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Bidding behavior in asymmetric auctions: An experimental study

Werner Güth[a], Radosveta Ivanova-Stenzel[b,*], Elmar Wolfstetter[b]

[a]Max Planck Institute for Research into Economic Systems, Kahlaische Str. 10, D-07745 Jena, Germany
[b]Department of Economics, Humboldt University Berlin, Spandauer Str. 1, D - 10099 Berlin, Germany

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Abstract

We review an asymmetric auction experiment. Based on Plum (Int. J. Game Theory 20 (1992) 393) private valuations of the two bidders are independently drawn from distinct but commonly known distributions, one of which first-order stochastically dominates the other. We test the qualitative properties of that model of asymmetric auctions, in particular whether the weak bidder behaves more aggressively than the strong, and then test bidders' preference for first- vs. second-price auctions.

JEL classification: D44; C91

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1. Introduction

As in most of the theoretical literature, auction experiments typically assume that bidders' valuations or signals are drawn from the same probability distribution

*Corresponding author.
E-mail addresses: gueth@mpiew-jena.mpg.de (W. Güth), ivanova@wiwi.hu-berlin.de (R. Ivanova-Stenzel), wolf@wiwi.hu-berlin.de (E. Wolfstetter).

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(see the survey by Kagel (1995)). However, this symmetry assumption is violated in many real-life auction environments, because bidders often know that and how they differ, for example, in the light of earlier experiences or due to collusion between subsets of otherwise symmetric bidders. The limited relevance of the symmetric auction framework is aggravated by the fact that many of the celebrated results of symmetric auctions, such as the revenue equivalence of a larger class of auction games, do not extend to asymmetric auctions.

There is a small theoretical literature on asymmetric auctions, which deals with various kinds of asymmetries: asymmetries between commonly known distribution functions from which valuations or signals are independently drawn, asymmetries induced by a known ranking of valuations (which involves a particular stochastic dependency), and asymmetries between valuation functions while maintaining the symmetry of the distribution from which private signals are drawn.

Several authors have followed the first approach to model bidder asymmetry, assuming that bidders' valuations are independently drawn from different probability distributions, which are common knowledge among them. In this spirit, Vickrey (1961) already considers an auction with two bidders in which one bidder's valuation is known to the other bidder with certainty,[1] and Griesmer et al. (1967) analyze first-price auctions with two bidders whose valuations are uniformly distributed on different supports.

More recently, Plum (1992) analyzes the two-bidder case for arbitrary continuous distributions and proves that the first-price auction has a unique pure strategy equilibrium with strictly monotone increasing bid functions. In addition, he explicitly solves the first-price auction game for a parametric class of distribution functions.

In a similar framework, Maskin and Riley (2000a) explain the properties of several asymmetric two-bidder examples, where a stochastic order stronger than first-order stochastic dominance is assumed. Maskin and Riley (2000b), Reny (1999), and Jackson et al. (2001) analyze the existence of pure strategy equilibria in first-price auctions for the general n bidder case, and Lebrun (1999) proves uniqueness of pure strategy equilibria in the general n bidder case.[2]

Asymmetries induced by a known ranking of valuations are analyzed by Landsberger et al. (2001). This asymmetry cannot be subsumed under the approach that assumes that valuations are independently drawn from commonly known distribution functions. Indeed, it generates distinct results. Elbittar (2002) tests experimentally the behavioral components of the comparative-statics predictions derived by Landsberger et al. (2001) regarding the bidders' behavior, the auctioneer's expected revenue, and the efficiency of the allocation. Part of his results support the

[1]This asymmetric auction game has only an equilibrium in mixed strategies (see also Holt and Solis-Soberon, 1992).

[2]Incidentally, these proofs of existence employ very different methods. Maskin and Riley (2000b) use topological methods developed by Dasgupta and Maskin (1986). Plum (1992) and Lebrun (1999) establish directly that a solution to a suitable set of differential equations exists. Reny (1999) employs his concept of "payoff secure" games. Finally, Jackson et al. (2001) view the tie-breaking rule as part of the solution of the game and show that there is always some tie-breaking rule for which an equilibrium exists.

theoretical predictions concerning bidders' behavior and efficiency. The bidder with the lower valuation bids more aggressively than the high-valuation bidder, and the achieved efficiency is lower than when there is no information about the ranking of valuations. Contrary to the theoretical predictions, revelation of the ranking of valuation does not always lead to higher expected revenue for the seller.

Another branch of the literature has also begun to analyze asymmetries between valuation functions in the affiliated and (almost) common-value framework, while maintaining the symmetry of the distribution from which private signals are drawn (see the example by Bikchandani (1988) and the associated experiment by Avery and Kagel (1998); see also the example by Bulow et al. (1999)).

In the literature, there are only a few asymmetric auction experiments. Most of them concern the common-value case (see the survey by Kagel (1995)). To the best of our knowledge, there are only two experiments that assume private values: the already mentioned Elbittar (2002) and the study by Pezanis-Christou (2002). The latter tests one particular version of the Maskin and Riley model (2000a) which assumes that one of two bidders has greater probability of not being able to bid. He shows that the first-price auction tends to generate higher revenue than the second-price auction, contrary to what theory predicts.

The present paper reviews a laboratory experiment of bidding behavior in asymmetric auctions, in which valuations are private information and are independently drawn from distinct but commonly known distribution functions. The experiment employs the functional specification used by Plum (1992) and Kalkofen and Plum (1996) for which explicit equilibrium solutions of bidding strategies are available. We explore whether actual bidding exhibits similar qualitative properties as the game-theoretic solution. In particular, we compare first- and second-price auctions, and ask:

- Does the weak bidder bid more aggressively than the strong bidder?
- Does the first-price auction raise more expected revenue for the seller than the second-price auction?
- Does the second-price auction generate higher payoffs to the strong bidder and the first-price auction to the weak bidder?
- Do bidders rank the two auctions accordingly?

More specifically, we conduct an experiment where subjects are assigned either the role of the weak or the strong bidder and play repeatedly both first-price and second-price auctions with a randomly chosen partner of the opposite type. One notable feature of our experiment is that, at some point, bidders are given the chance to select the auction rule before or after they learn their valuation. This allows us to examine subjects' preferences over the two auction rules and to check the consistency of their behavior with respect to the profitability of these rules. In particular, we let them bid for the right to choose the favored auction against a random price generator in the spirit of the Becker et al. (1963) mechanism which should induce participants to reveal their true willingness to pay for the right to dictate the auction rule (first- or second-price).

Our main results are that first-price auctions generate higher revenue to the seller. In second-price auctions, truthful bidding turns out to be a reasonably good prediction. In first-price auctions, the bidder with the more favorable valuation shades his bid more than the bidder with the lower valuation. However, both bidders bid significantly higher than they should, and the difference between the weak and the strong bidders' bid functions is not as large as predicted by theory. The payoffs earned by both bidders in second-price auctions far exceed that obtained in first-price auctions. When subjects can choose the auction rule, they overwhelmingly select the one that yields the higher payoff. As expected, strong bidders tend to bid more for the right to dictate the auction rule.

The remainder of the paper is organized as follows: The theoretical background and the experimental design are explained in Sections 2 and 3. In Section 4, we present the main experimental results. We close with a summary of the results in Section 5.

