Bidding Behavior in Multi-Unit Auctions — An Experimental Investigation*

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Abstract

We present laboratory experiments of different multi-unit auction mechanisms. Two units of a homogeneous object were auctioned off among two bidders with flat demand for two units. We test whether expected demand reduction occurs in open and sealed–bid uniform–price auctions. Revenue equivalence is tested for these auctions as well as for the Ausubel, the Vickrey and the discriminatory sealed–bid auction. Furthermore, we compare the five mechanisms with respect to the efficient allocation of the units.

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1 Introduction

Motivated by high profile auctions such as the FCC or the Treasury bill auctions, theoretical research was lately extended from single to multi-unit auctions. Friedman (1960) proposed to change the rules for the Treasury bill auctions from a discriminatory to a uniform price format which was then thought to be a generalization of the incentive compatible Vickrey auction to the multi-unit case. Recent research proved that this is not the case. Ausubel and Crampton (1998) showed that in the uniform price auction any bidder has an incentive to reduce demand on all except for the first unit since one of his bids may determine the price he has to pay for inframarginal units. Furthermore they showed that in many cases the discriminatory auction outperforms the uniform-price auction. Katzmann (1995), Noussair (1995), Engelbrecht–Wiggans and Kahn (1998), and Grimm et al. (2001) analyze auctions where bidders have demand for multiple units and give examples of equilibria that involve demand reduction.

A sealed–bid mechanism that generalizes the Vickrey auction for single units to the multi-unit case has already been presented in Vickrey (1961). It is basically a special case of the revelation mechanisms developed independently by Clarke (1971) and Grooves (1973). Ausubel (1997) proposed an open auction that implements the multi unit Vickrey auction in a way that is possibly most transparent to bidders.

In this paper we experimentally investigate bidding behavior in five different multi-unit auction formats: the discriminatory auction (DA), the uniform-price sealed–bid auction (UPS), the uniform-price open auction (UPO), the Vickrey Auction (VA) and the Ausubel Auction (AA). Our experiment consists of a series of two-unit, two bidder auctions. Bidders have a flat demand for two units. In this framework, in the most extreme case, demand reduction in equilibrium involves a zero bid on the second unit in the uniform price auctions. This implies a maximum difference between the theoretical prediction for the uniform price auction and the other auction formats in terms of revenue.

We find that revenue equivalence as predicted for the two uniform price auctions and for the non–uniform–price auctions, respectively, does not hold. The allocative efficiency is highest in the Ausubel auction. Overbidding occurs almost only in the uniform-price sealed–bid and the Vickrey auction. Demand reduction is more frequent in the uniform-price open auction than in the uniform price sealed–bid auction, but, interestingly, does also occur (though less frequently) in the Ausubel auction.

In the uniform price treatments bidders played both, UPO and UPS. Here we found that even pairs that coordinated on the payoff-dominant equilibrium that involves demand reduction in UPO, only rarely managed to do so in subsequent UPS. We observe, however, some tendency towards the payoff dominant equilibrium.

Closely related experiments were run by Kagel and Levin (2001) and List and Lucking–Reiley (2000). Kagel and Levin compare uniform–price sealed–bid and open auctions and the Ausubel auction and find systematic demand reduction in the uniform–price auctions. Their subjects also have flat demand for two units but bid against robot bidders with unit demand. List and Lucking–Reiley conduct field experiments, comparing the uniform–price sealed–bid and the Vickrey sealed–bid auction by selling sportscards in two–unit, two–person auctions. They also find demand reduction in uniform–price auctions, compared to Vickrey auctions. They cannot, however, control for the bidders' valuations.

2 Theoretical Background and Hypotheses

2.1 Equilibrium Analysis

We investigate bidding behavior in independent private value auctions with two bidders and two indivisible identical objects for sale. Each bidder demands at most two units. A bidder places the same value v_i on each unit. The bidders' valuations are drawn independently from the same uniform distribution on the interval $[0, V]$.

We consider five different auction formats. In the three sealed–bid auctions the bidders simultaneously submit sealed–bids for each of the units demanded and prices and allocations are determined according to the auction rules. The two open auctions start out with a price of zero and active bids on all units demanded. The price is increased and units are traded according to the rules of the mechanism as bidders drop out. In all auctions the two highest bids each win a unit.

Uniform–Price Sealed–Bid Auction [UPS] and Uniform–Price Open Auction [UPO]

In the uniform price auctions the price for all units equals the highest rejected bid. In our experiment, this is the third highest bid. In the uniform price sealed–bid auction, each bidder places two bids and the units are allocated to the two highest bids (or randomly in case of a draw). The uniform-price open auction starts out with a price of zero, with the price increasing continuously thereafter. Bidders start out actively bidding on two units each and may choose the price(s) where they drop out on one unit, or on both. Dropping out is irrevocable so that a bidder can no longer bid on a unit he has dropped out on. As soon as the number of active bids equals the number of units available, both items are sold to the bidder(s) holding the active bids at the price where the last bidder dropped out. Thus, the price is determined either by a second dropout of a player on one unit or by a player's simultaneous dropout on both units.

In both uniform price formats it is a weakly dominant strategy to bid one's valuation v_i on the first unit (i.e. the higher bid always equals the true valuation). A bid on the first unit will only determine the price if it is the highest rejected bid, i.e. if the bidder does not get a unit. Therefore, lowering the bid implies the risk of missing a profitable deal whereas overbidding might result in buying a unit at a loss. This is even more obvious in the open auction. If a player has already dropped out on one unit, dropping out on the other unit before the valuation is reached guarantees a profit of 0, whereas continuing might yield a positive profit, if the other bidder drops before the own valuation is reached. Staying in above the own valuation causes a loss as soon as the other bidder drops out.

However, lowering the bid on the second unit presents a trade-off. A lower bid on the second unit lowers the chance of winning two units but, at the same time, may reduce the price paid for the first unit. As it turns out, the uniform-price auctions have multiple equilibria. All equilibria that do not involve truthful bidding on the first unit are weakly dominated. Among those equilibria that involve truthful bidding on the first unit the following are the extreme cases: Truthful revelation on both units,

$$b_1(v_i) = b_2(v_i) = v_i \tag{1}$$

and full demand reduction on the second unit such that the bids on the second unit are zero,

$$\begin{aligned} b_1(v_i) &= v_i \\ b_2(v_i) &= 0. \end{aligned} \tag{2}$$

In the following we will refer to these equilibria as the truth-telling (TT) and the demand reduction (DR) equilibrium, respectively.

The remaining equilibria in undominated strategies are of the following form: Let (x_k, y_k), $k = 1, \ldots K$ be a sequence of non–overlapping intervals with $x_k \geq 0$, $y_K \leq V$, $x_k \leq y_k$, and $y_k \leq x_{k+1}$. Then, the equilibrium strategies are:

$$\begin{aligned} b_1(v_i) &= v_i \\ b_2(v_i) &= \begin{cases} x_k & \text{if} \quad v_i \in [x_k, y_k], \\ v_i & \text{otherwise.} \end{cases} \end{aligned} \tag{3}$$

This implies that a bidder bids truthfully if his valuation lies in a truthtelling interval $[y_k, x_{k+1}]$ and partially reduces demand if his valuation lies in a demand reduction interval $[x_k, y_k]$.

For the sealed–bid auction we show in appendix A.1 that it does not pay to deviate from this strategy given the other player plays it. For the open auction, the strategy is sequentially rational if player i's beliefs have the following form:[1]

(a) If the other player drops out on one unit at $z_k \in [x_k, y_k]$ his valuation is uniformly distributed[2] in the interval $[z_k, y_k]$,

(b) If the other player drops out on one unit at $z_k \in [y_k, x_{k+1}]$ his valuation is uniformly distributed in the interval $[z_k, \min\{x_{k+1}, v_i\}]$,

(c) The other player's valuation is always lower than one's own valuation if he already has dropped out on one unit.

(d) The players follow Bayes' rule given (a), (b), and the ex ante objective probability distribution of types.

It seems, however, highly unlikely that players can coordinate on such a more sophisticated equilibrium. There is no particular incentive to do so and they are more difficult to figure out than the TT– and the DR–equilibrium. Note that the latter are extreme cases in the sense that DR requires $K = 1$, $x_1 = 0$, and $y_1 = 1$ and TT results for $K = 0$ or for $x_k = y_k \; \forall k =, \ldots, K$. Furthermore, the DR equilibrium is the only Perfect Bayesian equilibrium of the open auction if the beliefs strictly follow Bayes' rule also off the equilibrium path (that is, if a bidder observes a dropout he infers only that the opponent's valuation is higher than the dropout price and updates the initial distribution accordingly). In particular, such beliefs imply that whenever one bidder drops out on one unit, the other immediately follows (see appendix A.4).

All other equilibria of the open auction require that the players believe to be able to infer information from the other bidders actions (off the equilibrium path) that exceeds the minimal requirement that players only play undominated strategies. Moreover, any equilibrium that involves bidding truthfully on the second unit requires type dependent beliefs. Intuitively, such equilibria may not seem completely implausible: If, for example $K = 1$, $x_k = 0$, and $y_k = V/2$ equilibrium beliefs would be that only players with low valuations drop out early, whereas players with high valuations always behave rather competitively. However, to make truthful bidding above $V/2$ an equilibrium strategy, beliefs have to depend on the own type (i. e. valuation), as in (b) and (c) above.

We show in appendix A.3 that among all equilibria of the uniform price auction the DR equilibrium yields the highest expected payoff to the bidders.

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[1] See also appendix A.2 for a detailed analysis.

[2] The uniform distribution is only one example that implies that the suggested equilibrium strategy is indeed a best reply.

Discriminatory Auction [DA]

In the discriminatory auction, the two highest bids win a unit each and the respective prices equal these bids.

An important observation in order to derive the optimal strategy is that with flat demand a bidder places the same bid on both units.[3] Suppose the other bidder places two different bids. Then, in order to win one unit a bidder has to overbid only the other bidder's lower bid and in order to get two units both his bids have to exceed the other bidder's higher bid. Therefore, a bid on the first unit solves the optimal tradeoff between the probability of winning (against the other bidder's lower bid) and profit in this case. Now observe that the probability of winning the second unit is even lower (one has to overbid the other bidder's higher bid) and therefore, the optimal tradeoff for the second unit cannot be solved at a lower bid. Thus, both bids will be equal since by definition the bid for the second unit cannot be higher than the bid for the first unit. If the other bidder chooses identical bids, the argument is even more obvious, since the tradeoff is the same for both units.

Thus, the equilibrium bid function on each unit solves

$$\max_b F(\sigma(b))[v_i - b(v_i)], \tag{4}$$

where $\sigma(b)$ is the inverse of the equilibrium strategy $b^\star(v)$. In the case of uniform distributed valuations on $[0, V]$ and two bidders the equilibrium bid functions are

$$b_1^\star(v_i) = b_2^\star(v_i) = \frac{1}{2}v_i. \tag{5}$$

Vickrey Auction [VA]

In the multi unit generalization of the Vickrey auction the price a bidder pays for a unit is determined by the bid (other than his own) that is displaced by his successful bid. In our framework this means that, if one player places the two highest bids, he pays the two bids of the other player since his lower bid displaces the higher bid of the other player and his higher bid displaces the lower bid of the other player. If each player placed one of the two highest bids, each pays the lower bid of the other player because his higher bid displaces the lower bid of the other player.

Thus, a bidder cannot influence the price he pays for any unit he obtains by changing his bids. Changing an unsuccessful bid will have no effect unless if this bid is increased and displaces another bid. In that case one obtains another unit and pays the displaced bid. This, of course,

[3] See Lebrun and Tremblay (2000) for a formal proof of this fact for much more general demand functions.

increases profits if and only if the displaced bid is lower than the own valuation v_i. Thus it is clearly weakly dominated to bid below v_i. But bidding above v_i on any unit is also dominated, since this might displace a bid that is also higher than v_i and hence cause a loss. Therefore, each bidder has the weakly dominant strategy to bid truthfully on both units. (For the general case see also Vickrey, 1961.)

Ausubel Auction [AA]

The Ausubel (or dynamic Vickrey) auction (Ausubel, 1997) is an open mechanism that implements the multi-unit Vickrey auction in a way that has the greatest potential for transparency to bidders. The auction starts out with a price of zero and then increases continuously. In the general case, at any price it is checked for each bidder whether the aggregate demand of the other bidders is smaller than the available number of units. If this is the case, he receives the available units at the current price.

In our case, the price is raised until one bidder (say, bidder i) drops out on one unit. At this point bidder j gets one unit for sure (in other words: he "clinched" one unit). This unit is traded immediately and bidder j pays the price at which he has clinched the unit. Then the auction continues at this price for the remaining item that is still unsold. From now on the two bidders are involved in a single-object English clock auction.

Under these rules the bidders have an incentive for full demand revelation on both units since the price paid for the first unit does not affect the price paid on the other unit. Thus,

$$b_1(v_i) = b_2(v_i) = v_i. \tag{6}$$

This equilibrium is obtained by iterated elimination of weakly dominated strategies. If one bidder has dropped out it is weakly dominated to drop out at a price other than v_i, since the drop out price only determines the price for the remaining unit and one can only lose by staying in above v_i and can miss a possible gain by dropping out before v_i is reached. Eliminating these strategies then yields that the price of the first drop out does not influence the result of the subsequent bidding process. Hence it is also weakly dominated to drop out on the first unit at a price other than v_i since this drop out price only determines the price for this unit. To make not dropping out at a price lower than v_i optimal requires, however, knowing that the other player will not play a dominated strategy (e.g. will not drop out immediately after). Hence the equilibrium is not in weakly dominant strategies, but the mechanism is only dominant solvable. The solution concept is thus weaker than in VA.

