

On some auction rules for amicable divorce in equal share partnerships*

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Abstract

Partnerships form and dissolve. Dissolution often requires a change in property rights, from joint to single ownership, in the hands of one of the partners. This calls for a dissolution rule that assigns full property rights to the partner who makes the best use of these assets, and assures fair compensation of those who give up their ownership rights. The present paper analyzes two simple auction rules to solve these problems.

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1. Introduction

Partnerships form and dissolve. Examples range from marriage to business partnerships. Business partnerships are a frequent form of organization in law firms and small scale joint ventures among professionals with complementary skills.

Typically, a partnership invests in specific assets. Therefore, when one partner requests dissolution of the partnership, the best use of these assets

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is to grant single ownership to one of the partners, who may then continue to run the business, either alone or with a new partner.

Similarly, when the owner of a business deceases, an inheritance rule may stipulate that the firm is awarded to one of the heirs, in order to maintain undiluted incentives, with the provision that the other heirs earn some fair compensation.

A dissolution rule determines the partner who shall become single owner, and the compensation to be paid to the other partner(s).

The legal literature on partnership dissolution generally advises to use a simple dissolution rule, known as “Texas shootout” or “buy-sell option” (see Mancuso und Laurence (2003)). There, the party who asks for dissolution must propose a price at which the other partner(s) may either sell their share(s) or buy the proposer’s share. These rules are now so predominantly used that “... *a lawyer’s failure to recommend or include them in modern joint venture agreements is considered ‘malpractice’ among legal scholars and practitioners*” (Brooks und Spier (2004)).

In contrast to the generally accepted legal advice, economists tend to favor a more even handed mechanism, such as the use of an auction, in which all partners are asked to bid for the other partners’ shares (see Cramton, Gibbons, und Klemperer (1987), McAfee (1992), and the survey by Moldovanu (2002)). However, as De Frutos und Kittsteiner (2004) have shown, the two mechanisms tend to have similar properties if the “buy-sell option” is preceded by an open, ascending-bid (English) auction, that determines who shall have the right to propose in the execution of a subsequent Texas shootout.

The present paper considers the dissolution problem of an equal share partnership, and solves the equilibrium of two simple auction rules. It is closely related to the literature on fair division games (see Güth und van Damme (1986))¹, on partnership dissolution (see Engelbrecht-Wiggans (1994), whose results are corrected here), and to Lengwiler und Wolfstetter (2004), where we analyze some mathematically similar auction problems.

Like most of the partnership dissolution literature, we ignore the impact of dissolution rules on the very formation of partnerships. This leads to a restrictive notion of efficiency. As was shown in Li und Wolfstetter (2004), standard dissolution rules are generally not efficient, because they give rise

¹ Güth und van Damme (1986) solve the fair division game, assuming a uniform distribution of valuations and complete sharing of the revenue; Güth, Ivanova-Stenzel, Königstein, und Strobel (2002) present experimental evidence on such games.

to either efficient dissolutions, combined with overinvestment or excessive dissolutions, combined with underinvestment. This should be kept in mind in the interpretation of the efficiency notion employed here.

2. The model

Consider a partnership with $n \geq 2$ partners, each holding an equal share $\frac{1}{n}$ of the partnerships' assets. Partners have decided to dissolve and award single ownership to one partner. The selection of single owner is determined by an auction, possibly run by a professional auctioneer. That auction shall satisfy the following requirements:²

1. Single ownership is awarded to the highest bidder.
2. The winner either pays a price equal to his bid (first-price auction) or to the second highest bid (second-price auction); losers pay nothing.
3. Each bidder, winner and losers alike, receives an equal share α of the price, P , as a transfer, where $\alpha \leq \frac{1}{n}$.
4. If a professional auctioneer is employed, he receives a transfer equal to $(1 - n\alpha)P$; if no auctioneer is employed, the price is completely shared by bidders, i.e. $\alpha = \frac{1}{n}$.

Each partner knows how much the partnership is worth to him if he acquires single ownership. That valuation, denoted by x , is known to him but not to others (private value assumption). However, bidders' have beliefs about others' private valuations. These are described by independently and identically distributed random variables, each drawn from the continuously differentiable probability distribution F , with support $(0, 1)$ and density function $f(x) := F'(x)$. In other words, we assume the *symmetric, independent, private values* framework.