2. Theoretical background

We consider a slightly simplified version of Plum (1992). Specifically, two risk-neutral bidders ($i = 1, 2$) compete for the purchase of a single item in either a first- or a second-price sealed-bid auction. Bidders' valuations are private information and independently drawn from uniform distributions with supports $[\alpha, \beta_1]$ and $[\alpha, \beta_2]$, with

$$\beta_2 = 200 > \beta_1 = 150 > \alpha = 50, \tag{1}$$

as illustrated in Fig. 1. Therefore, the random valuation V_2 is obtained by "stretching" the valuation V_1. Obviously, V_2 is more favorable than V_1, in the strong sense of first-order stochastic dominance.

Equilibrium bid functions: As shown by Plum (1992, pp. 401–403), the first-price sealed-bid auction has the following equilibrium bid functions:

$$b_i^f(v_i) = \alpha + \frac{v_i - \alpha}{1 + \sqrt{1 + \gamma_i c (v_i - \alpha)^2}} \quad \text{for } i = 1, 2, \tag{2}$$

$$\text{with } c := \frac{1}{(\beta_1 - \alpha)^2} - \frac{1}{(\beta_2 - \alpha)^2}; \quad \gamma_1 := -1, \ \gamma_2 := 1. \tag{3}$$

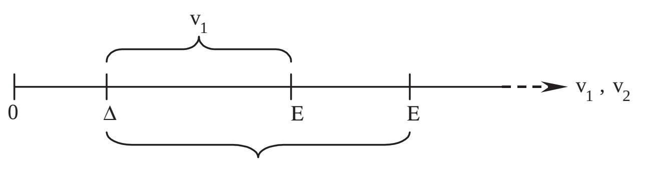

Fig. 1. Support of random valuations.

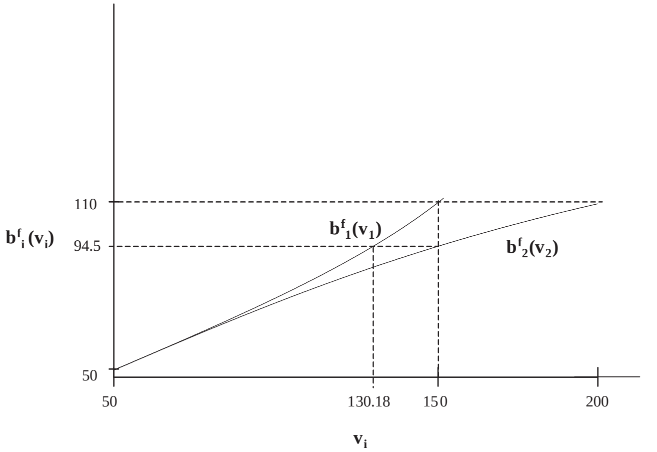

Fig. 2. Equilibrium bid functions in first-price sealed-bid auction.

These are plotted in Fig. 2, which shows that the bidder with the more favorable valuation (bidder 2) bids pointwise less than bidder 1. An immediate implication is that the first-price auction gives rise to inefficiency, because the bidder with the lower valuation wins the auction with positive probability (e.g., if $v_2 = 150$, the bidder with the lower valuation wins the auction for all V_1 in (130.18,150), as illustrated in Fig. 2).

Of course, truthful bidding is an equilibrium in weakly dominant strategies in the second-price sealed-bid auction. However, the strategy of bidder 2 is somewhat arbitrary if his valuation is greater than β_1. Indeed, in that case all bids $b \in [\beta_1, \beta_2]$ are optimal. This follows from the fact that by elimination of weakly dominated strategies, bidder 1 will never bid more than β_1.[3]

Bidders' equilibrium payoffs: Using Plum's solution of the two auction games, we now compute bidders' equilibrium payoffs, denoted by $u_i(v_i)$. These are needed as a benchmark in our experiment.

If the auction is second-price, the computation of $u_i^s(v_i)$ is straightforward. By a well-known result $u_i^{s\prime}(v_i) = F_j(v_i)$, therefore:

$$u_1^s(v_1) = \int_\alpha^{v_1} F_2(x)\mathrm{d}x \quad \text{and} \tag{4}$$

$$u_2^s(v_2) = \begin{cases} \int_\alpha^{v_2} F_1(x)\mathrm{d}x & \text{if } v_2 \leqslant \beta_1, \\ \int_\alpha^{\beta_1} F_1(x)\mathrm{d}x + v_2 - \beta_1 & \text{if } v_2 \in [\beta_1, \beta_2]. \end{cases} \tag{5}$$

[3]In the experiment, quite a few bidders 2 with valuations above β_1 did not bid truthfully, but submitted bids equal to β_1 and some even equal to β_2. This is consistent with equilibrium bidding.

If the auction is first-price, bidders i's equilibrium probability of winning is

$$\Pr(b_i^f(v_i) > b_j^f(V_j)) = \Pr(V_j < b_j^{f^{-1}}(b_i^f(v_i))) \\ = F_j(b_j^{f^{-1}}(b_i^f(v_i))). \tag{6}$$

Therefore, in order to compute $u_i^f(v_i)$, one needs to find the inverse $\phi_i(x)$ of the equilibrium bid function which is defined on bids x. This requires some transformation of variables. For this purpose, we define $B_i := b_i - \alpha$, $x_i := v_i - \alpha$, $d_i := \sqrt{1 + \gamma_i c x_i^2}$, and after a few manipulations, one obtains $x_i = 2B_i/(1 - \gamma_i B_i^2 c)$. Hence,

$$\phi_i(x) := b_i^{f^{-1}}(x) = \alpha + \frac{2(x-\alpha)}{1 - \gamma_i (x-\alpha)^2 c}. \tag{7}$$

Therefore, one obtains:

$$u_i^f(v_i) := F_j(\phi_j(b_i^f(v_i)))(v_i - b_i^f(v_i)) \\ = \frac{v_i - b_i^f(v_i)}{\beta_j - \alpha}\left(\frac{2(b_i^f(v_i) - \alpha)}{1 - \gamma_j (b_i^f(v_i) - \alpha)^2 c}\right). \tag{8}$$

Bidders' equilibrium payoffs are plotted for both auction rules in Fig. 3, which illustrates that the strong bidder 2 strictly prefers the second-price auction, whereas the weak bidder 1 strictly prefers the first-price auction:

$$u_1^f(v_1) > u_1^s(v_1) \quad \text{for all } v_1 \in (\alpha, \beta_1], \tag{9}$$

$$u_2^s(v_2) > u_2^f(v_2) \quad \text{for all } v_2 \in (\alpha, \beta_2]. \tag{10}$$

These opposite rankings are intuitively appealing, and they are consistent with a key result in Maskin and Riley (2000a).

Bidders' equilibrium payoffs behind the veil of ignorance: From the $u_i(v_i)$'s, one can compute bidders' equilibrium payoffs behind the veil of ignorance, i.e., the expected

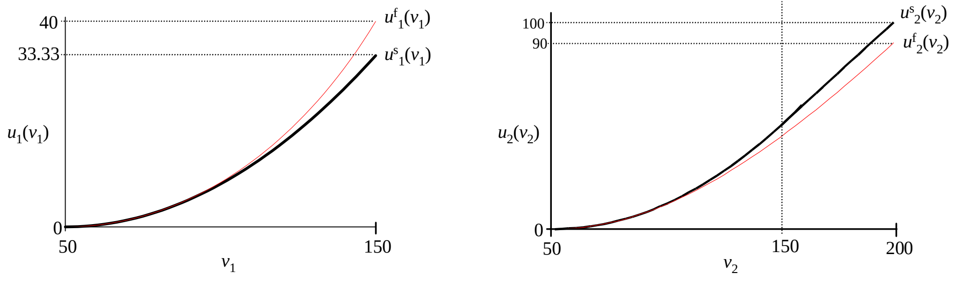

Fig. 3. Bidders' equilibrium payoffs.

payoffs determined before valuations are drawn:

$$U_i^s := E[u_i^s(V_i)] = \frac{1}{\beta_i - \alpha}\int_\alpha^{\beta_i} u_i^s(x)\mathrm{d}x, \tag{11}$$

$$U_i^f := E[u_i^f(V_i)] = \frac{1}{\beta_i - \alpha}\int_\alpha^{\beta_i} u_i^f(x)\mathrm{d}x. \tag{12}$$

For the assumed valuations of the parameters α, β_i this implies:

$$U_1^s = 11.111 < U_1^f = 12.295, \tag{13}$$

$$U_2^s = 36.111 > U_2^f = 32.474. \tag{14}$$

Of course, this ranking is implied by (9)–(10).