In contrast the mechanism might be more transparent. This leads to a potential trade off between transparency and the strength of the solution concept (see also Kagel et al., 2001).

2.2 Hypotheses Derived from the Theory

The theoretical analysis gives us several hypotheses to test. First, we expect to observe demand reduction on second unit bids in the uniform price auctions, whereas there should be no demand reduction in the discriminatory, the Vickrey and the Ausubel auction. Second, more generally, the bids on both units should be equal in the discriminatory, the Vickrey, and the Ausubel auction. Third, first unit bids in the two uniform-price auctions, the Vickrey and the Ausubel auction should equal the valuation. Fourth, revenues are expected to be significantly lower in the uniform price auctions than in the other three auctions. Revenues in the discriminatory, the Vickrey and the Ausubel auction are theoretically equivalent in our setting. In DA the price for both units is $\frac{1}{2}\max\{v_i, v_j\}$ and $E[\max\{v_i, v_j\}] = \frac{2}{3}V$ and in AA and VA the price is $\min\{v_i, v_j\}$ and $E[\min\{v_i, v_j\}] = \frac{1}{3}V$, so that the expected revenue is $\frac{2}{3}V$ in both cases. In contrast the expected revenue is 0 in the uniform-price auctions (if the DR–equilibrium is played).

Furthermore we have an equilibrium selection problem in the uniform-price auctions. These auctions have several equilibria, one of which payoff dominates the others from the bidders' viewpoint. This equilibrium involves a zero bid on the second unit. In the UPO this is the only Perfect Bayesian equilibrium where beliefs always strictly follow Bayesian updating. From these considerations two interesting questions arise: Do the bidders select the equilibrium that guarantees them the higher payoff in UPS and, do the bidders select this equilibrium more often if they played UPO before playing UPS?

3 Experimental Design

In each auction two units of a homogeneous object were auctioned off among two bidders with flat demand for two units. The choice to auction off two units per auction yields a simple payoff-dominant equilibrium in the case of uniform price auctions, as described above. This creates the most significant difference between equilibrium bidding in the uniform price auctions and the discriminatory, Vickrey, and Ausubel auctions. In each auction the bidders' private valuations for both units were drawn independently from the same uniform distribution on $[0, 100]$.[4] The bid-

[4]Valuations were in fact drawn from the set of integers in [0,100] and also bids were restricted to integers. While this restriction might be problematic in general, it does

ders were undergraduate students from Humboldt University Berlin, the University of Zürich, and the ETH Zürich. Pairs of bidders were randomly formed. In DA, VA and AA each pair played ten auctions under the same rules. In the uniform price auctions, in treatment UPOS each pair first played ten open auctions and then ten sealed–bid, in treatment UPSO vice versa.[5] Apart from that, in each session only one type of auction was conducted. For each treatment we had ten pairs, except for treatment DA, where we had nine.

Subjects were placed at isolated computer terminals, so that they could not determine whom they formed a pair with. Then the instructions (see appendix B) were read aloud. Before the start of a sequence of ten auctions, subjects played three dry runs, where they knew that their partner was simulated by a pre-programmed strategy. These strategies and the valuations of the subjects in the three dry runs were chosen such that it was likely that each subject was exposed to the results to win 0 units in one auction, 1 unit in another and 2 units in the third. The pre-programmed strategies did not reflect any characteristics of the equilibria (in particular complete demand reduction in the uniform price auctions) and the subjects were explicitly advised that they should not see these strategies as examples of a good or a bad strategy (because they only observed the bids, they could not really copy the programmed strategy anyhow). In the uniform price sessions subjects were informed that after the first ten auctions, ten further auctions under a different rule would be conducted, without explaining details at that point. After all pairs had finished the first ten auctions, the instructions for the second part were again read aloud. After each auction subjects were informed about their partners' bids, their own gains or losses and their own total profits so far.

In the open auction formats the price stayed at 0 for four seconds and then increased at a rate of 1 per second. Bidders could drop out on one or both (if no bidder had dropped out before) units at any time. After one player dropped out on one unit and the other player was informed about this, the price stayed at the drop-out level for four seconds and increased at a rate of 1 per second thereafter. If a bidder dropped out during these four seconds, the drop-out is regarded as at the same price but later than the first drop-out. At any time during the bidding process, the bidders could observe the current price, the number of items for sale and the number of active bids. If there were more than two active bids when the price raised above the maximum price of 100, then in UPO

not have any influence on the equilibria in our design, primarily because the grid is sufficiently fine.

[5] This is why we only played ten auctions per pair and auction type. We wanted the total number of periods not to exceed 20 to avoid subjects getting bored. Also we wanted to keep the incentives in each auction relatively high with a limited budget.

both bidders received one unit for a price of 100. In AA both bidders received one unit for a price of 100 if both still had two active bids, while the remaining unit was randomly allocated if one bidder had dropped out on one unit before. The sealed–bid were run in a straightforward way, i. e. both bidders simultaneously placed two bids. Subjects were informed that the order of the bids was irrelevant.

After each auction bidders were informed about the observed dropout prices, or all four bids in the open auctions and the sealed–bid auctions, respectively, as well as the resulting allocation, their profits and their aggregate profits.

The experimental software was developed in zTree (Fischbacher, 1999). The sessions lasted for about 60 to 80 minutes in the uniform price auctions and for about 30 to 50 minutes in the other treatments. At the end of each session, experimental currency units were exchanged in real currency at a rate of DM .04 (Berlin) or CHF .04 (Zürich) per ECU. Average earnings were DM 15.95 in AA, DM 12.60 in DA, DM 14.85 in VA, DM 32,73 in UPOS, and DM 30,10 in UPSO. In addition subjects received DM 5 (Berlin) or CHF10 (Zürich) as showed-up fee.

4 Experimental Results

Let us first define some useful terminology for describing observed behavior. The equilibria of the different auctions have some very basic common properties. First, it holds for all auction formats that overbidding one's valuation is always (weakly) dominated. In the uniform price auctions, bidding the valuation on the first unit is a weakly dominant strategy. Thus, any bidding behavior that at least resembles equilibrium behavior involves bidding one's valuation on the first unit and something between 0 and the valuation on the second unit. If subjects do so we call this *equilibrium–like* behavior.[6]

A common property of the equilibria of VA, AA and DA is that bids on both units must be equal. In the Vickrey auction bidding one's valuation on both units is a weakly dominant strategy. In the Ausubel Auction the same equilibrium strategy is derived by iterated elimination of weakly dominated strategies.

In treatments UPOS and UPSO the subjects played both uniform price auctions in sequence. For the general comparison of all five auctions we only consider the first set of auctions out of these sessions (denoted by UPO and UPS) since behavior in the second set of auctions is not independent of what happened in the first one. We analyze behavior in the

[6]Such a strategy would indeed be part of an equilibrium if the other player played accordingly.

second set of auctions (denoted by UPsO and UPoS) separately in subsection 4.7, looking at whether bidders learn to play the DR–equilibrium in the sealed–bid auction if they played the open version first.

4.1 Demand Reduction in UPO/UPS

In UPO behavior is consistent with *equilibrium–like* behavior in about half of the cases, and in about a third of the cases in UPS. Four of the pairs in UPO played almost the (DR–) equilibrium strategy, while the other pairs either bid roughly consistent with the TT–equilibrium or don't seem to have found a reasonable strategy. Figures 2.3 and 2.4 show dropout prices in UPO. "Double dropouts" are simultaneous dropouts of one bidder on both units. "First dropouts" are the first dropout of a bidder on a unit while "second dropouts" refer to the second drop-out in one auction, i. e. the price where the auction ends, not necessarily to the second drop-out of one player. Figure 2.4 shows that in four pairs both bidders almost always dropped out with one unit at price 0, independent of the valuation.[7] In contrast, Figure 2.3 depicts the overall behavior.

Figures 1.1 and 1.2 show the bids in UPS, where "unit1 bids" refers to the (weakly) higher, and "unit2 bids" to the (weakly) lower bid of a bidder. In UPS behavior was sometimes consistent with *equilibrium–like* behavior and sometimes not for each of the pairs. Only one subject consistently chose the TT–equilibrium strategy, which was, however, not part of an equilibrium either since the other subject was underbidding on the second unit most of the time.

In the open auction, one can observe whether bidders play according to the requirement of the DR equilibrium to drop out on one unit immediately once the other bidder dropped out. Bidders violated this requirement in 55 % of the cases where it was possible, i.e. they did not drop out immediately following the other bidder's drop-out.

4.2 Equality of Bids and Bid Spreads

According to the equilibrium prediction, in AA, VA, and DA the bidders should place equal bids on both units. We study the deviation from this prediction and also compare it to the bid spreading in UPS, where it is consistent with equilibrium behavior.

[7] We include only the observed bids in the Figures for the open auctions. For the unobserved bids, a lower threshold is given by the price were the auction ended. Hence the figures for the open auctions should not be directly compared to those for the sealed–bid auctions, because the latter show all bids, whereas the former show only the two lowest bids in each auction.

maxbid-minbid	UPO	UPS	AA	VA	DA
= 0	27%*	18%	64%*	49%	12%
< 10% Equ.	43%*	34%	68%*	62%	15%
≥ 40% Equ.	32%*	33%	18%*	14%	49%

Table 1: Share of bid pairs (*: of observable bid pairs) that are exactly equal, where the difference is smaller than 10, or larger than 40 percent.

AA Five pairs play almost exactly according to the equilibrium prediction, i. e. double dropouts at the valuation (see Figure 2.2). In some pairs bidders tried to cooperate by bidding 0 on both units if the valuation was relatively low (see the double dropouts at zero in Figure 2.1), or by using strategies that resembled the DR equilibrium strategies in a uniform price auction. These attempts to cooperate were in general not successful and were abandoned after some auctions.

In about half of the cases we can observe (or infer lower or upper bounds for) the difference between maximal and minimal bids in AA. In 64% of decidable cases the bids are exactly equal, in 68% the difference is smaller than 10% of the equilibrium bid (i.e. the valuation) and in only 18% of the cases the difference is greater or equal to 40% of the valuation (see Table 1). Most unobservable cases are those where the other bidder dropped out on two units, hence there is no indication that the undecidable cases correspond to large bid spreads, just the contrary.

VA Though only two out of ten pairs are very close to the equilibrium in VA, eight subjects play close to the weakly dominant strategy to place bids equal to the valuation for both units. The first and second unit bids are show in figures 1.3 and 1.4, respectively.

Similarly to AA, in VA most of the bid differences (62%) are smaller than 10% of the equilibrium bid (i.e. valuation), with 49% of bid pairs being exactly equal. In only 14% of cases the difference exceeds 40% of the valuation. The bid differences are thus substantially smaller than in DA and UPS (see Table 1). A linear regression of the bidspread yields, over all subjects and periods (with robust standard errors taking the dependence within a pair into account) a significantly ($p < .1$) negative coefficient ($-.87$) for period. Hence the bid spreading in VA clearly decreases over time. Analyzing the data for the individual bidders shows 4 (out of 20) bidders who place equal bids in all 10 auctions. According to a Kolmogorov-Smirnov test the hypothesis that both the higher and the lower bids (relative to equilibrium bids) are drawn from the same distribution can be rejected at the 5%-level for only 4 bidders (and for 6 bidders according to a Mann-Whitney test).

Estimating bid functions that are linear in the valuation yields over all subjects very similar results for the higher (coefficient for valuation .859, constant 7.74) and the lower (coefficient for valuation .826, constant 3.09) bid. For 10 subjects the coefficient for the valuation is within 10% deviation of the equilibrium prediction (i.e. 1) for the higher bid and for the lower bid for 11 subjects, for 9 respectively 8 even within 5%.

DA Bidders did not choose equal bids in DA. In only 12% of cases the bids were exactly equal and in only 15% (including the 12% equal bids) the difference was smaller than 10% of the equilibrium bid (i.e. 5% of the valuation, see Table 1). About half of these nearly equal bids were submitted by only two subjects. 49% of the bid spreads are larger than or equal to 40% of the equilibrium bid. Thus bid differences are even larger than in UPS, where this is implied by demand reduction. This is also vividly illustrated by comparing Figures 1.5 and 1.6. As shown by Figure 1.5, the majority of the first unit (i.e. higher) bids lies between the valuation and the equilibrium bid (valuation divided by 2), whereas a large share of the second unit (i.e. lower) bids lies substantially below the equilibrium bid. As can also be seen in Figures 1.5 and 1.6, except for one subject in one auction, we observe overbidding only for very small valuations and to a very small degree. It seems that it is obvious to bidders in DA that overbidding is dominated.