From the perspective of one bidder, the valuations of all rival bidders is a random sample of size $n - 1$. We denote the highest and second highest of these $n - 1$ valuations by the order statistics Y_1 resp. Y_2 . The probability distribution function of Y_1 is obviously $G(x) := \Pr\{Y_1 \leq x\} = F(x)^{n-1}$.

² In the following we always denote a random variable by a capital letter and its realization by a corresponding lower case letter.

The two auction games are solved by the following procedure: 1) we invoke the working hypothesis that the equilibrium bid function, β_1 (first-price auction), resp. β_2 (second-price auction), is symmetric and strict monotone increasing. And we then show that these assumptions confirm.

The strict monotonicity of bid functions assures that the highest bidder is the bidder with the highest valuation; similarly, the second- resp. third-highest bidder is the bidder with valuation Y_1 , resp. Y_2 . Therefore, if a bidder makes the bid $\beta(y)$, his bid is the highest and thus wins with probability

$$\Pr\{\beta(Y_1) < \beta(y)\} = \Pr\{Y_1 < y\} = G(y) = F(y)^{n-1}. \quad (1)$$

3. Solution of the first-price auction

Consider a bidder with valuation x who assumes that all rival bidders play the strict monotone increasing bid strategy β_1 . If that bidder computes his best reply bid, he can restrict bidding to $b \in [\beta_1(0), \beta_1(1)]$, because bidding outside that range is obviously not profitable. Therefore, bidding according to the strategy β_1 as if the valuation were equal to $y \in (0, 1)$, rather than the true valuation, x , captures all relevant deviating bids. We can thus write the payoff function of a bidder who has the valuation x , yet makes the bid $\beta_1(y)$, in the form (where $m_1(y)$ denotes bidders' expected net payment)

$$\begin{aligned} U_1(x, y) &:= G(y)x - m_1(y) \\ m_1(y) &:= G(y)(1 - \alpha)\beta_1(y) \\ &\quad - \alpha(1 - G(y))E[\beta_1(Y_1) \mid Y_1 > y]. \end{aligned} \quad (2)$$

In equilibrium, the bid function β_1 is such that it is each bidder's best reply to bid $\beta_1(x)$, rather than $\beta_1(y)$, $y \neq x$. Therefore, β_1 is an equilibrium if and only if $x = \arg \max_y U_1(x, y)$, $\forall x$.

Using the first-order condition for a best-reply, this leads to the differential equation

$$0 = \frac{\partial}{\partial y} U_1(x, y)|_{y=x} = (x - \beta_1(x))G'(x) - (1 - \alpha)\beta_1'(x)G(x). \quad (3)$$

Using the transformation $H(x) := G(x)^{\frac{1}{1-\alpha}}$, this differential equation can be written in the form

$$(H(x)\beta_1(x))' = xH(x). \quad (4)$$

Proposition 1 Consider first-price auctions. The equilibrium bid function, β_1 , and bidders' equilibrium expected payoffs, $u_1(x)$, are³

$$\beta_1(x) = x - \int_0^x \frac{H(y)}{H(x)} dy, \quad H(x) := F(x)^{\frac{n-1}{1-\alpha}} \quad (5)$$

$$= E_{Y_1 \sim H} [Y_1 \mid Y_1 < x],$$

$$u_1(x) := U_1(x, x) = \int_0^x G(y) dy + u_1(0) \quad (6)$$

$$u_1(0) = \alpha \int_0^1 \beta_1(y) dG(y). \quad (7)$$

Both β_1 and u_1 are strict monotone increasing.

PROOF Integration of (4) yields immediately (5). This strategy is obviously strict monotone increasing. In order to complete the proof, one must show that the first-order condition assures a global best reply. In the *section 6* we show that it does. (That proof applies to all auction rules considered here.)

Having established a strict monotone increasing, symmetric equilibrium, bidders' equilibrium payoff can be computed as follows: By a well-known result,⁴

$$m_1(x) = m_1(0) + \int_0^x y G'(y) dy. \quad (8)$$

In equilibrium, a bidder with valuation $x = 0$ never wins the auction, yet earns his share of the auction revenue; therefore,

$$m_1(0) = -\alpha \int_0^1 \beta_1(y) dG(y). \quad (9)$$

Combining (2), (8) and (9) gives (7), as asserted.