Bidders' value of dictatorship behind the veil of ignorance: In the second phase of the experiment, we give bidders the chance to select the auction rule before they know their valuation. Specifically, we let them bid for the right to choose the favored auction against a random price generator (Becker et al., 1963). After bids are made, one bidder is selected with probability 1/2 as potential "dictator" who may choose the auction rule, and a price is drawn at random. If the potential dictator's bid is at or above the random price, he must pay that price and choose the auction rule; otherwise the auctioneer selects the auction rule by the flip of a fair coin.

Obviously, truthful bidding is the weakly dominant strategy, and the value of the right to choose the auction, which we call value of dictatorship, L, is

$$L_i = \max\{U_i^s, U_i^f\} - \tfrac{1}{2}(U_i^s + U_i^f), \tag{15}$$

$$L_1 = 0.592 < 1.8185 = L_2. \tag{16}$$

Of course, if a bidder becomes dictator, he chooses the auction that yields the higher U_i. As one can see from (13) to (14), the weak bidder 1 prefers the first-price auction, whereas the strong bidder 2 favors the second-price auction. In view of $L_2 > L_1$, bidder 2 will win against the random price generator more frequently than bidder 1.

Value of dictatorship if bidders know their valuation: In the third phase of the experiment, we give bidders the chance to select the auction rule after they learn their valuation, i.e., after the veil of ignorance has been removed, again using the random price mechanism. In the spirit of L_i, one may define the value of dictatorship as follows:

$$l_i(v_i) = \max\{u_i^s(v_i), u_i^f(v_i)\} - \tfrac{1}{2}(u_i^s(v_i) + u_i^f(v_i)). \tag{17}$$

However, this ignores some subtle updating and strategic signaling issues. For example, in the event that the strong bidder 2 fails to dictate the preferred second-price format, player 1 should update his probability assessment of v_2, which in turn affects the equilibrium strategies of the subsequent first-price auction game and hence the equilibrium payoff $u_i^f(v_i)$ in (17). Therefore, (17) can only serve as a rough benchmark, if at all.

3. Experimental design

In the experiment, each subject played either the role of the weak or the strong bidder. In each auction, subjects confronted a randomly selected partner of the opposite type. Each experimental session was subdivided into three phases:

The first phase consisted of four cycles of 12 bidding rounds. In each cycle, six first-price auctions were followed by six second-price auctions. In each round, private valuations v_1 and v_2 were randomly drawn before bids $b_1(v_1)$ and $b_2(v_2)$ were made.

The second phase consisted of 16 bidding rounds. Prior to bidding, bidders had a chance to dictate the auction rule according to the following procedure: (1) Both bidders $i \in \{1, 2\}$ were asked to state their maximum willingness to pay for the right to dictate the auction rule (value of dictatorship). (2) An unbiased chance move selected one bidder as potential dictator. (3) Another chance move drew a random number from a uniform distribution with support $[0, 30]$. The potential dictator had to choose the auction rule if that random number was not greater than his declared maximum willingness to pay; otherwise the auction rule was selected by the flip of a fair coin. Subsequently, the auction was played as in the first phase.

The third phase differed from the second only in the timing of events; there, bidders knew their private valuations v_1 and v_2 before any decision was taken.

After each auction bidders were informed about the outcome: whether they won that auction, the price paid by the winner, their own private valuation, all bids, and their own payoff. In addition, they were given an account of their total profit up to this round, and their average profit in each auction type. At the end of each auction in phases two and three, the dictator also learned about his cost for choosing the pricing rule.

In the experiment, we neither excluded over- nor underbidding by letting v_1 vary from 50 to 150 and v_2 from 50 to 200, whereas bids $b_i(v_i)$ could vary from 0 to 250. The random price p of the second (third) phase was chosen from the range $0 \leqslant p \leqslant 30$ since the value of the right to dictate the pricing rule should always be nonnegative. Of course, neither v_i nor $b_i(v_i)$ can vary continuously in a computerized setup. In our experiment, both, v_i and b_i, were integers. The two private valuations v_1 and v_2 were independently drawn from a uniform distribution with support $\{50, 51, \ldots, 150\}$ and $\{50, 51, \ldots, 200\}$, respectively.

All experimental sessions were conducted with the use of a computer-based software system, created with z-Tree (Fischbacher, 1998). Participants were invited to register for the experiment (mainly by the distributing of leaflets in undergraduate courses of the economics faculty of Humboldt University, Berlin). After entering the computer laboratory, participants were seated at visually separated terminals where they could read the instructions (see the Appendix). They could privately ask for clarifications but not for advice. We performed eight sessions, seven with 14 participants and one with 12 participants each (due to a technical problem, only the results of the first phase could be saved for one of the eight sessions). All sessions lasted about 2 hours. In the experiment, we used a fictitious currency called Experimental Currency Unit (ECU). The cash rate of the ECU earned by each subject was: 100 ECU = 1.50 DEM (EUR 0.77). In addition, subjects received an endowment of 700 ECU to cover possible losses. Subjects' total earnings ranged between 13.18 DEM (EUR 6.74) and 66.78 DEM (EUR 31.14) with a mean of 32.73 DEM (EUR 16.73).

4. Results

In the following, we use the theoretical results of Section 2 as a benchmark to assess the actual bidding behavior in phases 1 and 2. The analysis of phase 3 is of an exploratory nature since the associated theoretical benchmark is debatable.

4.1. Efficiency, prices, and bidders' payoffs

The second-price auction is efficient since truthful bidding is an equilibrium in dominant strategies. However, the first-price auction is not efficient since the bidder with the lower valuation wins with positive probability (see Fig. 2). In the experiment, we measure efficiency ($\mathscr{E}$) as follows:

$$\mathscr{E} = \frac{v_{\text{buyer}}}{\max\{v_1, v_2\}}.$$

Although the efficiency of the second-price auction is not 100% (see Table 1), it is, however, higher than that of the first-price auction (see Fig. 8 and Table 4). As shown in Fig. 4, the tendency toward efficiency is more pronounced in second-price auctions when distinguished by phases. We observe on average a stable increase in efficiency over time.

Table 1 also lists average equilibrium prices (based on the actual valuations drawn)[4] and average observed prices of first- and second-price auctions, separately for phases 1, 2, and 3. A Sign Test ($p = 0.004$, one-tailed, $N = 8$, phase 1)[5] for all three phases reveals that the seller's revenues are higher in first-price auctions (see Table 4).[6] The cumulative distribution function of the observed price of the first-price auction shows consistently more mass on high prices than that of the second-price auction (first-order stochastic dominance), in all three phases (see Fig. 8). Note that, based on observed behavior, the seller benefits more from the first-price auction than predicted by the theory, i.e., the game-theoretic solution underestimates the scale of the comparative advantage of the first-price auction. Of course, risk aversion leads to less bid shading in the first-price auction but should not affect bidding in the second-price auction. This suggests that risk aversion may explain the higher than predicted profitability of the first-price auction.