A linear regression of the bidspread yields, over all subjects and periods (with robust standard errors) a negative coefficient ($-.05$) for period, which is, however, not significantly smaller than 0 ($p > .8$). Hence the bidspread on average decreases over time, but the effect is very small and insignificant. Analyzing the data for the individual bidders, a Kolmogorov-Smirnov test shows that the hypothesis that both the higher and the lower bids (relative to equilibrium bids) are drawn from the same distribution can be rejected at the 5%-level for 12 out of 18 bidders (and for 14 bidders according to a Mann-Whitney test). Hence bid spreading is clearly more prominent in DA than in VA, as is also revealed by a Mann-Whitney test comparing the average bid spreads (relative to equilibrium bids) of the individual subjects ($p < .001$). The same test shows that relative bid spreads in DA are even significantly larger than in UPS ($p < .05$).

These results are supported by estimating bid functions that are linear in the valuation. Over all subjects, when estimating the function for the higher bid the coefficient for the valuation is with .516 close to the equilibrium value of .5, while it is substantially smaller for the lower bid (.379). For the higher bid the coefficient for the valuation is within 10% deviation of the equilibrium prediction only for 7 out of 18 subjects and for the lower bid for only 5 subjects.

T-tests over all subjects and periods (with robust standard errors) show that the average higher bid is significantly higher ($p < .05$) than the equilibrium bid (difference 5.48). 11 (out of 18) subjects bid significantly ($p < .05$) above the equilibrium, 1 below. The average lower bid is 3.73 smaller than the equilibrium bid ($p < .1$). 2 subjects bid above equilibrium, 6 below ($p < .05$).

At a first glance, a possible explanation for the bidspreading seems to be risk aversion. However, for constant or increasing absolute risk aversion a lower bid on the second unit cannot be an equilibrium strategy since the lower bid competes with the higher bid of the other player, which rather implies bidding higher than lower on the second unit. Strongly decreasing risk aversion could explain the observed behavior in a single auction but would imply that bids decrease over time, which they do not. In addition second units below the risk-neutral equilibrium strategy ($b = \frac{v}{2}$) cannot be consistent with any form of risk aversion, because risk aversion always predicts bidding higher than the risk-neutral equilibrium on both units.[8] The data are consistent with some statements in the questionnaires that suggest that subjects were placing one bid as if they were highly risk averse and the other as if they were risk seeking ("a high secure bid and a lower bid that could yield a higher profit"). This seems to describe a highly myopic zero-profit aversion (since subjects try to secure a unit in each single auction). This behavior does not only lead to first bids being substantially higher than the equilibrium bid, but also to average bids above equilibrium.

UPS In UPS, 34% of bid spreads are below 10% of the valuation (including 18% of the pairs that are exactly equal). Only 33% of bid differences exceed 40% of the valuation (see Table 1). A Kolmogorov-Smirnov test applied to the individual bidders reveals that the hypothesis that both the higher and the lower bids (relative to equilibrium bids) are drawn from the same distribution can be rejected at the 5%-level for 13 out of 20 bidders (and for 14 bidders according to a Mann-Whitney test). A linear regression of the bidspread yields, over all subjects and periods (with robust standard errors taking the dependence within a pair into account) an insignificantly ($p > .5$) negative coefficient (-.23) for period. Hence the bid spreading in UPS slightly decreases over time (where the DR–equilibrium predicts bid spreading).

In UPoS, the sealed–bid auction played after the open auction, the

[8]Furthermore, according to Rabin (2000) and Rabin and Thaler (2001) risk aversion on such small stakes cannot be reconciled with the maximization of the expected utility of wealth. (According to Cox and Sadiraj, 2001, however, it is consistent with the maximization of the expected utility of income). Since no losses are involved (at least as long as the own valuation is not overbid), the explanation of myopic loss aversion given by Rabin and Thaler for low stake risk aversion does not explain our results either.

linear regression yields an insignificant positive coefficient (.52) for period. Hence the bid spread is increasing over time, approaching the DR–equilibrium prediction. According to a Kolmogorov-Smirnov test the hypothesis that both the higher and the lower bids (relative to equilibrium bids) are drawn from the same distribution can be rejected at the 5%-level for 14 out of 20 bidders (and for 15 bidders according to a Mann-Whitney test). The average bid spread in UPoS (21.63) is larger than in UPS (18.22). A two-sample t-test shows, however, that bid spread in UPoS is not significantly larger than in UPS ($p > .1$). A Mann Whitney test applied to the relative bid-spreads shows no significant difference ($p > .1$) either. Hence there is only very weak support for the hypothesis that bid spreading would be larger in UPoS than in UPS.

For UPS estimating bid functions linear in the valuation yields over all subjects a coefficient for the valuation of .83 for the higher bid and .63 for the lower bid. For 6 subjects the coefficient of valuation is for the higher bid within 10% deviation of the equilibrium prediction. While in UPoS the coefficient for the valuation is for the higher bid with .73 even further from the equilibrium prediction than in UPS, that for the minimal bid is with .43 closer to the DR–equilibrium prediction. For 8 subjects the coefficient of valuation for the higher bid is within 10% deviation of the equilibrium prediction.

4.3 Are First-Unit Bids Truthful in VA, UPO, UPS, and AA?

In AA most of the observed first unit bids are truthful except for pair 4 where one of the bidders tried to cooperate by quitting on both units immediately and then expecting the other bidder to do the same in the next round (see the double dropouts at 0 in Figure 2.1). Of 83 observable first-unit bids, 53% are exactly equal to the valuation and another 18% just one ECU above or below. Overbidding is very rare (9 bids that exceed the valuation by more than 1 ECU), probably because it is easy to see for bidders that it is dominated.

In UPO we only have few cases where we can observe both bids (56 out of 200 possible cases). If the bidders play the DR–equilibrium, they both drop out immediately and thus, we do not know what they would bid on the first unit. However, in half of the cases the outcome matches *equilibrium–like* behavior in the sense stated above (30.4% of first unit bids are exactly equal to the valuation and an additional 21.4% are one ECU above or below). Truthful bidding is more frequent in UPsO (out of 59 observable first-unit bids, 45.8% are exactly equal to the valuation and an additional 25.4% are one ECU above or below.) According to t-tests (with robust standard errors), relative underbidding on the first unit (where the first unit bids are observable) is not significantly different from 0 in any of the open auctions (AA, UPO, UPsO). A Mann-Whitney test reveals that relative underbidding is significantly larger in UPO and UPsO than in AA but does not significantly differ between UPO and UPsO (5% -level). Overbidding occurs even less than in AA.

In VA a high fraction of first unit bids is at the valuation (29.5% of bids are exactly equal to the valuation and an additional 10.5% are one ECU above or below), as can be seen in Figure 1.3. The average bid on the first unit exceeds the valuation by only .78 ($p > .7$, t-test with robust standard errors), 6 subjects significantly overbid (t-test, $p < .05$), 2 underbid. The average bid on the second unit is 5.49 below the valuation ($p > .11$), with 4 subjects significantly underbidding. 2 subjects bid exactly the valuation on both units in all auctions.

In UPS bidders frequently over- or underbid their valuations on the first unit, where overbidding is substantial, as can be seen in Figure 1.1. Bidders bid truthfully on the first unit in about a third of the cases (21.5% of bids are exactly equal to the valuation and an additional 13% are one ECU above or below), they either play the TT– or the DR–equilibrium strategy or something in between. Average overbidding on the first unit (5.55) just fails to be significant ($p \approx .103$, t-test with robust standard errors), with 6 subjects significantly overbidding and 5 subjects significantly underbidding. Overbidding even increases (insignificantly) over time. In contrast, in UPoS (compare Figures 3.1 and 3.3) the average first-unit bid is 5.02 below the equilibrium ($p > .3$), with 5 subjects significantly overbidding and 8 subjects significantly underbidding. A regression of the difference between the higher bid and the valuation shows that underbidding decreases significantly ($p < .1$) over time. One subject in UPS and two in UPoS bid their valuation on the first unit in all auctions. In total, even fewer first unit bids are truthful in UPoS than in UPS (20.5% of bids are exactly equal to the valuation and an additional 10.5% are one ECU above or below).

4.4 Allocative Efficiency

In equilibrium both units are allocated to the bidder with the higher valuation in AA, VA, and DA, but only one unit in the DR equilibrium of the uniform-price auctions. An efficient allocation requires to allocate both units to the bidder with the higher valuation, because independent of the price this maximizes social welfare, the sum of the bidders' profits and the auctioneer's revenue.[9] With allocative efficiency we hence refer

[9]This notion of efficiency is common in experimental auctions. It corresponds to non-experimental auctions, where the valuation represents, for example, an intrinsic valuation of the buyer for the good (in case of pieces of art) or prospective profits in a market (in case of licenses). In the experiment, however, both the valuation and the prices paid correspond to transfers between experimenter and subjects. Hence the total payoff (including the experimenter) is constant. This would not be reason to

simply to the number of efficiently (i.e. to the bidder with the higher valuation) allocated units. In the experiment in AA 84 % of the units are allocated efficiently, in VA 82,5 %, in DA 83,3 %, in UPO 74 % and in UPS 81 % (see also Table 4).[10] Thus in UPS the allocative efficiency is only slightly below that in AA, DA, and VA, although the predicted allocative efficiency in the DR–equilibrium is only half of that predicted in the three other auctions. In each of AA, VA, and UPS for exactly one pair all units are allocated efficiently. The low allocative efficiency in UPO is due to the coordination of some pairs on the DR–equilibrium.

Measuring allocative efficiency does not reflect the actual efficiency losses due to misallocations. If the "wrong" bidder obtains a unit, his valuation may be substantially or only slightly below the other bidder's valuation, causing either dramatic or small welfare losses. An alternative measure of efficiency is obtained by comparing the valuation for obtained units to the maximal possible valuation. This yields that the relative efficiency in DA is significantly higher than in VA (Mann-Whitney test based on aggregated data for each pair, $p < .1$) and UPO ($p < .05$) and that the relative efficiency in AA is significantly higher than in UPO ($p < .05$). A further measure is the efficiency loss (in terms of total welfare) relative to the maximal possible efficiency loss, i. e. the maximum that could be lost due to misallocation. This yields significantly higher relative efficiency losses for UPO than for AA and DA (Mann-Whitney, $p < .1$). These measures, that aim at minimizing the potential distortions that could result from the randomly chosen valuations, thus confirm the results obtained by measuring the efficiently allocated units.

To maximize efficiency thus neither open auctions nor non–uniform–price auctions are preferable in general. The allocative efficiency is highest in AA, though with only a marginal advantage compared to DA and VA. Looking at the individual decisions, however, reveals that the effi-

worry, if experimental subjects did not care for efficiency. There is, however, evidence that experimental subjects directly care for efficiency and not only for their own payoff and the fairness of the allocation (see Charness and Rabin, 2001, and Engelmann and Strobel, 2002). It then matters, whether the subjects include the experimenter in their considerations. If not (which is usually assumed), the efficiency measure corresponds to the joint payoff of the bidders, i. e. $(v_1 - p_1) + (v_2 - p_2)$ (with v_i being the valuation of the bidder who obtained unit i, and p_i the price paid for this unit), whereas the standard measure of efficiency simply amounts to $v_1 + v_2$. This could lead efficiency minded subjects to cooperate in the experiment (causing efficiency losses according to our standard measure), but not outside the laboratory. If they include the experimenter, efficiency concerns are irrelevant since the total payoff is constant, unless subjects assume that they have higher marginal utility from the payoff then the sponsor of the experiment, or if they also consider the experimenters' future salary increases due to ground-breaking publications. In the latter case, they should try to produce interesting results.

[10]According to a Mann-Whitney Test (using the share of efficiently allocated units in each pair as observations) none of the differences is significant.

ciency losses in DA are more robust than in AA. Misallocations in DA are primarily caused by bid spreading, a robust effect in most pairs. In contrast, misallocations in AA are caused by three phenomena. First, by two subjects who did not seem to follow a clear strategy, second by situations of minimal differences between valuations (and thus minimal efficiency losses), and third by attempts to cooperate, which tend to disappear over time. Hence the causes of misallocations in AA are either not robust or imply small efficiency losses. Thus based on the experimental behavior rather than the experimental allocation, AA seems to be preferable to DA in terms of efficiency, with VA somewhere in between.

The experimental behavior also suggest that increasing the number of bidders would probably result in an advantage for AA over DA. On the one hand, underbidding in attempts to cooperate is likely to decrease in AA for more than two bidders. On the other hand, given the bid spreading in DA (which might prevail also for more players), the probability that first unit bids of bidders with low valuation are higher than second unit bids of bidders with high valuation and hence for misallocations becomes substantial for higher numbers of bidders. The latter effect might also occur if valuations are drawn from a rougher grid.

Since DR is in line with equilibrium behavior in UPO, but is only an attempt to cooperate in AA, it seems likely that with random matching DR will occur less in the latter. In UPO, in contrast, under random matching, one bidder might teach a series of other bidders the DR–equilibrium. Hence we believe that the advantage of AA over UPO with respect to efficiency would be larger under random matching than under fixed matching, so that the fixed matching employed in our experiment is a tougher test for AA.

4.5 Auctioneer's Revenues

The expected equilibrium revenues of the auctioneer are equal in VA, DA and AA. The empirical (see Table 2.1) revenues in AA range from 44 % to 115 % of the equilibrium revenues in the individual pairs with 84,74 % over all pairs. In VA empirical revenues are between 41 % and 131 % with 95,58 % over all pairs. In contrast, in DA the empirical revenues reach between 83 % and 145 % of the equilibrium revenues in the individual pairs and 110,72 % over all pairs. The difference in relative revenues between AA and DA is significant both according to a t-test and a Mann-Whitney test ($p < .05$). The revenues in DA are larger than in equilibrium (Mann-Whitney, $p < .05$), those in AA are slightly smaller (t-test, $p < .1$). Thus our results are clearly in contrast to the predicted revenue equivalence.