Note that even a bidder with valuation $x = 0$ has a positive payoff, $u_1(0) > 0$, unlike in the standard first-price auction without revenue sharing. \square

³ $E_{Y_1 \sim H} [Y_1 \mid Y_1 < x]$ denotes the conditional expected value of the order statistic Y_1 , assuming Y_1 is drawn from the c.d.f. H .

⁴ This standard result applies to symmetric, pure strategy equilibria of all symmetric, independent, private values auctions that award the auctioned object to the highest bidder. See Wolfstetter (1999, Proposition 8.2) or Krishna (2002, Proposition 3.1).

Corollary 1 β_1 can be viewed as the solution of a standard first-price auction, where Y_1 is drawn from the fictitious c.d.f. $H := F^{\frac{n-1}{1-\alpha}}$ rather than the true c.d.f. $G := F^{n-1}$. Bidding is uniformly higher than in a standard first-price auction without revenue sharing.

PROOF By a well-known result, the equilibrium bid function in a first-price auction without revenue sharing is $\tilde{\beta}_1(x) = E_{Y_1 \sim G}[Y_1 | Y_1 < x]$. Therefore, β_1 can be viewed as the solution of a standard first-price auction, where Y_1 is drawn from the c.d.f. H rather than the true distribution G . Evidently, for $\alpha > 0$, H first-order stochastically dominates G ; therefore, by the well-known “*First-Order Stochastic Dominance Theorem*” (see Wolfstetter (1999, Lemma 4.1))

$$\begin{aligned} \beta_1(x) &= E_{Y_1 \sim H}[Y_1 | Y_1 < x] \\ &> E_{Y_1 \sim G}[Y_1 | Y_1 < x] \\ &= \tilde{\beta}_1(x). \end{aligned} \tag{10}$$

□

Corollary 2 Suppose no auctioneer is employed and bidders share the entire revenue ($\alpha = \frac{1}{n}$). Then, β_1 is equivalent to a standard first-price auction with one additional bidder.

PROOF If $\alpha = \frac{1}{n}$, one obtains $H(x) = F(x)^{\frac{n-1}{1-\alpha}} = F^n$. Therefore, H can be viewed as the c.d.f. of the order statistic Y_1 for an underlying sample of n rather than $n - 1$ valuations, i.e. as if $n + 1$ rather than n bidders would participate. □

4. Solution of the second-price or English auction

In a private values context, second-price and open, ascending-bid (English) auctions are equivalent.

In second-price auctions one must distinguish between three payoff relevant events:

1. the event of winning the auction, which occurs if one’s bid is the highest;

2. the event of losing and making the price, which occurs if one's bid is the second-highest;
3. the event of losing without making the price, which occurs if one's bid is the third- or lower-highest.

The case of two bidders is somewhat special, because in that case the “third-highest” bid is equal to the reserve price (or minimum bid), which we normalize to zero. Therefore, the cases of two and more than two bidders require a separate analysis.

4.1 Solution for the case of two bidders

If $n = 2$, the payoff function of a bidder with valuation x who bids $\beta_2(y)$, and assumes that all other bidders play the strict monotone increasing strategy β_2 is

$$U_2(x, y) = F(y)x - m_2(y) \quad (11)$$

$$m_2(y) = F(y)(1 - \alpha)E[\beta_2(Y_1) | Y_1 < y] - (1 - F(y))\alpha\beta_2(y).$$

Again, β_2 is an equilibrium if and only if $x = \arg \max_y U_2(x, y), \forall x$. Using the first-order conditions for a best-reply, this gives rise to

$$0 = \frac{\partial}{\partial y} U_2(x, y)|_{y=x} = (x - \beta_2(x))F'(x) + \alpha(1 - F(x))\beta_2'(x). \quad (12)$$

After a bit of rearranging, one obtains the differential equation

$$\beta_2(x) - x = \alpha \frac{1 - F(x)}{F'(x)} \beta_2'(x), \quad (13)$$

which, using the transformation $K(x) := (1 - F(x))^{\frac{1}{\alpha}}$, can be written as

$$(\beta_2(x)K(x))' = xK'(x). \quad (14)$$

Proposition 2 *Consider a second-price auction with two bidders. The equilibrium bid function, β_2 , and bidders' equilibrium expected payoffs, $u_2(x)$, are*

$$\beta_2(x) = x + \int_x^1 \frac{K(y)}{K(x)} dy, \quad (15)$$

$$u_2(x) := U_2(x, x) = \int_0^x F(y) dy + u_2(0) \quad (16)$$

$$u_2(0) = \alpha \int_0^1 K(y) dy. \quad (17)$$

Both β_2 and u_2 are strict monotone increasing, and $\beta_2(x) > x, \forall x > 0$.