[4]The expected equilibrium prices are 90.46 for the first-price and 88.89 for the second-price auction. In order to compute these values, one needs to apply formulas (a) and (b) in Theorem 5 of Kalkofen and Plum (1996).

[5]For phases 2 and 3: $p = 0.008$ ($N = 7$).

[6]On this and other non-parametric test used see Siegel and Castellan (1988).

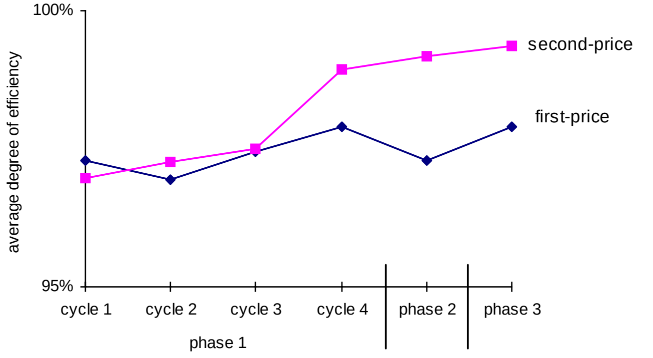

Fig. 4. Average degree of efficiency $\mathscr{E}$ over time.

Table 1
Average degree of efficiency ($\mathscr{E}$), percentage of efficient allocations ($\mathscr{E}_A$%), average prices (P_O: observed, P_P: predicted), and average payoffs of bidder 1 (U_1) and bidder 2 (U_2)

Price rule		First-price						Second-price					
		$\mathscr{E}$	$\mathscr{E}_A$%	P_O	P_P	U_1	U_2	$\mathscr{E}$	$\mathscr{E}_A$%	P_O	P_P	U_1	U_2
Phase	1	0.97	84	103.87	90.4	6.37	22.75	0.98	88	87.06	88.7	11.23	35.36
	2	0.97	85	104.70	90.6	6.65	21.68	0.99	94	90.69	89.6	11.12	34.42
	3	0.98	87	98.74	88.5	7.35	22.15	0.99	94	89.97	88.5	11.72	33.83

According to Table 1 both bidders' average payoffs are higher in the second-price auction than in the first-price auction. Whereas the payoffs in phases 1 and 2 of the second-price auction are close to their benchmarks in all sessions,[7] in the first-price auction they are systematically below their benchmarks (13)–(14). The second-price auction generates significantly higher payoffs to both bidders and in all three phases.[8] The possibility to dictate the auction type in the second (third) phase has no substantial impact on the payoffs.[9]

4.2. Bidding behavior

We now analyze the data to assess how well theory predicts observed bidding. We focus on the first-price auction, using a range of tools, from simple measures to nonparametric estimates of bid functions.

[7] We have confirmed that the distributions of actually selected values v_1 and v_2 do not differ significantly from the a priori distributions.

[8] One-tailed Sign Test based on session data; for phase 1: $p = 0.004$ ($N = 8$), phase 2: $p = 0.008$ ($N = 7$), phase 3: $p = 0.063$ ($N = 7$).

[9] For an exact comparison one should deduct the payments for the right to choose from bidders' payoffs in phases 2 and 3. However, this adjustment does not significantly change these payoffs.

In the second-price auction, nearly half of all observed bids in all three phases are equal to subjects' valuations (39% in phase 1, 47% in phase 2, and 48% in phase 3). There is, however, some slight overbidding (the average degree of bid shading is in the range −3% to −5% for bidder 1 and −1% to −2% for bidder 2). This could be due to the fact that (i) sealed-bid procedures slow down learning since they provide only little feedback; and to (ii) hostile behavior of the weak bidder (who overbids more often and who might feel underprivileged). Nevertheless, truthful bidding turns out to be a reasonably good prediction.[10] Therefore, a detailed further review of the data on the second-price auction is omitted.

Empirical test of basic properties of equilibrium bidding: Turning to the first-price auction, for a first impression we plot the distributions of the individual bids for each bidder type in Fig. 5 where the reference lines indicate truthful bidding and benchmark bidding, respectively. In the first-price auction, bidding above benchmark is rational for risk-averse bidders; however, bidding higher than one's valuation cannot be rationalized. The two lines are the boundaries for rational bidding. The overwhelming majority of the observations is inside this area. The few cases of bidding higher than one's valuation tend to occur at the beginning of the experiment and vanish afterwards.

According to the theory the strong bidder should never bid above the maximum bid of the weak bidder. A weaker requirement is that the strong bidder should never bid more than 150, the highest possible valuation of the weak bidder, for valuations in the interval [150, 200]. A closer look at the data shows that bidder 2 behaves in line with this prediction. In phase 1, only 11% of the bids submitted by bidders with valuations $V_2 \geqslant 150$ were above 150, and that share decreases over time (in the second part of phase 1, the share goes down to 9%; in phase 2: 9%, phase 3: 7%). However, these nonrationalizable bids come from a few bidders.[11]

In Table 2, we compare the average degrees of bid shading (observed and predicted) between the two bidders for all phases conditioning on the event that valuations are drawn from the stated subsets.[12] Thereby the degrees of bid shading are defined as $[v_i - b_i(v_i)]/v_i$, $i = 1, 2$.

As predicted, the observed bid shading increases in valuations for both bidders (except for bidder 1 in phase 3); also the strong bidder shades more than the weak bidder in each corresponding range of valuations. In the lowest range [50, 100], the observed bid shading is higher than predicted, whereas the reversed holds true in all other ranges. This is due to the fact that bidders shade considerably at all valuations

[10] Harstad (2000) has observed that subjects tend to overbid less in second-price auctions if they first gain experience in first-price auctions. In our experiment, subjects had to play second-price auctions after participating in several first-price auctions.

[11] For example, in the second part of phase 1, altogether 13 (out of 55) bidders submit bids above 150, but only six of them do this more than once and only four of them submit bids that exceed 151.

[12] Note that, if one uses the unconditioned average, that measure would overestimate the extent of the strong bidder's bid shading. This is due to two effects: (1) relative to the weak bidder, the strong bidder shows more mass on high valuations; (2) the degree of bid shading is monotonically increasing in valuations.

