Incomplete Information in Rent-seeking Contests

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Abstract

We consider a variant of the Tullock lottery contest. Each player's marginal cost of effort is drawn from a potentially different continuous distribution. In order to study the impact of incomplete information we compare three informational settings to each other: players are either completely informed, privately informed about their own costs, or ignorant of all cost realizations. For the first and the third setting we determine the unique pure-strategy Nash equilibrium. Under private information we prove existence of a pure-strategy Bayesian Nash equilibrium and identify a sufficient condition for uniqueness. Assuming that cost distributions all have the same mean, we show that under ignorance of all cost realizations ex ante expected aggregate effort is lower than under both private and complete information. Focusing on the standard lottery contest and assuming costs are all drawn from the same distribution, ex ante expected rent dissipation, however, is larger in the latter settings. Between complete and private information there is neither a general ranking in terms of effort nor in terms of rent dissipation.

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1 Introduction

Many economic situations can be described as contests among players who invest costly effort to increase their probability of winning a prize. Examples are rent-seeking, lobbying, R&D races, election or advertising campaigns, litigation, and, of course, also military conflict as well as sports.[1] In all these situations, contestants might often be unsure about the abilities of their rivals for exerting effort or might not know their rivals' values for the prize. In addition, players might even be uncertain about their own ability or value. In this paper, we study how uncertainty and private information affect the outcome of a contest compared to the complete information case.

Contests have been modeled in a variety of ways. A distinction may be made between perfectly and imperfectly discriminating contests, depending on whether the player who invested the highest effort wins with certainty or not. The (first-price) all-pay auction is a prominent example of the former and has been thoroughly studied both with completely and incompletely informed contestants.[2] One of the most popular imperfectly discriminating contests is the rent-seeking contest by Tullock (1980). In the simplest version of that model, the winning probability of player i amounts to $x_i \big/ \sum_j x_j$ where x_i denotes i's effort. This is also known as the lottery contest.[3] A vast literature has developed extending Tullock's model in numerous directions. Yet, in contrast to the all-pay auction, there are only very few studies that depart from the basic assumption of players being all completely informed about every aspect of the game. Clearly, the case of incomplete information in rent-seeking contests deserves greater attention.

Some progress has been made in the analysis of Tullock contests under incomplete information for the case where there are only two players who both privately know their type (i.e., their valuation for the prize or their cost per unit of effort). Hurley and Shogren (1998a) numerically study the equilibrium of the lottery contest assuming types are drawn from two different discrete distributions. A more tractable distributional assumption allows Malueg and Yates (2004) to obtain a closed form solution for equilibrium efforts in the Tullock contest when there are only two possible types for both players. A closed form for a different binary distribution is found by

[1]See Konrad (2009) for a recent survey on contest theory and its application to those examples.

[2]See, e.g., Baye, Kovenock, and de Vries (1996) for the complete and Krishna and Morgan (1997) for the incomplete information case.

[3]The winning probability is equivalent to that in a lottery where each player i buys an amount x_i of lottery tickets and puts them into a box from which the winner is drawn.

Münster (2009) when studying a repeated lottery contest. The only known closed-form equilibrium for a continuous type distribution is due to Ewerhart (2010) who, for this purpose, constructs a rather special distribution function. When avoiding such very specific distributional assumptions, the analysis of Tullock contests with private information has to be done without relying on closed-form expressions for equilibrium efforts. For a more general binary distribution, Katsenos (2010) explores a lottery contest that is preceded by a signaling stage. Fey (2008) offers a proof of existence of a symmetric pure-strategy equilibrium for a lottery contest with types drawn from a continuous uniform distribution. The only author to consider more than two players is Ryvkin (2010) who extends Fey's proof to that case, at the same time also relaxing the assumptions on the contest success and distribution function.[4]

