the first auction are independent draws from the uniform distribution on the interval $[0,2]$, so that it is the incumbent's valuation that comes from the unit interval in the second auction.

Notice that the second auction is essentially the same under either scale assumption—only the labels of the incumbent and the contestant are reversed. Consequently, we first consider the second auction equilibrium in the English and then the Dutch format for the case of economies. For both formats we derive the expected revenue and value of incumbency. The case of diseconomies follows mutatis mutandis (one only needs to reverse subscripts and the sign of the value of incumbency).

The logic behind Corollary 3 applies here as well, so that the format of the first auction does not affect the expected revenue and price in the first auction. Hence, the first auction expected revenue and price are $1/3+\Delta$ in the case of economies, and $2/3+\Delta$ in the case of diseconomies, where Δ is determined by the format of the continuation game.

The Asymmetric English Auction

Bidders bid their valuation in the second auction. This yields (expected) payoffs:

$$U_I(v_I) = v_I - (\min\{v_I, 1\})/2,$$
$$U_C(v_C) = v_C^2/4.$$

Therefore, bidders' ex ante expected payoffs are

$$E[U_I(V_I)] = \int_0^1 (v_I^2/4)\,dv_I + \int_1^2 [(v_I - 0.5)/2]\,dv_I = 7/12,$$
$$E[U_C(V_C)] = \int_0^1 (v_C^2/4)\,dv_C = 1/12.$$

The value of incumbency is

$$\text{(A.1)} \qquad \Delta_E^{\text{econ}} = E[U_I(V_I)] - E[U_C(V_C)] = 1/2\left(= -\Delta_E^{\text{disecon}}\right).$$

The expected revenue of the seller is equal to the expected equilibrium price of the second auction. It is equal to

$$\text{(A.2)} \qquad E[P_2] = (1/3)(1/2) + (1/2)(1/2) = 5/12.$$

The Asymmetric Dutch Auction

The equilibrium strategies of the asymmetric Dutch auction are:

$$b_I(v_I) = v_I/\left(1+\sqrt{1+3/4v_I^2}\right), \quad \text{for } v_I \in [0,2], \quad \text{and}$$
$$b_C(v_C) = v_C/\left(1+\sqrt{1-3/4v_C^2}\right), \quad \text{for } v_C \in [0,1].$$

These strategies yield the (expected) payoffs:

$$U_I(v_I) = (v_I - b_I(v_I))\,Pr\{b_I(V_I) > b_C(V_C)\} = v_I^2/\left(\sqrt{1+3/4\,v_I^2}+1\right), \quad \text{and}$$
$$U_C(v_C) = v_C^2/\left(2\sqrt{1-3/4\,v_C^2+2}\right).$$

The corresponding ex ante expected equilibrium payoffs are

$$E[U_I(V_I)] = \int_0^2 v_I^2/\left(\sqrt{1+3/4v_I^2}+1\right)1/2\,dv_I = 2/3\sqrt{3}\ln\left(2+\sqrt{3}\right),$$
$$E[U_C(V_C)] = \int_0^1 v_C^2/\left(2\sqrt{1-3/4v_C^2}+2\right)dv_C = 1/2 - 2/9\sqrt{3}\pi.$$

This yields a value of incumbency of

$$\text{(A.3)} \qquad \Delta_D^{\text{econ}} = 2/9\sqrt{3}\left(\ln\left[26+15\sqrt{3}\right]+\pi\right) - 1/2 \approx 0.41\left(\approx -\Delta_D^{\text{disecon}}\right).$$

The associated probability distributions of equilibrium bids are

$$G_I(x) = 4x/(4+3x^2),$$
$$G_C(x) = 8x/(4-3x^2), \quad x \in [0, 2/3],$$

which yields a second auction expected revenue and price of

$$\text{(A.4)} \qquad E[P_2] = \int_0^{2/3} x\,d\,(G_I G_C) = (18+32\ln(5/6))/27 \approx 0.45.$$

Results

Recall that in the first auction the expected equilibrium revenue and price are independent of the format of the first auction. Therefore, the expected equilibrium revenue and hence price in the first auction is $1/3+\Delta$ in the case of economies, and $2/3+\Delta$ in the case of diseconomies.

Using (A1) and (A2) it follows that

$$E\left[P_1^E\right] \gtrless E\left[P_2^E\right] \iff \Delta^E \gtrless 0.$$

Therefore, Theorem 1 holds without qualification.

Regarding the Dutch format, equations (A3) and (A4), yield

$$E\left[P_1^D\right] \gtrless E\left[P_2^D\right] \iff \Delta^D \gtrless 0.$$

Therefore, the spirit of Theorem 2 is preserved. Theorem 3 is now strengthened in the sense that diseconomies now unambiguously lead to declining expected prices.

Finally, note that

$$\Pi^E \gtrless \Pi^D \iff \Delta \gtrless 0.$$

Therefore, the first part of Theorem 4 holds unconditionally, and the second part is now the converse of the first part.

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