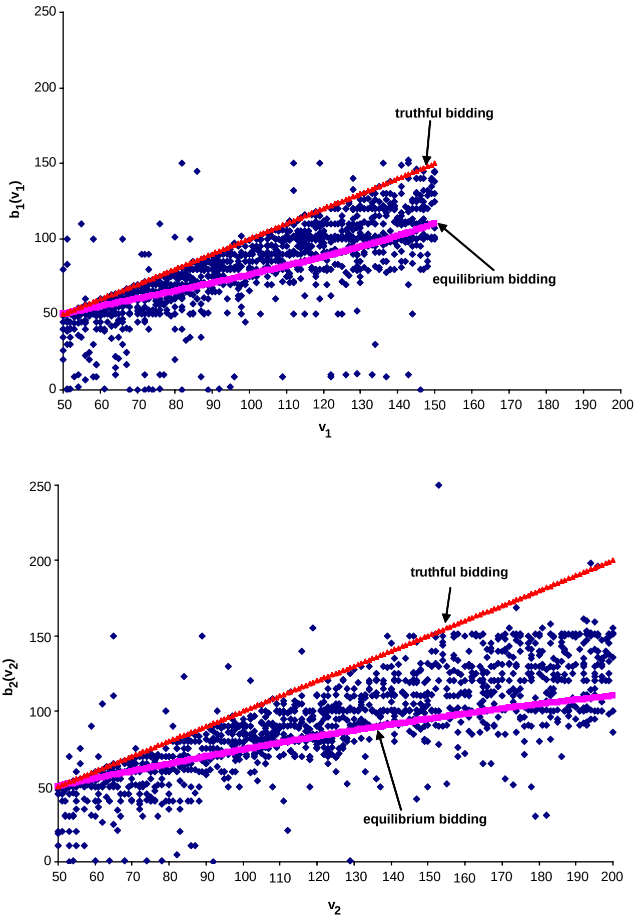

Fig. 5. Scatterplots of individual bids for both bidders in first-price auction (phase 1).

Table 2
Average degree of bid shading in first-price auction (theoretical predictions in brackets)

Bidder		Weak (1)		Strong (2)		
v_i		$v_1 \in [50, 100]$	$v_1 \in (100, 150]$	$v_2 \in [50, 100]$	$v_2 \in (100, 150]$	$v_2 \in (150, 200]$
Phase	1	0.18 (0.15)	0.20 (0.27)	0.21 (0.16)	0.24 (0.32)	0.31 (0.41)
	2	0.18 (0.15)	0.19 (0.27)	0.20 (0.14)	0.25 (0.32)	0.30 (0.41)
	3	0.21 (0.15)	0.19 (0.27)	0.22 (0.15)	0.27 (0.32)	0.32 (0.41)

even though according to the theory they should bid 50 at $v = 50$ and shade relatively little for low valuations close to 50. Note that such an asymmetry between shading at low and high valuations cannot show up in settings where bidders cannot bid below the minimum of the support of valuations, as is the case in most standard auction experiments. Over time, there is no considerable change in the gap between the observed and predicted bid shading.

For first-price auctions it is interesting to analyze how the degree of bid shading depends on v_i. In Fig. 6, the top diagram plots the equilibrium degrees of bid shading and the bottom diagram the corresponding average observed degrees in phase 1. The equilibrium as well as the observed bids are computed as the average bids for the respective v_i-intervals. One qualitative aspect of the top diagram seems to hold also for the bottom one, namely a general tendency of bidder 1 to shade less. According to theory the gap between the two bidders' degrees of bid shading should increase in valuation. This is confirmed by the data only for valuations between 100 and 130. At low valuations ($50 \leqslant v \leqslant 90$) the theory predicts that the gap vanishes. However, in this region the data show a noticable gap. A statistical test at different v_i-levels shows significantly higher average bid shading for bidder 2 when valuations

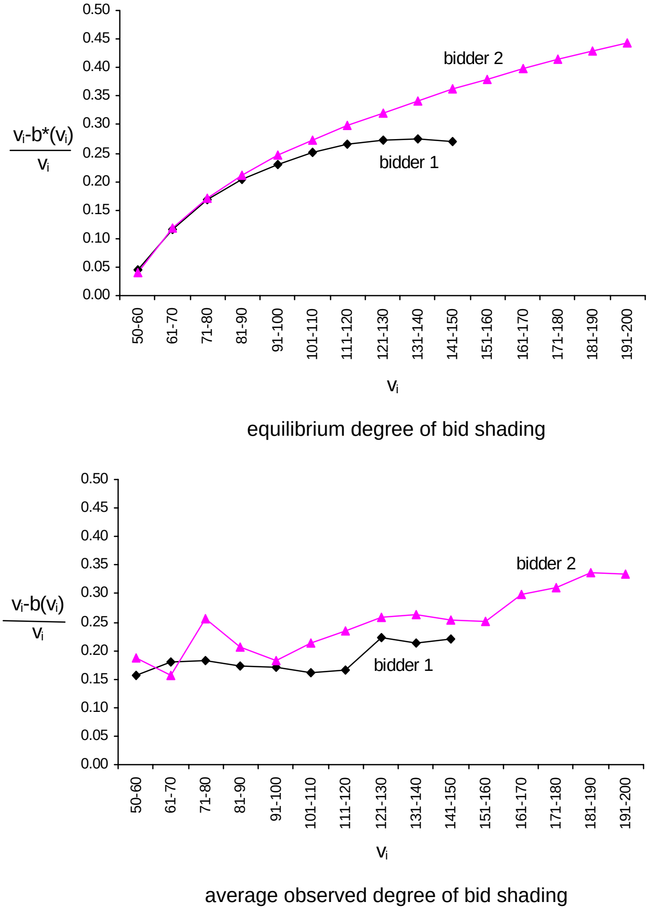

Fig. 6. Degree of bid shading in first-price auction (phase 1).

are rather large ($100 < v_i \leqslant 140$); for low valuations the results are significant only for the region $70 < v_i \leqslant 80$.[13]

Nonparametric estimates of bid functions: A more ambitious test of the predictive power of the theory employs estimates of bid functions and compares them to the corresponding equilibria. We did this by estimating the following nonparametric kernel regression (see Pagan and Ullah, 1999):[14]

$$b_i = f_i(v_i) + \varepsilon_i \quad \text{with} \tag{18}$$

$$E(\varepsilon_i | v_i) = 0, \ i = 1, 2. \tag{19}$$

The advantage of this method is that it does not impose a particular functional form.

The results are plotted in Fig. 7.[15] The solid lines are the estimated bid functions, the two surrounding dashed lines indicate the 95% uniform confidence bands, whereas the dotted lines represent truthful bidding and equilibrium bidding, respectively.

Conforming with theory, bid functions as well as the corresponding degrees of bid shading tend to increase, and the weak bidder bids more aggressively than the strong. However, there is a considerable quantitative gap between the estimated and the corresponding equilibrium bid functions. Both, the weak and the strong bidder bid far higher than they should, and the equilibrium bid functions are almost consistently outside the confidence bands. Although statistically significant, the gap between the weak and the strong bidders' bid functions is remarkably small. Moreover, for high valuations ($v_2 > 150$), the estimate for the strong bidder is above the maximum bid of the weak bidder, which is a blatant violation of rational behavior. As one can see from Fig. 7, the upper bound of the 95% confidence band of the estimated bid function for the weak bidder is equal to 135. Remarkably, 30% of all bids submitted by strong bidders with a valuation above 150 are higher than 135. All of this suggests that the theory cannot very well rationalize observed behavior.

However, this conclusion must be qualified in several regards. Overbidding is typically observed not only in the symmetric but also in the two other asymmetric auction experiments (see Elbittar, 2002; Pezanis-Christou, 2002). The standard explanation is the presence of risk aversion.[16] Obviously, in our framework risk aversion will also contribute to raise equilibrium bids, moving closer to the observed data, although the precise effect depends on the assumed utility functions. We also point out that the estimates are based on pooled data. Therefore, they reflect the behavior of the entire population rather than of individual bidders.

[13] One-tailed Wilcoxon Test, $p < 0.074$.

[14] $\hat{E}(b_i | v_i = a) = \frac{\sum_{i=1}^{N} K((v_i - a)/h) b_i}{\sum_{i=1}^{N} K((v_i - a)/h)}$, where K is the Gaussian kernel function and h is the bandwidth parameter chosen by crossvalidation.