In the uniform price auctions the (DR–) equilibrium revenues are 0. The empirical revenues are naturally higher. To compare the revenues in the two uniform price auctions, we measure the revenues relative to the

TT–equilibrium revenues (which correspond to the expected equilibrium revenues in the other auctions). In UPO the revenues range from 1 % to 100 % of the (TT–) equilibrium revenues, with 55,28 % over all pairs. In UPS they range from 89 % to 161 % and reach 106,74 % over all pairs. The difference is clearly significant (t-test or Mann-Whitney test, $p < .01$). Furthermore, the revenues are significantly smaller than in equilibrium in UPO (t-test or Mann-Whitney test, $p < .01$) but not significantly different in UPS. Thus revenue equivalence does not hold for the two uniform price auctions either.

In line with the equilibrium prediction the relative revenues in UPO are significantly lower than in VA, AA, and DA (t-test or Mann-Whitney, $p < .05$), but in contrast to the equilibrium prediction, the relative revenues in UPS are even slightly larger than those in AA (t-test, $p < .1$).

4.6 Bidder Payoffs

The pair that plays closest to the (DR–) equilibrium in UPO naturally receives almost the (DR–) equilibrium payoff (see Table 3.1, UPO, pair 4), while the other pairs and all pairs in UPS obtain payoffs substantially below the equilibrium payoff in most of the auctions. In some auctions, however, the latter pairs obtain above equilibrium payoffs due to underbidding of subjects with low valuations. In UPO the bidder payoffs range from 38% to 102% of the DR–equilibrium payoff, with an average of 68%. In UPS they range from 51% to 88% with an average of 69%. While bidder profits in both UPS and UPO are significantly lower than the DR–equilibrium payoff (Mann-Whitney or t-test, $p < .05$), in UPS they are even lower than the TT–equilibrium payoffs (t-test, $p < .05$) and payoffs relative to the TT–equilibrium payoffs are significantly smaller in UPS than in UPO (Mann-Whitney or t-test, $p < .05$).[11]

In AA five pairs play almost exactly according to the equilibrium prediction (see Figure 2.2), in some pairs bidders tried to cooperate by bidding 0 on both units if the valuation was relatively low (see the double dropouts at zero in Figure 2.1), or by using strategies that resembled the DR equilibrium strategies in a uniform price auction which results in payoffs that exceed equilibrium payoffs substantially (see Table 3.1, AA). The extreme excess profits of pair 4 (they achieve 289% of equilibrium payoffs) are partly a coincidence. In several auctions the valuations of both bidders in this group were very close, so that the equilibrium payoffs were very small. Attempts to cooperate through demand reduction or generous dropping out at a low price with both units led to payoffs substantially above the equilibrium. Also, the low bidder payoffs in pair

[11] The comparison between UPS and UPO yields different results depending on which equilibrium is used as a benchmark because the valuations are randomly drawn and hence different in the two treatments.

10 (54% of equilibrium payoffs) are partly driven by a chance event. One bidder always had the lower valuation. This seems to have caused some frustration which resulted in overbidding, that was possibly driven by spite or just by a desire to experiment. The other pairs' payoffs range from 91% to 138% with an overall average of 107%.

In VA the bidder profits are close to the equilibrium with an average of 91% over all pairs. While four pairs are within 5% deviation of the equilibrium payoff, one pair only achieves 43%, and another 164% (see Table 3.1).

In DA, since average bids are above equilibrium, the bidders' payoffs were consistently lower than under the equilibrium prediction in most pairs (see Table 3.1). Seven out of nine pairs are below 90% of the equilibrium payoffs, one pair obtains 113% and the average is 82% of the equilibrium.

The bidder profits in DA are significantly lower than the equilibrium payoffs (Mann-Whitney or t-test, $p < .05$) and than those in UPO (Mann-Whitney or t-test, $p < .05$) and AA (Mann-Whitney test $p < .05$).

4.7 Effects UPO ↔ UPS

One might suspect that subjects learn how to play the payoff dominant DR–equilibrium of the uniform price auction better in the open version. Thus, the subjects who played UPO and UPS played another ten auctions in the other uniform price format. We will refer to the ten open auctions that are played after the sealed–bid auctions as UPsO and to the sealed–bid auctions played after the open auctions as UPoS.

Three of the pairs that played UPO first cooperated almost from the start and continued to do so until the end of the open auctions. Figure 5 depicts for UPOS the percentage of the (DR–) equilibrium profit each of those three pairs reached per auction. While the pairs always realized roughly the DR equilibrium profit in UPO, this did not carry over to the subsequent sealed–bid auctions with the same pricing rule. There the bidders' profits differed substantially from equilibrium profits across all pairs.

If one looks at all pairs, the bidder profits between UPS and UPoS (see Table 3.2) do not differ significantly, indicating that bidders do not learn to play the DR equilibrium in UPS even if they played UPO before. However, the auctioneer's revenue was much lower in UPoS than in UPS (71,33 % vs. 106,74 % of the TT–equilibrium revenue, see Table 2.2 and 2.1) (Mann-Whitney or t-test, $p < .05$, in UPoS it is also significantly lower than in the TT–equilibrium), and also allocative efficiency is significantly lower in UPoS than in UPS (67.5% vs. 81%, Mann-Whitney test, $p < .05$). This, on the other hand, indicates that behavior got closer to playing the DR equilibrium.

Moreover, looking at the scatter diagrams reveals a better understanding of UPoS. Figure 3.1 and 3.2 depict the first and second unit bids in UPoS. Compared with Figure 1.1 and 1.2 (UPS), bids seem to be closer to equilibrium like behavior. This is even more the case for those pairs that seem to have found the low price equilibrium in the open auction they played before (see Figures 3.3 and 3.4). Bid spreads, however, are not significantly different in UPoS and UPS.

For the pairs that played UPS first and then UPsO we got an unexpected result. As one might suspect, bidders do not learn to play the DR equilibrium in the UPS design. Surprisingly this seems to extend partially to UPsO. In the following ten open auctions only three pairs found the cooperative equilibrium. Bidder profits fluctuated a lot but are on average close to those in UPO (69,71% vs. 67,83% of the DR–equilibrium profits, see Tables 3.1 and 3.2). Bidder profits (relative to the TT–equilibrium) increase, however, significantly from UPS to UPsO (t-test, $p < .05$). The auctioneer's revenues are higher (but not significantly) in UPsO than in UPO (65,07 % vs. 55,28 % of the TT–equilibrium revenue, see Tables 2.2 and 2.1). The allocative efficiency does not significantly differ between UPO (74%) and UPsO (79%).

Another interesting observation is that bidders violated the requirement of the DR equilibrium to drop out on one unit immediately once the other bidder dropped out more often in UPsO (66 % of the cases where it was possible). In UPO, it was violated in only 55 % of the cases.

Figure 4.1 depicts the observed bids in the UPsO. As Figure 4.2 shows, there are still three pairs who play close to the equilibrium prediction. Figure 4.3 depicts the behavior of those pairs in the preceding UPS treatment.

4.8 Learning

It is striking that there seems to be almost no learning within one auction format. In cases where bidders play close to equilibrium strategies, they start doing so in the first three auctions, but no pair learns to do so later. Also time trends lead partially towards the equilibrium (e.g. bid spreading in VA decreases and increases in UPoS, underbidding on the first unit decreases in UPoS), but partially away from the equilibrium (e.g. bid-spreading in UPS decreases and overbidding increases in UPS). That we do not find much learning is certainly partially so because we only play ten auctions. The virtual absence of learning trends after the third auction is still surprising, though. Furthermore, most subjects played the auctions very fast and took very little time to review the results. Hence it appears to us, that plainly increasing the number of auctions would not have produced much more learning. This would rather require to slow the subjects down, for example by imposing a minimal time they are shown the feedback.

4.9 Questionnaires

In all treatments there are participants who indicate in post-experimental questionnaires that they tried to cooperate as well as participants who explicitly behaved competitively or even spiteful. There is no indication that subjects realized that demand reduction is an equilibrium in the uniform-price auctions. In the uniform-price auctions as well as in the Vickrey and Ausubel auctions, several subjects realized that demand reduction is (weakly) payoff dominating all (other) equilibria and in the former some realized that in UPO cooperation is easier than in UPS, while none made an explicit reference to equilibria. Many subjects note avoiding losses as a primary aim or as a constraint on their attempts to maximize their payoffs.

5 Conclusions

The results of our experiments agree with some of the theoretical predictions, while they clearly contradict others. Demand reduction occurs in the uniform-price auctions, though it also does to a lesser extent in the Ausubel auction. The allocative efficiency is lowest in UPO, and highest in AA, where the latter differs only slightly from UPS, VA, and DA with respect to the number of efficiently allocated units, but indicates the least robust causes of misallocations. The revenue equivalence of the Ausubel auction, the Vickrey auction and the discriminatory sealed–bid auction is clearly rejected, as it is for the two uniform-price auctions. In clear contrast to the theory, the auctioneer's revenues do not primarily depend on the pricing-rule, but whether the auction is open or sealed–bid.

Part of the results do not come as a surprise, though not predicted by the equilibrium analysis. Overbidding is more frequent in UPS and in VA than in UPO and AA, since it is less salient that overbidding is dominated. Coordination on the DR–equilibrium seems to be much easier in the uniform-price open auction than in the sealed–bid auction, because one player can signal by dropping out. Bidding above the equilibrium strategy is much more frequent in the discriminatory than in the Ausubel auction, since in the latter case this involves overbidding, which is easier recognized as not optimal, than the optimal bids in the discriminatory sealed–bid auction are figured out. The last two behavioral effects cause the auctioneer's revenues to be higher in the sealed–bid auctions than in the open auctions.

Our primary results are well in line with those of Kagel and Levin (2001). They also find more demand reduction in the uniform price open auction than in the uniform price sealed–bid auction and much less demand reduction in the Ausubel auction. Furthermore, they also find much more overbidding in the uniform price sealed–bid auction than in the two open auctions. In their experiment as well as in ours the UPS yields higher revenues to the auctioneer but lower allocative efficiency than the Ausubel auction. So we provide further indication for the theoretically unanticipated trade-off between revenue and efficiency. Thus their results do not seem to critically depend on the simulation of other participants by computers. The primary properties of the auction mechanisms they investigate carry over to our interactive environment, in particular the superior performance of the Ausubel auction. However, in contrast to Kagel and Levin, in our experiment there seems to be surprisingly little learning both within and across auction rules. Those subjects who figure out the equilibria do so almost at once. This is particularly surprising given that our interactive environments seems more complicated and since we did not provide hints against overbidding.

In line with our observation that the pricing-rule is less important for the revenues than whether the auction is sealed–bid or open, List and Lucking-Reiley (2000) find little differences in revenues between VA and UPS. As we do, they also find more overbidding on the first unit in UPS compared to VA.[12] Our results also confirm the observation of List and Lucking-Reiley that the bid spreading is larger in UPS than in VA, and confirm that this leads to (slightly) more misallocations. What is surprising, though, is that bid spreading is most extreme in DA, where it is not consistent with equilibrium behavior. This seems to be caused by a dislike for zero profits which leads subjects to increase the probability of acquiring at least one unit at the expense of expected profits. This zero profit aversion has no distorting effect in the other auction mechanisms, since the probability of acquiring at least one unit (without making losses) is maximized by bidding the valuation on the first unit, consistent with equilibrium behavior. The bid spreading in DA cannot be completely explained with risk aversion, because the majority of second-unit bids is below the equilibrium, inconsistent with risk aversion. One might argue, that this result, that bidding behavior that at a first glance looks perfectly consistent with the usual overbidding in first-price single unit auctions, cannot be explained by risk aversion, might even question whether risk aversion is the correct explanation for the overbidding in single unit auctions.

In line with the results in Kagel et al. (2001), we find that there is a trade-off between the stricter equilibrium concept of VA and the possi-

[12] In a related experiment, Engelbrecht-Wiggans, List and Lucking-Reiley (1999), find that this effect disappears with 3 or 5 bidders. They also find, consistent with the theoretical prediction, that demand reduction is still present, but reduced if the number of bidders is increased.

bly more transparent mechanism in AA. The total allocative efficiency is almost identical. Inefficient allocations in AA seem indeed partially caused by bidders hoping that the second bidder plays the weakly dominated strategy to drop out after a drop out of the first bidder (which the second bidder then sometimes does), whereas inefficient allocations in VA result from bids that more often deviate from the valuation, which may possibly be due to the fact that in VA it was less transparent to the bidders that bidding the own valuation was dominant.

Manelli et al. (2001) compare VA and AA in a design where three bidders compete for three units and each can buy three units, but the third unit has a value of 0. They study both auctions with and without a common value component. In one of their experiments (ASU), they find that due to overbidding in VA, the revenues are higher than in AA. Since some bidders in AA bid aggressively on the third unit, causing efficiency losses, the total efficiency is roughly equal. These results are in line with ours. In contrast, in their second experiment (UI), where overbidding is weaker, in VA the efficiency is higher but the revenues are lower than in AA.