PROOF Integration of (14) and a bit of rearranging gives

$$\begin{aligned} \beta_2(x)K(x) &= \beta_2(0)K(0) + \int_0^x yK'(y) dy \\ &= \beta_2(0) + xK(x) - \int_0^x K(y) dy. \end{aligned} \quad (18)$$

Setting $x = 1$, one finds $\beta_2(0) = \int_0^1 K(y) dy$. Therefore,

$$\beta_2(x) = x + \int_x^1 \frac{K(y)}{K(x)} dy, \quad (19)$$

$$\begin{aligned} u_2(x) &= F(x)x - m_2(x) \\ &= \int_0^x F(y) dy + \alpha\beta_2(0) \\ &= \int_0^x F(y) dy + \alpha \int_0^1 K(y) dy. \end{aligned} \quad (20)$$

□

Corollary 3 *Evidently, the equilibrium bid function exhibits bidding more than one's valuation. Also, since $u_2(0) > 0$, bidders' equilibrium payoff is uniformly higher than in a second-price auction without revenue sharing (there, bidders' equilibrium expected payoff is equal to $\int_0^x F(y) dy$).*

The solution of β_2 is the same as that found in Engelbrecht-Wiggans (1994, Corollary2b).

4.2 Solution for the case of more than two bidders

If there are more than two bidders ($n > 2$), the payoff function $U_2(x, y)$ is

$$\begin{aligned}
U_2(x, y) &:= G(y)x - m_2(y) \\
m_2(y) &:= G(y)(1 - \alpha)E[\beta_2(Y_1) \mid Y_1 < y] \\
&\quad - \Pr\{Y_2 < y < Y_1\}\alpha\beta_2(y) \\
&\quad - \Pr\{Y_2 > y\}\alpha E[\beta_2(Y_2) \mid Y_2 > y].
\end{aligned} \tag{21}$$

In order to compute $U_2(x, y)$ one needs to know the joint density function of the order statistics Y_1, Y_2 , denoted by $f_{Y_2 Y_1}$, and the density function of Y_2 , denoted by f_{Y_2} ; these are equal to (see David (1970, Ch. 2))

$$f_{Y_2 Y_1}(r, z) = (n-1)(n-2)F(r)^{n-3}f(r)f(z), \quad \text{for } r \leq z, \tag{22}$$

$$\begin{aligned}
f_{Y_2}(r) &= (n-1)(n-2)F(r)^{n-3}f(r)(1-F(r)) \\
&= (n-1)\frac{d}{dr}F(r)^{n-2}(1-F(r)),
\end{aligned} \tag{23}$$

and, of course, $f_{Y_2 Y_1}(r, z) = 0$ for $r > z$. Using these, one can rewrite (21), after a bit of rearranging, in the form

$$\begin{aligned}
U_2(x, y) &= G(y)x - (1 - \alpha) \int_0^y \beta_2(z) dG(z) \\
&\quad + \alpha(n-1)\beta_2(y)(1-F(y))F(y)^{n-2} \\
&\quad + \alpha(n-1) \int_y^1 \beta_2(r) \frac{d}{dr}F(r)^{n-2}(1-F(r)) dr.
\end{aligned} \tag{24}$$

Again, β_2 is an equilibrium if and only if $x = \arg \max_y U_2(x, y), \forall x$. Using the first-order conditions for a best-reply, one obtains, after some rearranging, the differential equation

$$\beta_2(x) - x = \alpha \beta_2'(x) \frac{1-F(x)}{F'(x)}. \tag{25}$$

Proposition 3 Consider a second-price auction with more than two bidders. The equilibrium bid function, β_2 , and bidders' equilibrium expected payoffs, $u_2(x)$, are

$$\beta_2(x) = x + \int_x^1 \frac{K(y)}{K(x)} dy, \quad K(x) := (1 - F(x))^{\frac{1}{\alpha}}, \quad (26)$$

$$u_2(x) := U_2(x, x) = \int_0^x G(y) dy + u_2(0), \quad (27)$$

$$u_2(0) = \alpha(n-1) \int_0^1 \beta_2(z)(1-F(z)) dF(z)^{n-2}. \quad (28)$$

Both β_2 and u_2 are strict monotone increasing, and $\beta_2(x) > x, \forall x > 0$.