[15] The bid and value ranges are rescaled by division by 100.

[16] This interpretation has, however, triggered an ongoing debate (see the December 1992 issue of *American Economic Review*).

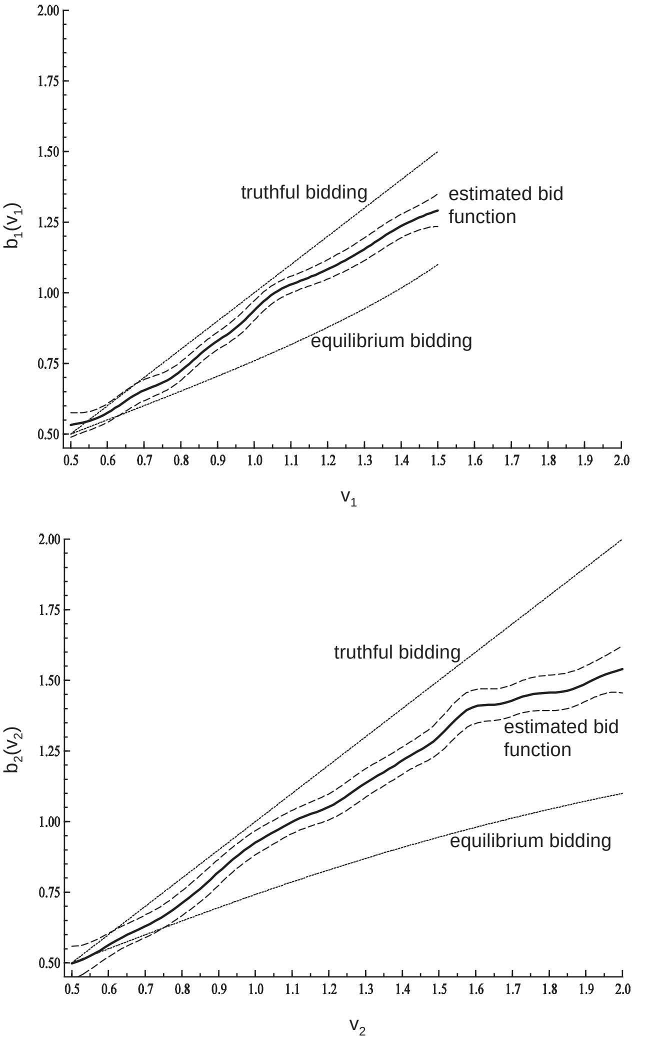

Fig. 7. Nonparametric regression of bids on values for each bidder type, first-price auction (95% uniform confidence bands, Gaussian kernel function, crossvalidation bandwidth selection method).

4.3. The willingness to pay for the right to choose the auction rule

In phases 2 and 3 of the experiment, subjects were asked to bid for the right to choose the auction rule, the details of which have already been explained in Section 3. We now compare these bids with both the equilibrium and the observed values of dictatorship, where the latter are based on the average payoffs earned in the two auctions according to definition (15).

In the first part of Table 3, $\mathscr{L}_i$ denotes the average willingness to pay for the right to choose, L_i, the equilibrium, and D_i, the observed value of dictatorship for the

Table 3
Average values of dictatorship and frequencies of the dictator's choice of the second-price auction (SPA)

	Value of dictatorship			Frequency of choice of SPA		
	$\mathscr{L}_i$	L_i	D_i	r_O	r_{P1}	r_{P2}
Bidder 1	3.7005	0.592	3.3142	80%	0%	81%
Bidder 2	6.4062	1.8185	6.3937	86%	100%	87%

second part (last eight rounds) of phase 2.[17] As one can see from Table 3, during the second part of phase 2, the strong bidder bids on average more for the right to dictate than the weak bidder ($\mathscr{L}_2 > \mathscr{L}_1$). This is in line with the predicted rankings of the values of dictatorship: $L_2 > L_1$ and $D_2 > D_1$. Relative to the prediction of the theory, both bidders overbid considerably ($\mathscr{L}_i > L_i$), see also Table 4. However, the bids $\mathscr{L}_i$ are remarkably close to the observed values of dictatorship D_i, although there is a high dispersion of $\mathscr{L}_i - D_i$, $i = 1, 2$, around its mean. The standard deviation of $\mathscr{L}_1 - D_1$ is 5.75 and that of $\mathscr{L}_2 - D_2$ is 6.72. This indicates that there is considerable over- and underbidding.

The second part of Table 3 displays the frequency of the dictator's choice of the second-price auction (SPA) with r_O denoting the observed frequency, r_{P1} the frequency predicted by the theory, and r_{P2} the frequency that would be observed if each subject chose the auction that yielded him the higher average payoff in the first phase.[18] Again, we see a gap between r_O and r_{P1} while r_O and r_{P2} are almost identical. This phenomenon can be easily explained. As we have noted before, in a second-price auction bidders tend to bid according to the equilibrium, whereas in the first-price auction, there is considerably less bid shading than in equilibrium. As one can see from Table 1, this insufficient bid shading occurs to such an extent that the second-price auction yields a higher payoff to both bidders. For bidder 1 this entails that he prefers the second-price auction even though in equilibrium he shows the reverse preference. An alternative explanation suggests that bidders opt for the second-price auction on the grounds of its strategic simplicity.

We now turn to phase 3 of the experiment, in which subjects learned their valuation before bidding for the right to choose. As mentioned in Section 2, we have some doubts about how to properly characterize the value of dictatorship. Therefore, we only report a few results.

Since bidders already know their private valuations, the values of dictatorship should, of course, depend on these valuations. If one uses the definition (17), it is clear that the equilibrium values of dictatorship are monotonically increasing in

[17] During the first part of phase 2, bids change considerably over time. We focus on the second part of phase 2 because from round 8 onwards, bidding for dictatorship remains fairly stable, which indicates that the learning process has been more or less completed.

[18] In the aggregate, the second-price auction generated a higher average payoff. However, for some bidders the first-price auction was on average more profitable. This explains why in Table 3 r_{P2} is below 100%.