An interesting aspect is that statements in the post-experimental questionnaires are similar in the uniform-price auctions and in the Ausubel auction. Several participants tried to cooperate by reducing demand and they observed that this worked fine in UPO, but less so in UPS and even less in AA. It seems, however, that they all failed to realize that cooperation was stable when it was an equilibrium. Hence the equilibrium prediction well organizes the data for some pairs although they do not think in these terms. They try to cooperate and are successful (if it is an equilibrium) or not, without understanding why. This is, of course, interesting from a general perspective. Equilibria can yield good predictions even in case they are possibly too sophisticated for subjects to be figured out if equilibrium choices can result from less sophisticated thought processes.

There are some policy conclusions. If the objective of the auctioneer is a combination of the maximization of efficiency and of the own revenues, the uniform price open auction is clearly not preferable. Demand reduction leads both to a reduction of revenues and to a misallocation of one unit.[13] If the primary aim is the efficient allocation, the Ausubel auction seems to be best suited, while if the focus is on revenues, the sealed–bid auctions perform best due to frequent overbidding in the case of the uniform price auction and to bids generally exceeding equilibrium

[13] Goswami, Noe, and Rebello (1996) find that in a setting with a high number of bidders (11) and units (100) and identical valuations, nonbinding preplay communication facilitates demand reduction in a uniform-price sealed–bid auction and shifts behavior towards the equilibrium in a discriminatory auction. This indicates that outside the laboratory, where communication is more likely, the differences between auction formats may even be stronger than in our (and others') results.

bids in the discriminatory auction. A mechanism that is easy to understand seems best suited to allocate the units efficiently, while one where optimal strategies are difficult to figure out may be better to maximize revenues. The latter, of course, also involves more risks for the auctioneer.

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A Equilibria of UPS and UPO

A.1 Equilibria of UPS

We first show that the strategies

$$b_1(v_i) = v_i \tag{7}$$
$$b_2(v_i) = \begin{cases} x_k & \text{if } \; v_i \in [x_k, y_k], \\ v_i & \text{otherwise.} \end{cases}$$

with (x_k, y_k), $k = 1, \ldots K$ being a sequence with the following properties: $x_k \geq 0$, $y_K \leq V$, $x_k \leq y_k$, and $y_k \leq x_{k+1}$ are an equilibrium of UPS. We show that, given player j plays (7) it does not pay for player i to deviate from this strategy. In order to simplify the calculations we show this for $V = 1$. Clearly, this also extends to arbitrary values of V.

First note that in both uniform price formats it is a weakly dominant strategy for the bidders to bid their valuation v_i on the first unit (i.e. their higher bid always equals their true valuation). Their bid on the first unit will only determine the price if it is the highest rejected bid, i.e. if the bidder does not get a unit. Like in the standard argument for the second price auction for the single unit case, lowering the bid implies the risk of missing a profitable deal whereas overbidding might result in buying a unit at a loss. This is even more obvious in the open auction. If a player has already dropped out on one unit, dropping out on the second unit before the valuation is reached guarantees a profit of 0, whereas continuing might yield a positive profit, if the other bidder drops before the own valuation is reached. Staying in above the own valuation causes a loss as soon as the other bidder drops out. It remains to be shown that the bid on the second unit as stated in (7) is indeed part of an equilibrium strategy.

In the following we show that if bidder j plays according to (7), it is indeed a best reply for bidder i to play (7) as well. We proceed in two steps. First, suppose bidder i's valuation is in a demand reduction interval, i. e. $v_i \in [x_k, y_k]$. Then

(a) If $v_j > y_k$ bidder i's profit is zero if he plays (7) and he cannot increase his profit by deviating.

(b) If $v_j \in [x_k, y_k]$ then for bidder i any bid below x_k or above v_i

clearly does not pay. Consider $b_2(v_i) = z$ with $z \in [x_k, v_i]$. We get

$$\begin{aligned}\pi_i(b_2(v_i) = z) &= Pr\{v_j > z | x_k \le v_j \le y_k\}(v_i - z) \\ &\quad + Pr\{v_j \le z | x_k \le v_j \le y_k\} 2 \left(v_i - E[v_j | x_k \le v_j \le z]\right) \\ &= \frac{y_k - z}{y_k - x_k}(v_i - z) + \frac{z - x_k}{y_k - x_k} 2 \left(v_i - \frac{z + x_k}{2}\right) \\ &= (v_i - z) + \frac{(z - x_k)}{y_k - x_k}(v_i - x_k).\end{aligned}$$

Taking the derivative with respect to z yields $-1 + \frac{v_i - x_k}{y_k - x_k}$ which is negative for $v_i < y_k$. Therefore, it does not pay to deviate from $b_2 = x_k$ if the competitor's valuation is in the same interval, which yields $\pi_i(b_2(v_i) = x_k) = v_i - x_k$.

(c) if $v_j \in [y_{l-1}, x_l]$, $l \le k$ the price on any unit bidder i obtains is always v_j. Since bidder i clearly prefers getting two units at price v_j to getting one unit, it does not pay to deviate from (7) either.

(d) if $v_j \in [x_l, y_l]$, $l < k$, then for bidder i bidding below x_l clearly does not pay. Any bid above y_l yields the same outcome, namely obtaining both units for v_j. Consider $b_2(v_i) = z$ with $z \in [x_l, y_l]$. We get

$$\begin{aligned}\pi_i(b_2(v_i) = z) &= Pr\{v_j > z | x_l \le v_j \le y_l\}(v_i - z) \\ &\quad + Pr\{v_j \le z | x_l \le v_j \le y_l\} 2 \left(v_i - E[v_j | x_l \le v_j \le z]\right) \\ &= \frac{y_l - z}{y_l - x_l}(v_i - z) + \frac{z - x_l}{y_l - x_l} 2 \left(v_i - \frac{z + x_l}{2}\right) \\ &= (v_i - z) + \frac{(z - x_l)}{y_l - x_l}(v_i - x_l).\end{aligned}$$

Taking the derivative with respect to z yields $-1 + \frac{v_i - x_l}{y_l - x_l}$ which is positive for $v_i > y_l$. Since any bid above y_l yields the same outcome (and $v_i \ge y_l$), it does not pay to deviate from $b_2 = x_k$ if the competitor's valuation is in a lower demand reduction interval.

Hence against no possible type of opponent deviating from (7) pays for a bidder with a valuation in a demand reduction interval. It remains to show that there is also no incentive to deviate from (7) if the bidder's valuation is in a truthtelling interval, i. e. $v_i \in [y_{k-1}, x_k]$. For $v_j > x_k$, $v_j \in [y_{l-1}, x_l]$, $l < k$, and $v_j \in [x_l, y_l]$, $l < k$ the same aguments as in (a), (c), and (d) apply. If $v_j \in [y_{k-1}, x_k]$ bidder i will receive no unit if $v_i < v_j$. If $v_i \ge v_j$ the price on any unit bidder i obtains will be v_j. So he prefers getting two units to getting one unit. Therefore, deviation from (7) never pays if the other player plays this strategy.

A.2 Perfect Bayesian Equilibria of UPO

In order to show that the equilibria of UPS, as given in (7), are also perfect Bayesian equilibria (PBE) of UPO we need to specify beliefs about the other bidder's valuation that make the strategy (7) sequentially rational given the system of beliefs and that are derived by Bayes' rule at least on the equilibrium path. The following beliefs support the strategy (7) as a PBE:

- As long as the other player does not drop out on any unit the belief is equivalent to the ex ante expected probability distribution, i. e. uniform.

- If the other player drops out on one unit at $z_k \in [x_k, y_k]$, $k = 1, \ldots K$, bidder i updates his subjective distribution function as follows:

$$\mu_i(v|z_k) = \begin{cases} 1 & \text{for} \quad v \in [y_k, 1], \\ \frac{v-p}{y_k-p} & \text{for} \quad v \in [p, y_k], \\ 0 & \text{for} \quad v \in [0, p], \end{cases}$$

  with $p \in [z_k, y_k]$ being the current price in the auction.

- If the other player drops out on one unit at $z_k \in [y_k, x_{k+1}]$, $k = 1, \ldots K$, bidder i updates his subjective distribution function as follows:

$$\mu_i(v|z_k) = \begin{cases} 1 & \text{for} \quad v \in [\min\{x_k, v_i\}, 1], \\ \frac{v-p}{\min\{x_k, v_i\}-p} & \text{for} \quad v \in [p, \min\{x_k, v_i\}], \\ 0 & \text{for} \quad v \in [0, p]. \end{cases}$$

  with $p \in [z_k, x_{k+1}]$ being the current price in the auction.

- If the current price p exceeds the the upper limit u of the interval where the other bidder dropped out on one unit bidder i updates his subjective distribution function as follows:

$$\mu_i(v|z_k) = \begin{cases} 1 & \text{for} \quad v \in [v_i, 1], \\ \frac{v-p}{v_i-p} & \text{for} \quad v \in [p, v_i], \\ 0 & \text{for} \quad v \in [0, p]. \end{cases}$$

  with $p \in [u, v_i]$ being the current price in the auction.

- If the other bidder drops out on both units the auction is over and beliefs can be arbitrary.

Note that if there are truth-telling intervals or if the price increases above the upper limit of the drop-out interval, beliefs depend on the own type (i. e. valuation).

A.3 Payoff Dominance of the DR–Equilibrium

We show that among all equilibria of the uniform price auction, the DR–equilibrium yields the highest expected profit to the bidders. We first compare the profits of the DR– and the TT–equilibrium:

$$\begin{aligned}
\pi_i^{DR} &= v_i \qquad \text{and} \\
\pi_i^{TT} &= Pr\{v_j \le v_i\} 2 \left(v_i - E[v_j | v_j \le v_i]\right) \\
&= v_i^2 .
\end{aligned}$$

Thus, $\pi_i^{DR} \ge \pi_i^{TT}$ whenever $v_i \le 1$ which is always true. The expected payoff difference is $\pi_i^{DR} - \pi_i^{TT} = v_i(1 - v_i)$. Now consider one of the intermediate equilibria. They yield the same payoff as the TT–equilibrium unless both valuations are in the same demand reduction intervall $[x_k, y_k]$. Then the payoff is by $\Delta\pi_i$ higher than the payoff of the truthtelling equilibrium, where

$$\begin{aligned}
\Delta\pi_i &= Pr\{v_j \in [x_k, v_i]\} \left(v_i - x_k - 2\left(v_i - E[v_j | v_j \in [x_k, v_i]]\right)\right) \\
&\quad + Pr\{v_j \in [v_i, y_k]\}(v_i - x_k) \\
&= (v_i - x_k)\left(v_i - x_k - 2\left(v_i - \frac{v_i + x_k}{2}\right)\right) + (y_k - v_i)(v_i - x_k) \\
&= (y_k - v_i)(v_i - x_k).
\end{aligned}$$

Therefore, we always get $\Delta\pi_i \le \pi_i^{DR} - \pi_i^{TT}$, which proves that the DR–equilibrium is payoff dominant.

A.4 DR–Equilibrium in UPO

In the uniform-price open auction it is, as shown above, a weakly dominant strategy to remain active on the first unit until one's true value v_i is reached. Furthermore, if beliefs strictly follow Bayes' rule also off the equilibrium path (that is, if a bidder observes a dropout he infers only that the opponent's valuation is higher than the dropout price and updates the initial distribution accordingly), only the DR–equilibrium survives (is sequentially rational). Each bidder's payoff if both drop out at price zero is $v_i - 0$.

To see this suppose that at any current price $p \in [0, 1]$ bidder i believes bidder j's valuation to be uniformly distributed on $[p, 1]$, unless j dropped out on both units. In particular, the beliefs do not change if j dropped out on only one unit. Suppose bidder j dropped out at price $y \ge 0$. Now bidder i has two options: Dropping out on one unit as well guarantees him profit $v_i - y$. In contrast, staying active with the second unit until some price $x > y$ yields, (if the other player uses the weakly dominant strategy to bid his valuation on the first unit) with (subjective) probability $\frac{1-x}{1-y}$ a profit of $v_i - x$ (if player j's valuation exceeds x), whereas with probability $\frac{x-y}{1-y}$ player j will drop out at a price below x which implies an expected price of $\frac{x+y}{2}$ and thus an expected profit $2(v_i - \frac{x+y}{2})$. Hence the total expected payoff from staying active until a price $x > y$ is

$$\begin{aligned} \pi_i(b_2(v_i) = x) &= \frac{1-x}{1-y}(v_i - x) + 2\frac{x-y}{1-y}(v_i - \frac{x+y}{2}) \\ &= v_i - x + \frac{x-y}{1-y}(v_i - y) < v_i - y \\ \text{for } x &> y \text{ and } v_i < 1. \end{aligned}$$

Thus whenever player j drops out at y, it is optimal for i to drop out at y as well (unless $v_i = 1$ which happens with probability 0). Hence in any continuation game that is reached after one player drops out,[14] the other drops out as well.