PROOF The differential equation (25) is the same as (13). Therefore, for given α the equilibrium bid function is the same as in the case of $n = 2$. Note, however, that α tends to decrease in n , because $n\alpha \leq 1$.

In order to compute $u_2(x)$, recall that $m_2(x) = m_2(0) + \int_0^x yG'(y)dy$. A bidder with valuation $x = 0$ wins the auction with probability zero, and makes the price, as a loser, with probability zero as well. Therefore, using (23),

$$\begin{aligned} m_2(0) &= -\alpha \int_0^1 \beta_2(z) f_{Y_2}(z) dz \\ &= -\alpha(n-1) \int_0^1 \beta_2(z)(1-F(z)) dF(z)^{n-2}. \end{aligned} \quad (29)$$

and the assertion follows immediately from the definition of $u_2(x)$. \square

The solution is not the same as that proposed in Engelbrecht-Wiggans (1994, Corollary 2b), which is incorrect.

5. Failure of revenue equivalence

By the well-known “revenue equivalence theorem”, two auction rules are payoff equivalent to all n bidders and to the auctioneer if and only if they exhibit the same allocation rule, and if the worst-off type (i.e., the bidder with valuation $x = 0$) earns the same equilibrium payoff, $u(0)$, (see Myerson (1981)).

Both auction formats have the same allocation rule, because the auction is won by the bidder with the highest valuation. Therefore both auctions are efficient. However, it is not immediately clear whether $u_1(0) = u_2(0)$; therefore it is not clear which auction format generates higher payoffs either to bidders or to the auctioneer.

Of course, if no auctioneer is employed and paid to run the auction, so that $\alpha = \frac{1}{n}$, one must have $u_1(x) = u_2(x)$. Because if one of these equilibrium payoffs were greater for some x , it would be uniformly greater for all x , due to $u_i(x) = \int_0^x G(y)dy - m_i(0)$, $i \in \{1, 2\}$; but this contradicts the efficiency of the two auctions.⁵

Therefore revenue equivalence is only open to doubt if a professional auctioneer is employed and paid part of the revenue, i.e. if $\alpha < \frac{1}{n}$.

Proposition 4 *Suppose $\alpha < \frac{1}{n}$. Then, for the same α first- and second-price auctions are generally not payoff equivalent to bidders and the auctioneer.*

PROOF Recall that for all x : $u_2(x) - u_1(x) = u_2(0) - u_1(0)$. Therefore we only need to show that $u_1(0) = u_2(0)$ does not hold true for all parameters and probability distributions.

In Table 1 we summarize several examples, with different combinations of n and α . Evidently, all examples that assume $\alpha < \frac{1}{n}$ violate payoff equivalence.

Since both auctions are efficient (i.e., the bidder with the highest valuation wins the auction), bidders prefer the second-price auction if and only if $u_2(0) > u_1(0)$ and the auctioneer has exactly the opposite preference order. \square

Example 1 *Table 1 compares $u_1(0)$ and $u_2(0)$ for different combinations of n and α . All examples assume a uniform distribution. The first example illustrates the fact that both auctions are revenue equivalent if no auctioneer is employed ($\alpha = \frac{1}{n}$); all other examples indicate that revenue equivalence fails if the auctioneer receives part of the revenue ($\alpha < \frac{1}{n}$).*

These examples suggest that, for a given sharing parameter α , the second-price auction tends to yield higher equilibrium payoffs to bidders, and hence a lower equilibrium payoff to the auctioneer. This suggests that an auctioneer who is paid a given share of the equilibrium price should propose a first-price auction, whereas bidders should insist on a second-price auction.

However, this conflict of interest occurs only for a *given* compensation scheme. By appropriately choosing *different* sharing parameters one can always make first- and second-price auctions payoff equivalent.

⁵ Due to the efficiency of the two auctions, the expected equilibrium payoffs, $E[u_i(X)]$, must be same, and equal to the expected value of the highest valuation of the entire sample of n valuations.