Table 4
Summary statistics

	Auction type	Phase		All	Session							
					1	2	3	4	5	6	7	8
Mean efficiency rate	First price	1		0.97	0.97	0.97	0.98	0.96	0.98	0.98	0.98	0.97
		2		0.97	0.97	0.98	0.98	0.97	–	0.96	0.97	0.98
		3		0.98	0.97	0.98	0.99	0.96	–	0.97	0.99	0.99
	Second price	1		0.98	0.97	0.97	0.97	0.98	0.98	0.98	0.99	0.97
		2		0.99	0.99	0.99	0.99	1.00	–	1.00	1.00	0.98
		3		0.99	0.99	1.00	0.99	1.00	–	1.00	1.00	0.99
Percentage of Pareto opt. allocations	First price	1		84.1%	84.5%	81.0%	85.4%	80.4%	85.1%	87.5%	86.3%	82.7%
		2		85.1%	80.7%	92.9%	84.4%	86.1%	–	86.7%	81.4%	85.4%
		3		87.2%	84.1%	85.5%	95.9%	79.4%	–	83.3%	91.7%	88.0%
	Second price	1		88.4%	87.5%	86.9%	85.4%	90.5%	89.3%	90.5%	92.9%	83.9%
		2		93.5%	92.7%	92.9%	92.2%	93.4%	–	95.5%	97.1%	90.6%
		3		93.9%	91.2%	96.5%	93.6%	97.4%	–	95.3%	93.8%	88.1%
Mean price	First price	1		103.87	103.80	108.86	101.93	92.80	107.93	106.89	107.84	100.65
		2		104.70	103.51	103.79	103.25	95.00	–	108.02	106.47	110.48
		3		98.74	98.61	104.27	102.63	77.21	–	100.15	102.21	98.92
	Second price	1		87.06	83.82	93.65	90.61	70.18	88.68	89.74	89.02	91.29
		2		90.69	88.49	95.19	94.23	72.58	–	90.00	94.62	102.13
		3		89.97	96.75	92.32	96.19	74.76	–	88.16	92.33	94.26
Std. Dev.	First price	1		25.80	27.62	28.52	21.06	22.20	26.35	24.86	23.66	27.04
		2		26.52	28.40	23.36	27.98	27.84	–	29.39	23.65	23.62
		3		26.25	27.79	24.89	21.57	25.58	–	25.98	24.05	27.48
	Second price	1		33.79	33.01	29.95	33.67	46.36	30.72	29.35	26.14	32.22
		2		33.78	32.75	30.07	29.51	45.16	–	28.80	23.89	33.59
		3		32.09	27.87	24.37	27.55	41.09	–	28.17	27.61	35.91
Mean bid shading $(v-b)/v$	First price Bidder 1	1	$v \in [50,\ 100]$	0.17	0.21	0.11	0.15	0.39	0.13	0.09	0.12	0.19
			$v \in (100,\ 150]$	0.20	0.22	0.14	0.22	0.33	0.16	0.13	0.16	0.25

Table 4 (*continued*)

	Auction type		Phase		All	Session							
						1	2	3	4	5	6	7	8
		Bidder 2	1	$v \in [50, 100]$	0.21	0.23	0.12	0.18	0.33	0.16	0.12	0.18	0.29
				$v \in (100, 150]$	0.24	0.26	0.21	0.27	0.34	0.20	0.18	0.21	0.25
				$v \in (150, 250]$	0.31	0.28	0.29	0.34	0.42	0.27	0.31	0.27	0.30
	Second price	Bidder 1	1	$v \in [50, 100]$	−0.05	−0.08	−0.13	−0.08	0.33	−0.05	−0.15	−0.06	−0.16
				$v \in (100, 150]$	−0.01	0.02	0.02	−0.12	0.08	0.00	0.00	−0.02	−0.06
		Bidder 2	1	$v \in [50, 100]$	−0.02	0.13	−0.20	0.01	0.16	−0.15	−0.07	−0.07	−0.03
				$v \in (100, 150]$	−0.02	0.06	−0.04	−0.02	0.00	−0.17	−0.03	−0.03	0.06
				$v \in (150, 250]$	0.04	0.13	0.08	0.02	0.03	−0.05	0.00	0.04	0.04
Std. Dev.	First price	Bidder 1	1	$v \in [50, 100]$	0.23	0.28	0.17	0.13	0.31	0.12	0.25	0.13	0.22
				$v \in (100, 150]$	0.15	0.17	0.13	0.08	0.21	0.11	0.09	0.13	0.15
		Bidder 2	1	$v \in [50, 100)$	0.24	0.24	0.27	0.12	0.28	0.17	0.23	0.16	0.29
				$v \in (100, 150]$	0.13	0.15	0.13	0.10	0.18	0.12	0.10	0.07	0.14
				$v \in (150, 250]$	0.14	0.11	0.19	0.11	0.12	0.14	0.13	0.09	0.15
	Second price	Bidder 1	1	$v \in [50, 100)$	0.38	0.33	0.36	0.39	0.48	0.38	0.38	0.16	0.30
				$v \in (100, 150]$	0.19	0.16	0.17	0.26	0.29	0.09	0.16	0.06	0.14
		Bidder 2	1	$v \in [50, 100]$	0.38	0.33	0.48	0.26	0.39	0.35	0.29	0.22	0.54
				$v \in (100, 150]$	0.21	0.23	0.14	0.12	0.11	0.36	0.13	0.17	0.20
				$v \in (150, 250]$	0.17	0.11	0.17	0.19	0.13	0.23	0.15	0.16	0.12
Meanwillingness to pay for the right to choose		Bidder 1	2		4.92	8.35	4.67	2.78	3.48	–	7.40	4.15	3.28
			3		4.10	4.10	3.08	3.53	6.02	–	5.64	2.97	3.28
		Bidder 2	2		7.55	7.79	6.79	4.83	11.29		4.64	9.89	7.23
			3		7.88	8.63	9.51	6.24	9.69	–	7.00	8.69	5.16
Std. Dev.		Bidder 1	2		6.62	8.84	6.20	5.06	3.71	–	8.06	5.71	5.02
			3		6.49	5.18	6.07	5.47	7.04	–	8.02	5.03	7.28
		Bidder 2	2		7.39	6.64	7.41	5.42	8.00		5.73	7.40	8.18
			3		9.53	9.71	10.86	9.08	10.49	–	9.00	8.88	7.57

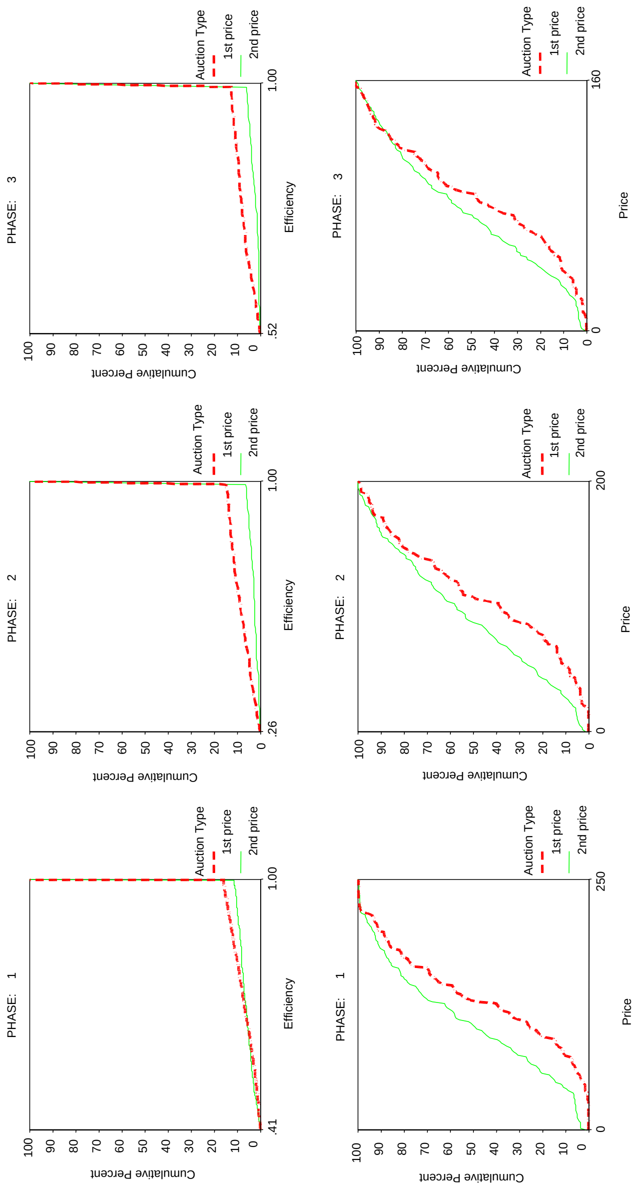

Fig. 8. Cumulative distributions of the degree of efficiency $\mathscr{E}$ and prices (for both auction types).

valuations. This is in accord with the observed average bids for the right to choose (see Fig. 9). Furthermore, as we compare bids across bidder types, we find that for the same valuation average bids are remarkably similar and do not change significantly over time (see Fig. 10).