Hence, if bidder i drops out at 0, bidder j drops out as well and i receives $\pi_i = v_i - 0$. If instead player i stays active until $z > 0$, player j would follow (drop out immediately) if this price is ever reached. However, he might also drop out on one unit at a price $y < z$. In this case, as shown above, it is optimal for player i to drop out at y as well, which yields a lower payoff than dropping out at 0 if $y > 0$ (remember that j would have followed a dropout at 0). If his valuation is lower than z and he did not drop out on one unit before, bidder j drops out on both units at his valuation v_j. Bidder i's expected payoff given that j does not drop out on single units below z is given by

$$\begin{aligned} \pi_i(b_2(v_i) = z) &= Pr\{v_j \leq z\}2(v_i - E[v_j|v_j \leq z] \\ &= z2(v_i - \frac{z}{2}) \\ &= zv_i + z(v_i - z), \end{aligned}$$

which is lower than v_i (the profit of a dropout at 0, since $(1-z)v_i > z(v_i - z)$.

Therefore, no matter if j drops out on one or two units, i's expected profit always falls short of his profit from a single dropout at 0. This shows that with the above mentioned requirements on the beliefs, dropping out at zero is the only PBE.

[14]Note that there are no real subgames, since there is incomplete information about the valuation of the other player.

B Instructions (Ausubel Auction)

Please read these instructions carefully. If there is something you do not understand, please raise your hand. We will then answer your questions privately. The instructions are identical for all participants.

In the course of the experiment you will participate in 10 auctions. In each auction you and another bidder will bid for two units of a fictitious good. This other bidder will be the same in each auction. Each unit that you acquire will be sold to the experimenters for your private resale value v. Before each auction this value **per unit**, v, will be randomly drawn independently for each bidder from the interval $0 \leq v \leq 100$ ECU (Experimental Currency Unit). Any number between 0 and 100 is equally probable. The private resale values of different bidders are independent. **In each auction any unit that you acquire will have the same value for you. This value will be drawn anew before each auction.**

Before each auction you will be informed about your resale value **per unit**, v. Each participant will be informed only about his or her own resale value, but not about the other bidder's resale value.

After a short break the auction starts:

The price **per unit** will successively be increased in steps of 1, beginning at a price of 0. At the beginning of the auction you are active on both units. At any time you can drop out on *one* unit by clicking the button *"drop out 1"* or you can drop out on *both* units simultaneously by clicking the button *"drop out 2"*.

If one of the bidders clicks the button *"drop out 2"*, the other bidder obtains both units for the price where the first bidder dropped out and the auction is finished (since then there are only two active bids left).

If one bidder drops out on one unit, the other immediately obtains one unit (since the first bidder has only one active bid left and can thus acquire at most *one* unit) for the price where the first bidder dropped out.

Then the auction continues at the price where the first unit was given away. Now only **one** unit is auctioned off and both bidders have only **one** active bid. If now one bidder drops out for this unit, the other bidder obtains this unit for the price where the bidder dropped out and the auction is finished.

If upon reaching the maximal price of 100 ECU there are four active bids left, both bidders receive one unit for a price of 100 ECU. If upon reaching the maximal price of 100 ECU there is only one unit given away, (both bidders still have one active bid), then the other unit will be randomly allocated for a price of 100 ECU among the two bidders.

Your profit per unit acquired is your resale value minus the price at which you obtained the unit.

If you do not obtain a unit you neither receive nor pay anything. Hence your profit is 0.

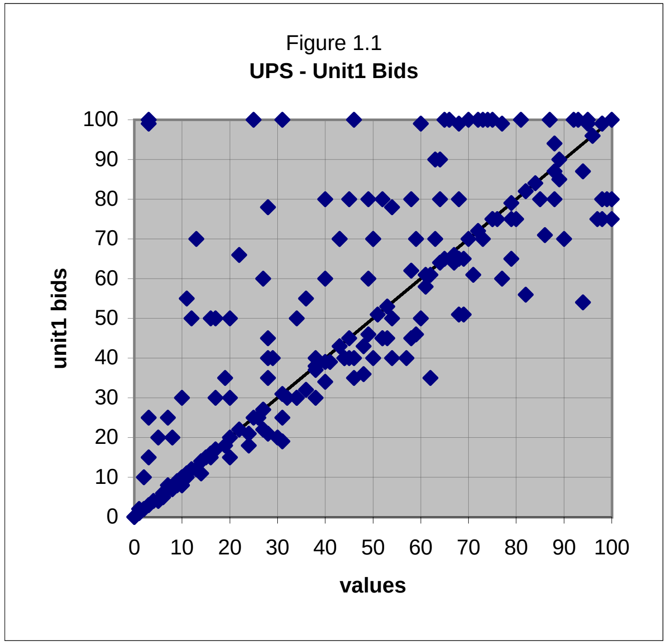
Note that you can make losses as well. It is always possible, however, to bid in such a way that you can prevent losses for sure.

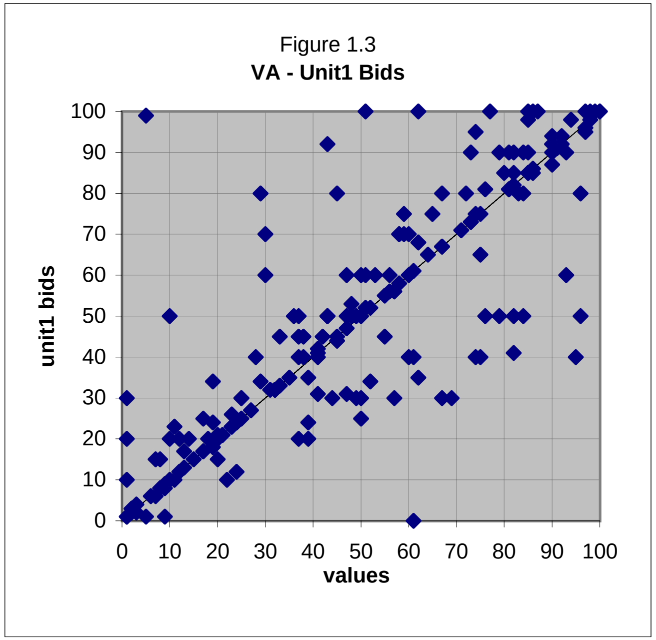
You will make your decision via the computer terminal. You will not get to know the names and code numbers of the other participants. Thus all decisions remain confidential.

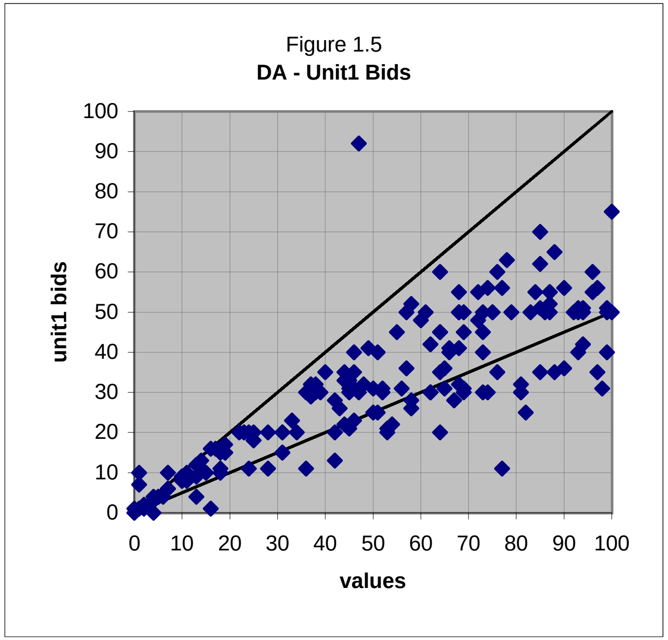
One ECU corresponds to 0,04 DM. You will obtain an initial endowment of 5 DM. If you make losses in an auction these will be deducted from your previous gains (or from your initial endowment). You will receive your final profit in cash at the end of the experiment. The other participants will not get know your profits.

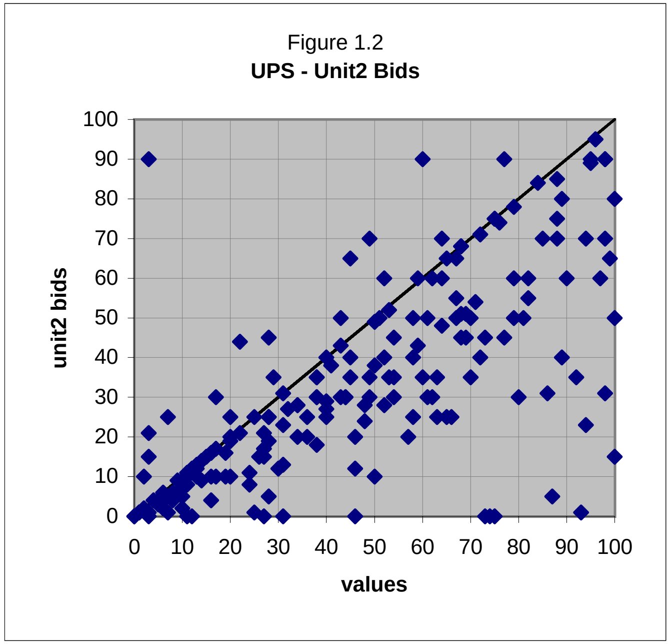
If there is something you have not understood, please raise your hand. We will then answer your questions privately.