In this paper, we analyze a contest among $n \geq 2$ players where player i's winning probability is given by the contest success function $(x_i + \sigma) \Big/ \left(\sum_j x_j + n\sigma \right)$ with $\sigma \geq 0$. This variant of the lottery contest has been proposed by Amegashie (2006). He argues that the parameter σ allows for varying the amount of noise in the contest success function in a tractable way. Alternatively, σ can also be interpreted as a commonly known amount of effort that each player did already invest at an earlier stage (e.g., in order to enter the contest). Myerson and Wärneryd (2006) refer to a similar contest success function. They suggest extending the Tullock contest by a parameter σ and taking the limit as $\sigma \to 0$ in order to remedy the problem that the Tullock contest success function is not defined when all players choose zero effort.[5]

We introduce uncertainty to our contest by assuming each player's constant marginal cost of effort to be drawn from a (possibly different) probability distribution. Varying the amount of information contestants have regarding cost realizations, we obtain three different informational settings. Either all players are completely informed about all marginal costs, each player privately knows his own marginal cost, or all players are unaware of the realization of all marginal costs (including their own). The analysis of the third type of contest, to which we will refer as the no in-

[4]There is also a small literature on Tullock contests where only one of two players is privately informed, including Hurley and Shogren (1998b) who assume private values and Wärneryd (2003) who considers common values. Wärneryd (2009) extends the latter study to more than two players.

[5]For the Tullock contest it is usually assumed that if $x_i = 0$ for all i, either all players are equally likely to win or nobody wins. Such an assumption, however, is not part of the axiomatization of the Tullock contest provided in Skaperdas (1996, Theorem 2), hence the suggestion by Myerson and Wärneryd (2006). Moreover, note that unless $\sigma = 0$ the contest we consider in this paper violates the homogeneity axiom that is part of Skaperdas' axiomatization of the Tullock contest. Yet our contest does belong to the more general class of contests axiomatized in Skaperdas (1996, Theorem 1).

formation contest, is closely related to that of the complete information contest. Because payoffs are linear in the cost parameter, the contest where players are ignorant of all costs is equivalent to a contest where players are completely informed and costs are equal to their expected values. In the course of the paper, we analyze equilibria for all three types of contests. This enables us to compare the informational settings to each other with respect to the amount of effort and rent dissipation they induce.

We assume that all differences among players are captured in their marginal costs. This is, of course, only one of many possible ways to model incomplete information and asymmetries among players. An other common choice in the literature is to assume that players differ in their valuations for the prize rather than in their costs. Yet, as many authors have pointed out, contests with asymmetric valuations and asymmetric marginal costs are, in fact, equivalent. For a given informational setting results for one model readily extend to the other model using a simple transformation of variables. Under private and complete information, this transformation of variables is identical. To the no information contest, however, a different transformation has to be applied. Consequently, the equivalence of models with uncertain costs and uncertain values does not hold when comparing the no information contest to the other two contests. In Section 6 we carefully analyze this issue and discuss to what extent our results under cost uncertainty also hold when valuations are randomly drawn.

In the complete information contest, realized costs typically differ across players. For $\sigma = 0$ such asymmetric rent-seeking contests have been widely studied. We extend the analysis of the complete information case to $\sigma > 0$. In doing so we complement the discussion in Amegashie (2006) by completely determining the pure-strategy Nash equilibrium and formally proving its uniqueness. For the uniqueness proof we follow Cornes and Hartley (2005) and modify their approach to accommodate $\sigma > 0$. Moreover, we formulate the equilibrium strategies for $\sigma \geq 0$ in a way that differs from the formulation in the literature on asymmetric contests. This turns out to be very useful for comparing different informational settings to each other.