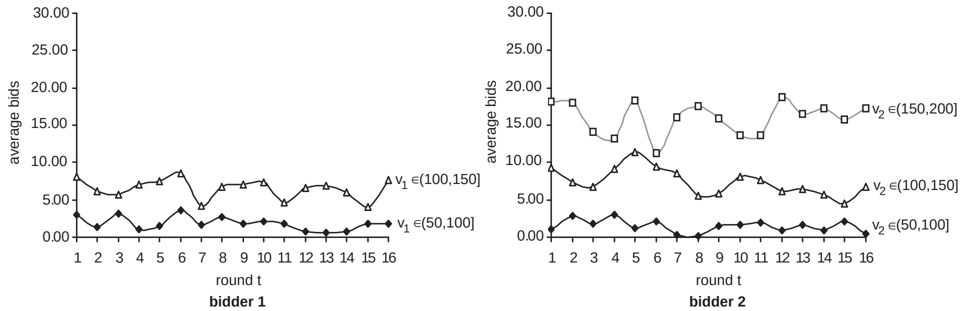

Fig. 9. Average bids for the right to choose in phase 3 over time.

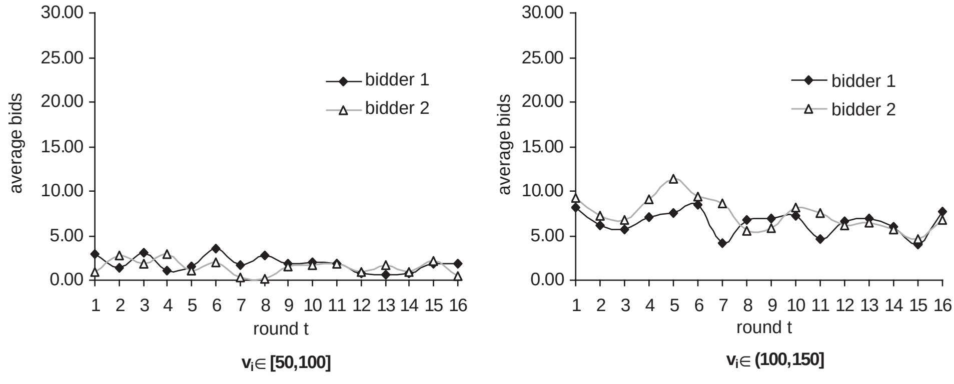

Fig. 10. Average bids for the right to choose in phase 3 for different v_i-intervals over time.

5. Concluding remarks

Although auctions are a familiar topic in experimental economics, most experiments deal with the case of symmetry. Our experiment studies auctions where private valuations are independently drawn from distinct but commonly known distributions, one of which first-order stochastically dominates the other. Our results (phases 1 and 2) are consistent with several qualitative predictions of the tested theory. In particular, we observe that:

- prices are significantly higher in first-price auctions,
- in first-price auctions weak bidders bid more aggressively than strong bidders,[19]
- in second-price auctions bids are close to bidders' valuations,
- strong bidders tend to bid more for the right to dictate the auction rule.

[19]This finding is similar to Elbittar (2002) who, however, considers a different kind of asymmetry between bidders.

However, there are also notable differences between observed and predicted bidding. For example, in the first-price auction, the estimated and the corresponding equilibrium bid functions tend to be far apart, both bidders shade significantly less than they should,[20] and the difference between the weak and the strong bidders' bid functions is too small.

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The authors gratefully acknowledge the very helpful and constructive advice by Charles Bellemare and two anonymous referees. This paper is part of the EU-TMR Research Network ENDEAR. Support from the *Deutsche Forschungsgemeinschaft*, SFB Transregio 15, "Governance and Efficiency of Economic Systems" is gratefully acknowledged.

Appendix

Instructions:[21]

Phase 1:

Please read these instructions carefully. They are identical for all participants.

During the experiment, you will take part in several auctions. In every auction a fictitious commodity is for sale which you can resell to the experimenters. You are one of two bidders. In each auction there are two bidders, 1 and 2, with different value ranges [50, 150] and [50, 200]. Each participant belongs either to type 1 or to type 2 and keeps his own type throughout the entire experiment. Both bidders know for certain both value ranges. In each auction the private reselling value v of each bidder is independently drawn from the interval $50 \leqslant v_1 \leqslant 150$ for bidder 1 and from the interval $50 \leqslant v_2 \leqslant 200$ for bidder 2, respectively, where all integers between 50 and 150 for bidder 1 and between 50 and 200 for bidder 2, respectively, are equally likely. Each bidder may place integer bids in the range from 0 to 250. The bidder with the highest bid buys the commodity and pays a price according to the pricing rule. Then he sells the commodity to the experimenter and receives his reselling value. The other bidder does not pay nor receive anything. If both bids are equal, the buyer is chosen by the flip of a fair coin.

There are two different auction rules. In the first-price auction the price is the highest bid; in the second-price auction it is the second highest bid. We denote the highest and the second highest bids by b_w, b_{2w}.

First-price auction:

- Price = highest bid ($p = b_w$)
- Bidder with highest bid becomes buyer. He pays p.

[20] Such overbidding pattern was, incidentally, also observed in a somewhat different context by Pezanis-Christou (2002).

[21] This is a translated version of the instructions. For the original instructions (in German), please contact one of the authors.

- Profit of buyer: $v_w - p$
- Profit of non-buyer: 0

Second-price auction:

- Price = second highest bid ($p = b_{2w}$)
- Bidder with highest bid becomes buyer. He pays p.
- Profit of buyer: $v_w - p$
- Profit of non-buyer: 0

In each auction, the bidder groups are formed randomly. After you have placed your bid, you are informed whether or not you are the buyer, about the price which has to be paid by the buyer, your private reselling value, your own bid, the bid of the other bidder, how much you have earned in this auction, your total profit up to this time, and your average profit in each auction type (per auction type). Altogether, you will play 48 successive auctions, which consist of four cycles of 12 bidding rounds. In one cycle, each participant first plays six times the first-price auction and then six times the second-price auction.

All valuations are denoted in a fictitious Experimental Currency Unit (ECU). The exchange rate from ECU to DEM is: 100 ECU = DM 1.50. You receive an initial endowment of DEM 10.50 (700 ECU) to cover possible losses.

Any decision you make is anonymous and neither your co-bidders nor the experimenter can detect which decisions were taken by you. If you have questions, please raise your hand. We will then answer your questions in private communication.

Phases 2 *and* 3:

You will play another 32 auctions. In the first 16 auctions (phase 2), bidders can choose the auction rule according to the following procedure:

(1) you are asked to state your maximum willingness to pay for the right to dictate the auction rule;

(2) an unbiased chance move selects either you or your co-bidder as potential dictator;

(3) another chance move draws a random number from a uniform distribution with support $[0, 30]$. The potential dictator has to choose the auction rule if that random number is not greater than his declared maximum willingness to pay; otherwise the auction rule is selected by the flip of a fair coin. Subsequently, the auction is played as in the first phase.

At the end of each auction in this phase, the dictator also receives information about his costs for choosing the auction type.

In the remaining 16 auctions (phase 3), you are informed about your private reselling value v before choosing the auction type as described above.

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