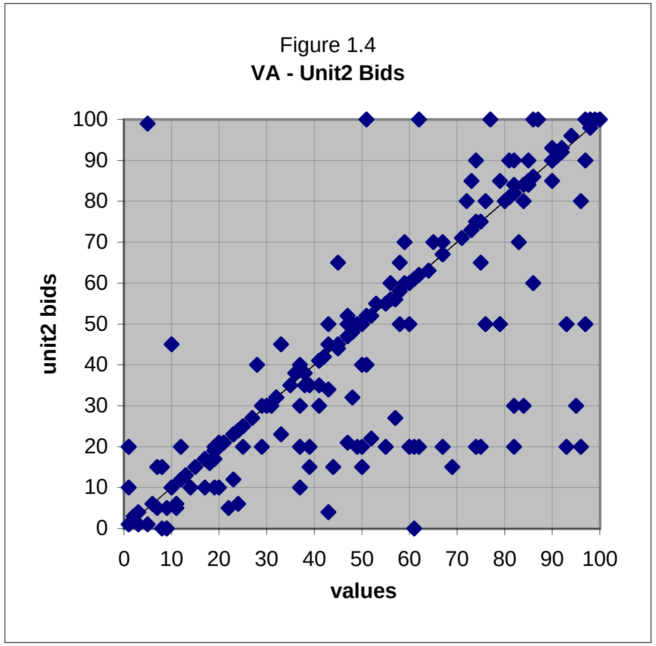
Scatter Diagrams, Sealed-Bid Auctions

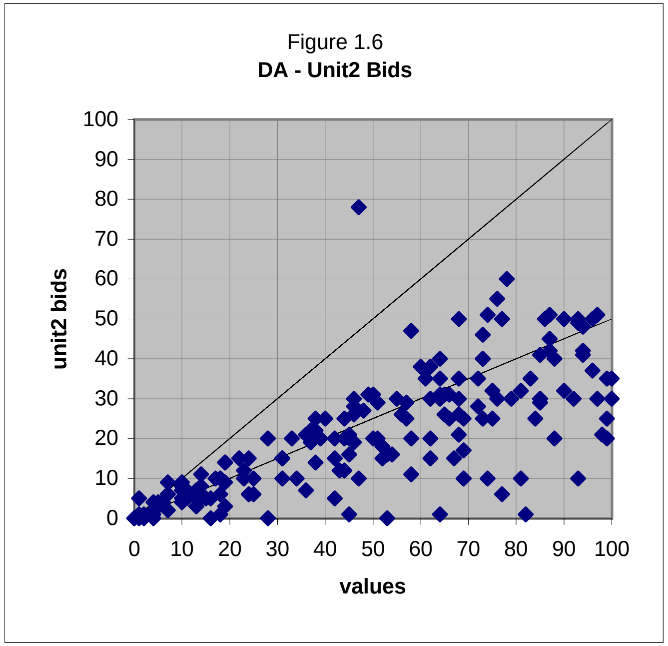




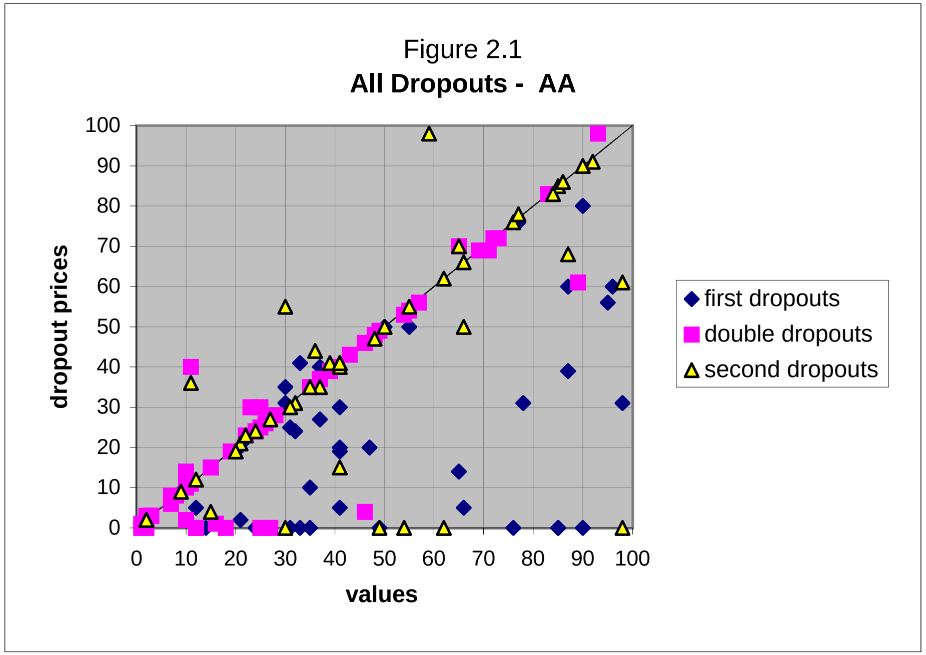


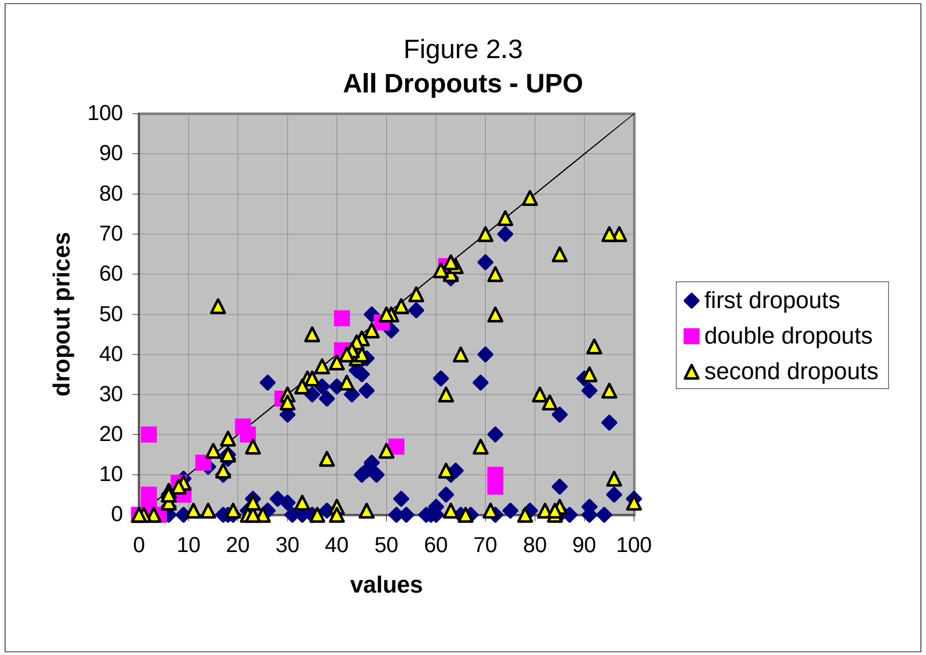


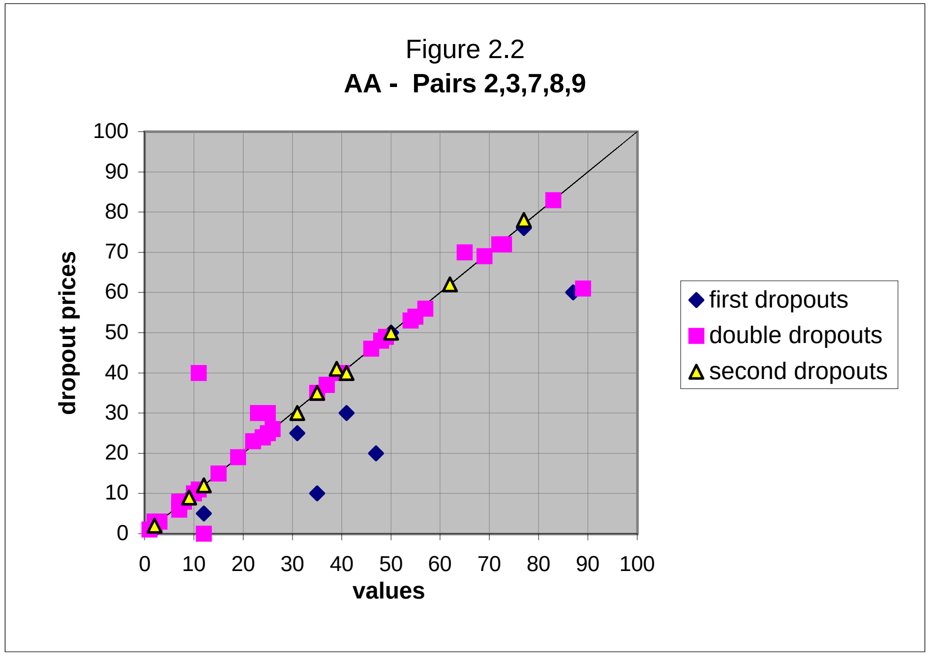


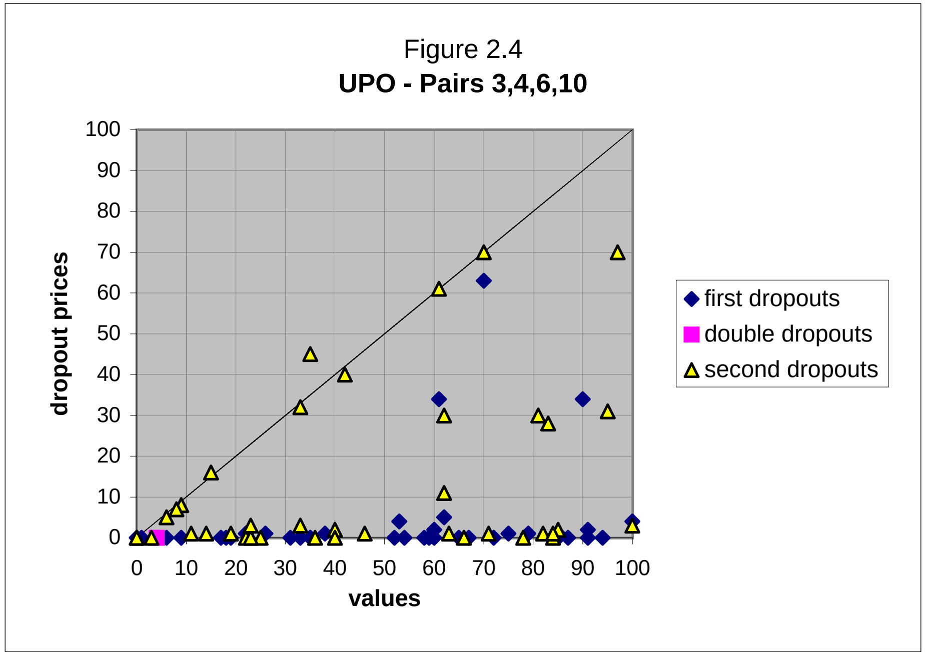


Scatter Diagrams - Open Auctions









UPoS - All Pairs

Figure 3.1
UPoS - Unit1 Bids

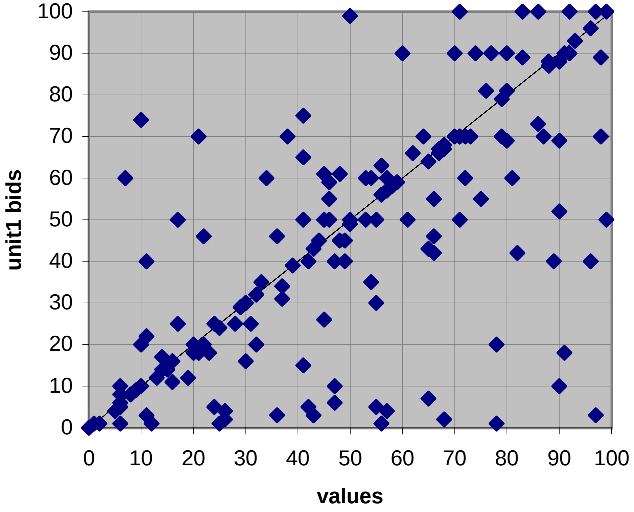
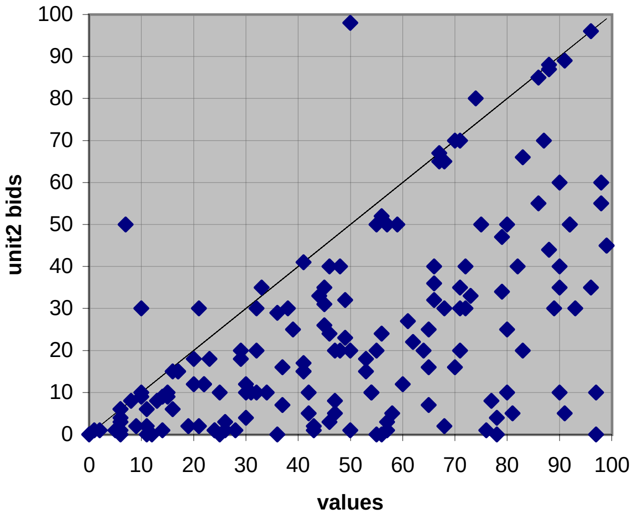


Figure 3.2
UPoS - Unit2 Bids



UPoS - pairs that played close to DR in the open auction

Figure 3.3
UPoS - Unit1 Bids (Pairs 3,4,6,10)

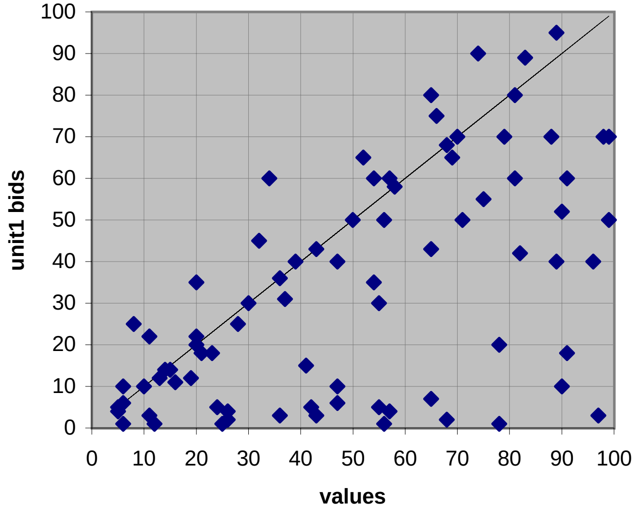
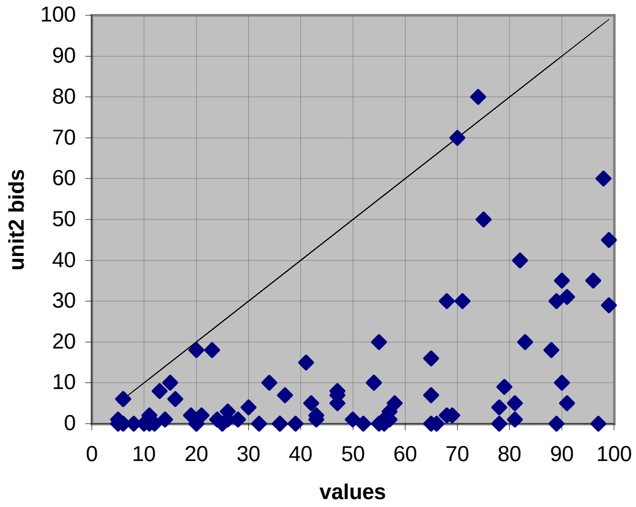
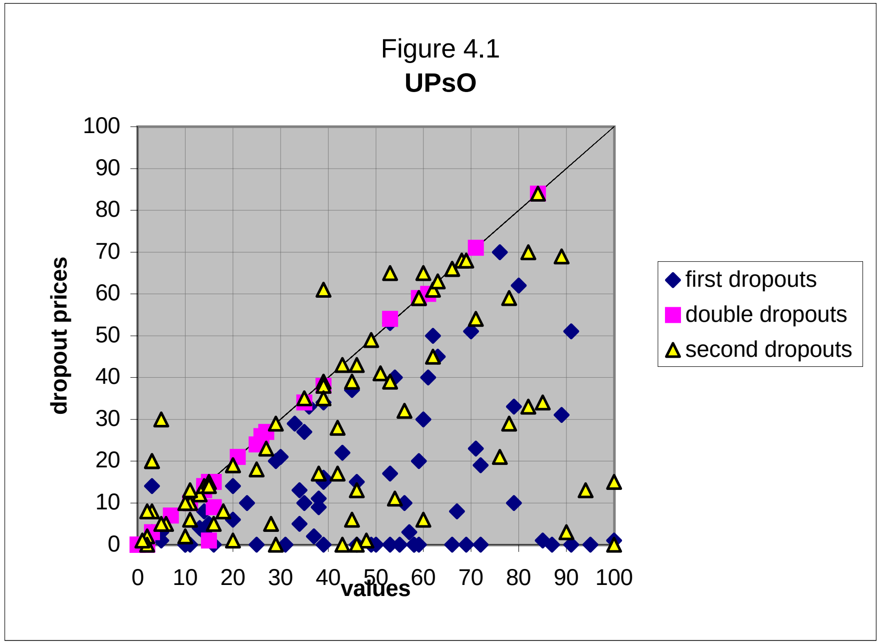
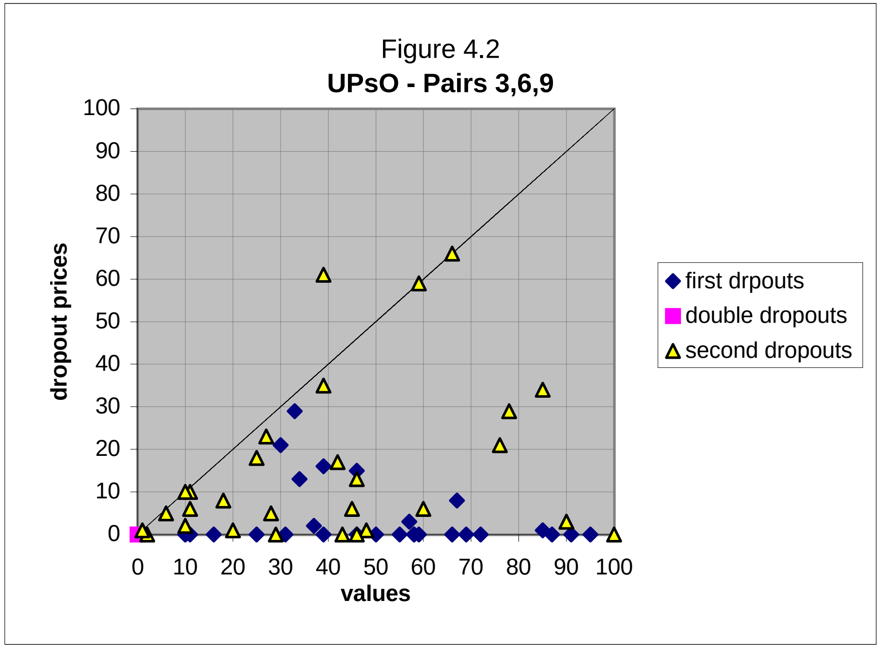