Studying the private information contest we assume costs to be independently drawn from continuous distributions. We prove the existence of a Bayesian Nash equilibrium in monotone pure strategies, provided that $\sigma > 0$. In addition, we show that the Bayesian Nash equilibrium is unique if σ is above a certain (strictly positive) threshold. In contrast to Fey (2008) and Ryvkin (2010) who develop their own existence proof, our proof of existence and uniqueness is a straightforward application of general results for Bayesian games derived by Athey (2001) as well as Mason and Valentinyi (2010). The class of contests considered by Ryvkin (2010) does not include the lottery contest with $\sigma > 0$, but his assumptions on the contest success function are in other respects less restrictive than ours. Our distributional assumptions, however, are more general: Ryvkin assumes that costs are all drawn from the same distribution whereas we allow for the cost distributions to differ across players.

Combining the equilibrium strategies determined under complete and no information with results characterizing equilibrium strategies under private information we find the following. If players are uncertain about the costs of all players, i.e., if they engage in the no information contest, ex ante expected aggregate effort is lower than under complete information. Assuming costs are drawn from continuous distributions that all have the same mean, expected aggregate effort in the no information contest is also lower than in the private information contest.

As efforts are lowest under no information, one would perhaps think that also rent dissipation must be lowest in that setting. Surprisingly, this need not be the case. Focusing on the standard lottery contest ($\sigma = 0$) and assuming costs are all drawn from the same continuous distribution function, we show that ex ante expected rent dissipation is indeed larger under no information than under both complete and private information. A general property of the equilibrium of the private and the complete information contest is that players with low realized costs invest more effort than players with high costs. In the no information contest, however, all players invest the same amount of effort if expected costs are the same for all players. Hence, a given level of expected aggregate effort induces higher expected rent dissipation under no information than under private or complete information. For $\sigma = 0$ it turns out that this effect increases expected rent dissipation under no information to such an extent that the informational environment with the lowest effort is at the same time associated with the highest rent dissipation.

In contrast to the clear-cut results we obtain when comparing the no information contest to the other two contests, we find that there is no general ranking between private and complete information, neither in terms of efforts nor in terms of rent dissipation. As equilibrium efforts under private information cannot be expressed in closed form, we approximate these efforts numerically. A short discussion of the numerical methods we use can be found in Appendix B. Our numerical examples show that whether private or complete information leads to higher efforts and rent dissipation depends on the distribution of types, on the number of players, and on the parameter σ.

In the literature, several different benchmarks have been used to compare private information contests to. In this paper we follow both Hurley and Shogren (1998a) as well as Malueg and Yates (2004). Hurley and Shogren (1998a) compare private information to no information and report that ex ante expected efforts are lower in the former setting when there is uncertainty regarding players' valuations for the prize. Whereas we find their ranking to be reversed under cost uncertainty, we also prove that Hurely and Shogren's numerical result generally holds for a whole class of two-player contests with uncertain valuations. Malueg and Yates (2004) compare private to complete information and find that interim (and, a fortiori, ex ante) expected efforts are exactly the same in both settings. It turns out that Malueg and Yate's finding is not robust when departing from their specific distributional assumption.

Contrary to our result that there is no general ranking between private and complete information, Ryvkin (2010) finds that in two-player rent-seeking contests private information generally leads to lower effort than complete information. The reason for this contradiction is that Ryvkin uses a different complete information benchmark. He compares the equilibrium effort of a player with a given cost realization under private information to the equilibrium effort in the complete information contest where both players have the same cost realization. Whereas under private information players' costs are typically asymmetric, Ryvkin does not allow for any asymmetry under complete information. His comparison therefore captures not only the effect of changing the information structure, but also of imposing symmetry at the same time.

Overall the impact of incomplete information on the rent-seeking contest we study in this paper is quite different than on the perfectly discriminating all-pay auction. On the one hand, in the all-pay auction private and complete information can be ranked clearly: Morath and Münster (2008) show that expected efforts are generally higher under private information than under complete information. On the other hand, there is no general ranking in terms of expected efforts between no information and the other two settings for the all-pay auction with uncertain costs. For the two-player all-pay auction with uncertainty regarding valuations, however, Morath and Münster (2010) find expected efforts to be higher under no information than under private information. Hence, in the all-pay auction with value uncertainty a contest organizer who directly benefits from players' efforts would ex ante prefer no information over the other two informational settings. In contrast, in the rent-seeking contest with cost uncertainty we analyze in this paper the no information contest is the worst option for the contest organizer.