	$n = 2, \alpha = 1/2$	$n = 2, \alpha = 1/3$	$n = 5, \alpha = 1/6$
$u_2(0)$	1/6	1/10	16/96
$u_1(0)$	1/6	1/12	16/145
$u_2(x) - u_1(x)$	0	> 0	> 0

Table 1: Illustration of bidders' equilibrium payoffs

6. Second-order conditions

When we derived the equilibrium, we employed only the first order condition for a best reply. Here we complete the proofs of equilibrium by showing that $U(x, y)$ is increasing in y for $y < x$ and decreasing for $y > x$.⁶ Naturally, this proves that bidders' payoff is maximized at $y = x$. In other words, for a bidder with valuation x it never pays to make a deviating bid $\beta(y) \neq \beta(x)$, if all other bidders play the candidate equilibrium strategy β . This proof applies to both auctions.

Consider a bidder with valuation x who assumes that all rival bidders play the strict monotone increasing candidate equilibrium strategy β . Without loss of generality that bidder considers only deviating bids, $\beta(y)$, for $y \in [0, 1]$. By construction and the monotonicity of β one has

$$\frac{\partial}{\partial y} U(x, y)|_{y=x} = 0, \quad \forall x, \quad (30)$$

$$\frac{\partial^2}{\partial y \partial x} U(x, y) = G'(y) > 0, \quad \forall x, y. \quad (31)$$

Now consider deviating bids, i.e. either $\beta(y) < \beta(x)$ or $\beta(y) > \beta(x)$. Then, by the above,

$$y < x \Rightarrow \frac{\partial}{\partial y} U(x, y) > \frac{\partial}{\partial y} U(y, y) = 0, \quad (32)$$

$$y > x \Rightarrow \frac{\partial}{\partial y} U(x, y) < \frac{\partial}{\partial y} U(y, y) = 0. \quad (33)$$

⁶ In the optimization literature, this property of a function is known as its "pseudo-concavity".

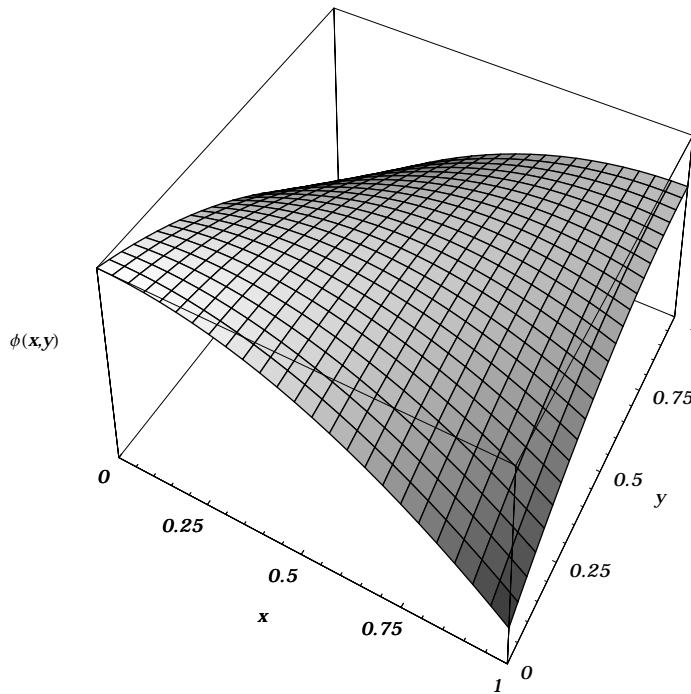


Figure 1: $\phi(x,y) := U_2(x,y) - u_2(x)$ for $n = 2, F(x) = x$, and $\alpha = \frac{1}{2}$.

In words, $U(x,y)$ is increasing in y for all $y < x$, and decreasing for all $y > x$. Therefore, bidding according to the candidate equilibrium strategy, $\beta(x)$ is a global best reply; hence, β an equilibrium.

An illustration of this property is in Figure 1, assuming a second-price auction with two bidders, a uniform distribution, and $\alpha = \frac{1}{2}$. In this figure we plot the gain from deviating from the equilibrium strategy. All the image values off the main diagonal of the domain depict the gain from making the deviating bid $\beta(y)$ instead of the equilibrium bid $\beta(x)$. Evidently, no deviation pays.

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