Figure 3.4
UPoS - Unit2 Bids (Pairs 3,4,6,10)



UPsO - all pairs and pairs that played close to DR





UPSo - bids in the preceeding sealed-bid auction by pairs that played close to DR in the open auction

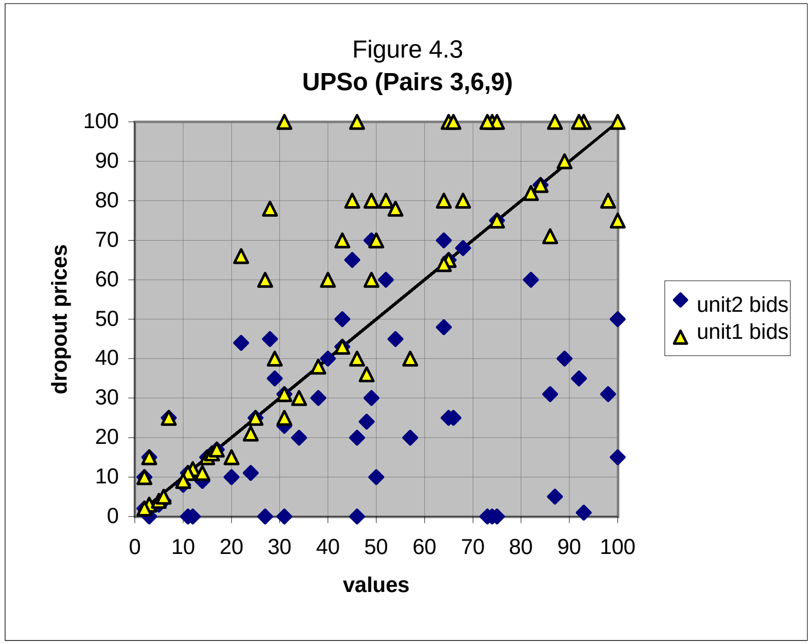


Table 2.1 **Auctioneer's revenues** (Equilibrium Revenue = TT Eq. Revenue in all auctions to make revenues comparable)

UPS

Pair	TT-Equilibrium Revenue (ER)	Revenue (R)	R - ER	(R * 100)/ER
1	488	682	194	139,75
2	548	486	-62	88,69
3	498	478	-20	95,98
4	356	572	216	160,67
5	496	502	6	101,21
6	618	760	142	122,98
7	760	710	-50	93,42
8	498	576	78	115,66
9	546	498	-48	91,21
10	502	404	-98	80,48
TOTAL	5310	5668	358	**106,74**

VA

Pair	Equilibrium Revenue (ER)	Revenue (R)	R - ER	(R * 100)/ER
1	614	568	-46	92,51
2	848	861	13	101,53
3	550	604	54	109,82
4	608	252	-356	41,45
5	770	643	-127	83,51
6	520	547	27	105,19
7	858	829	-29	96,62
8	576	753	177	130,73
9	476	444	-32	93,28
10	872	895	23	102,64
TOTAL	6692	6396	-296	**95,58**

DA

Pair	Equilibrium Revenue (ER)	Revenue (R)	R - ER	(R * 100)/ER
1	734	879	145	119,75
2	615	655	40	106,50
3	700	1017	317	145,29
4	628	639	11	101,75
5	714	829	115	116,11
6	687	835	148	121,54
7	652	542	-110	83,13
8	571	612	41	107,18
9	592	517	-75	87,33
TOTAL	5893	6525	632	**110,72**

UPO

Pair	TT-Equilibrium Revenue (ER)	Revenue (R)	R - ER	(R * 100)/ER
1	814	556	-258	68,30
2	562	562	0	100,00
3	654	72	-582	11,01
4	614	4	-610	0,65
5	732	485	-247	66,26
6	778	179	-599	23,01
7	1014	745	-269	73,47
8	546	370	-176	67,77
9	588	530	-58	90,14
10	768	405	-363	52,73
TOTAL	7070	3908	-3162	**55,28**

AA

Pair	Equilibrium Revenue (ER)	Revenue (R)	R - ER	(R * 100)/ER
1	618	271	-347	43,85
2	518	525	7	101,35
3	770	723	-47	93,90
4	638	279	-359	43,73
5	914	630	-284	68,93
6	892	674	-218	75,56
7	600	658	58	109,67
8	530	562	32	106,04
9	636	613	-23	96,38
10	804	929	125	115,55
TOTAL	6920	5864	-1056	**84,74**

Table 2.2 **Auctioneer's Revenues - UPsO and UPoS**
(Eq. Revenue = TT Eq. Revenue in all auctions to make revenues comparable)

UPoS

Pair	TT-Equilibrium Revenue (ER)	Revenue (R)	R - ER	(R * 100)/ER
1	594	762	168	128,28
2	782	672	-110	85,93
3	480	180	-300	37,50
4	646	42	-604	6,50
5	772	570	-202	73,83
6	618	478	-140	77,35
7	682	412	-270	60,41
8	682	776	94	113,78
9	540	492	-48	91,11
10	608	184	-424	30,26
TOTAL	6404	4568	-1836	**71,33**

UPsO

Pair	TT-Equilibrium Revenue (ER)	Revenue (R)	R - ER	(R * 100)/ER
1	628	376	-252	59,87
2	828	541	-287	65,34
3	474	98	-376	20,68
4	752	591	-161	78,59
5	608	255	-353	41,94
6	928	855	-73	92,13
7	712	599	-113	84,13
8	508	377	-131	74,21
9	738	195	-543	26,42
10	478	443	-35	92,68
TOTAL	6654	4330	-2324	**65,07**

Table 3.2 **Bidders' Profits - UPsO and UPoS**
(Eq. Profit = DR Eq. Profit)

UPoS

Pair	DR Equilibrium Profit (EP)	Profit (P)	P - EP	(P * 100)/EP
1	962	380	-582	39,50
2	1007	400	-607	39,72
3	942	825	-117	87,58
4	943	1029	86	109,12
5	1090	647	-443	59,36
6	1046	795	-251	76,00
7	989	829	-160	83,82
8	1072	629	-443	58,68
9	787	540	-247	68,61
10	1052	951	-101	90,40
TOTAL	9890	7025	-2865	**71,03**

UPsO

Pair	DR Equilibrium Profit (EP)	Profit (P)	P - EP	(P * 100)/EP
1	1075	895	-180	83,26
2	963	375	-588	38,94
3	837	758	-79	90,56
4	1154	757	-397	65,60
5	966	691	-275	71,53
6	1150	402	-748	34,96
7	984	530	-454	53,86
8	909	807	-102	88,78
9	1067	874	-193	81,91
10	958	926	-32	96,66
TOTAL	10063	7015	-3048	**69,71**

Table 3.1 **Bidder Profits** (Equilibrium Profit = DR Equilibrium Profit in UPO and UPS)

UPS

Pair	DR-Equilibrium Profit (EP)	Profit (P)	P - EP	(P * 100)/EP
1	969	637	-332	65,74
2	916	738	-178	80,57
3	978	726	-252	74,23
4	772	497	-275	64,38
5	866	734	-132	84,76
6	1077	682	-395	63,32
7	1003	533	-470	53,14
8	803	409	-394	50,93
9	852	548	-304	64,32
10	826	726	-100	87,89
TOTAL	9062	6230	-2832	**68,75**

VA

Pair	Equilibrium Profit (EP)	Profit (P)	P - EP	(P * 100)/EP
1	898	941	43	104,79
2	564	391	-173	69,33
3	584	470	-114	80,48
4	514	843	329	164,01
5	392	410	18	104,59
6	832	805	-27	96,75
7	484	316	-168	65,29
8	648	281	-367	43,36
9	832	790	-42	94,95
10	636	555	-81	87,26
TOTAL	6384	5802	-582	**90,88**

DA

Pair	Equilibrium Profit (EP)	Profit (P)	P - EP	(P * 100)/EP
1	734	553	-181	75,34
2	615	545	-70	88,62
3	700	329	-371	47,00
4	628	520	-108	82,80
5	714	535	-179	74,93
6	687	489	-198	71,18
7	652	737	85	113,04
8	571	490	-81	85,81
9	592	656	64	110,81
TOTAL	5893	4854	-1039	**82,37**

UPO

Pair	DR-Equilibrium Profit (EP)	Profit (P)	P - EP	(P * 100)/EP
1	1183	617	-566	52,16
2	898	538	-360	59,91
3	1011	883	-128	87,34
4	933	954	21	102,25
5	1072	592	-480	55,22
6	1055	783	-272	74,22
7	1168	443	-725	37,93
8	884	764	-120	86,43
9	981	688	-293	70,13
10	1121	729	-392	65,03
TOTAL	10306	6991	-3315	**67,83**

AA

Pair	Equilibrium Profit (EP)	Profit (P)	P - EP	(P * 100)/EP
1	826	1017	191	123,12
2	462	453	-9	98,05
3	728	767	39	105,36
4	188	543	355	288,83
5	458	632	174	137,99
6	582	644	62	110,65
7	652	592	-60	90,80
8	1008	976	-32	96,83
9	922	920	-2	99,78
10	540	293	-247	54,26
TOTAL	6366	6837	471	**107,40**

Figure 5

UPOS - Pairs 3,4,6

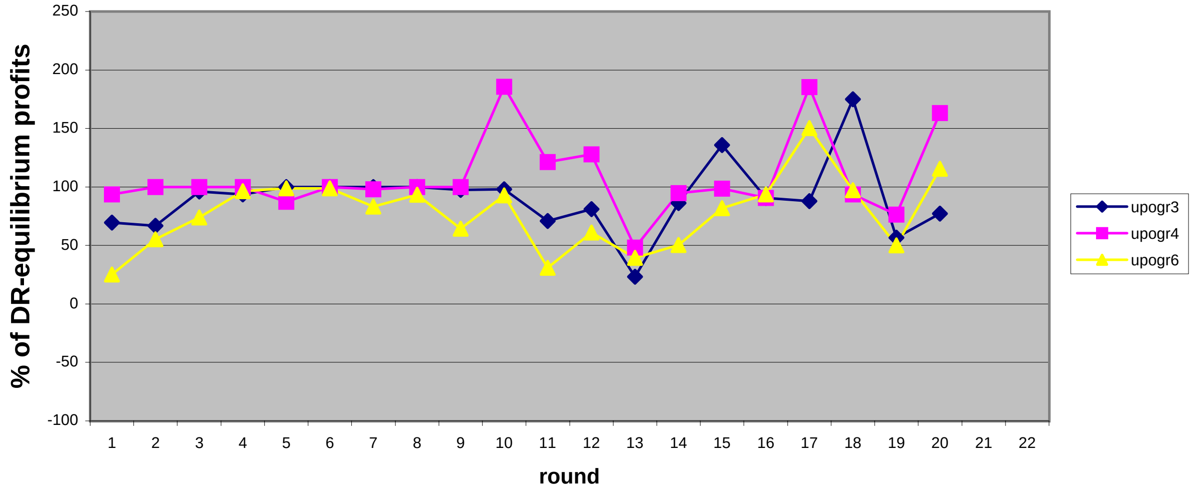


Table 4 **Efficiently Allocated Units**

VA		**AA**		**UPO**		**DA**		**UPS**	
All pairs	165	All pairs	168	All pairs	148	All pairs	150	All pairs	162
rel.	0,83	rel.	0,84	rel.	0,74	rel.	0,83	rel.	0,81
max	20	max	20	max	19	max	18	max	20
min	13	min	11	min	10	min	15	min	13
(second lowest)	14	(second lowest)	15	(second lowest)	11	(second lowest)	15	(second lowest)	14
				UPsO				**UPoS**	
				All pairs	158			All pairs	135
				rel.	0,79			rel.	0,675
				max	19			max	18
				min	11			min	10