The paper is organized as follows. Section 2 describes the basic assumptions of the model. In Section 3 we analyze the complete and the no information contest. Section 4 is devoted to the private information case. In Section 5 we compare expected efforts and rent dissipation in the different informational settings. A variant of the model where values rather than costs are randomly drawn is considered in Section 6. Section 7 concludes. Some of the proofs are relegated to Appendix A, whereas Appendix B contains notes on the numerical methods we apply.

2 The Model

There are $n \geq 2$ risk neutral players who compete in a contest for a single prize of value 1. Each player i invests a level of effort $x_i \geq 0$. Efforts are chosen simultaneously. Depending on the efforts of all players, the probability of player i winning the prize is given by the contest success function

$$p_i(\mathbf{x}) := \begin{cases} \dfrac{x_i + \sigma}{\sum_{j=1}^n x_j + n\sigma} & \text{if } \sum_{j=1}^n x_j + n\sigma > 0, \\ \dfrac{1}{n} & \text{otherwise} \end{cases} \tag{1}$$

where $\mathbf{x} := (x_1, x_2, \ldots, x_n)$ and $\sigma \geq 0$. Providing effort is costly. There are no fixed costs and each player i has constant marginal cost $c_i > 0$. Player i's payoff from taking part in the contest is therefore

$$u_i(\mathbf{x}, c_i) := p_i(\mathbf{x}) - c_i x_i.$$

Note that, instead of interpreting $p_i(\mathbf{x})$ as the probability of winning, we could also think of it as the share of the prize player i obtains, assuming the prize is divisible.

Let us now introduce uncertainty into our model. We assume that, for each player i, the parameter c_i is the realization of a random variable C_i which takes on values in $\mathscr{C}_i \subseteq [\underline{c}_i, \overline{c}_i]$ where $0 < \underline{c}_i < \overline{c}_i < \infty$. The joint distribution of $C_1, C_2, \ldots, C_n$ is commonly known to all players.

Consider the following timing. There is a point in time, T_1, where each player i privately learns the realization of his cost c_i. At some later point in time, T_2, all players are informed about the realizations of all cost parameters $\mathbf{c} := (c_1, c_2, \ldots, c_n)$. The time after T_2, between T_1 and T_2, and before T_1 is usually referred to as *ex post*, *interim*, and *ex ante*. Depending on the time at which we assume the contest to take place, we have the following three different types of contests.

Suppose the contest takes place ex post. As all players are informed about $\mathbf{c}$, we have a game of complete information which we will refer to as the *complete information contest*. Given $\mathbf{c}$, a pure-strategy Nash equilibrium of this game specifies an equilibrium effort level $x_i^{\text{CI}}(\mathbf{c})$ for each player i such that

$$x_i^{\text{CI}}(\mathbf{c}) \in \arg\max_{x_i} u_i\Big(x_i, \mathbf{x}_{-i}^{\text{CI}}(\mathbf{c}), c_i\Big) \quad \forall i, \tag{2}$$

where $\mathbf{x}_{-i}^{\text{CI}}(\mathbf{c}) := \Big(x_1^{\text{CI}}(\mathbf{c}), \ldots, x_{i-1}^{\text{CI}}(\mathbf{c}), x_{i+1}^{\text{CI}}(\mathbf{c}), \ldots, x_n^{\text{CI}}(\mathbf{c})\Big)$.

If the contest takes place ex ante, players have no information concerning cost parameters $\mathbf{c}$ other than the joint distribution they are drawn from. We will call this variant the *no information contest*. In a pure-strategy Nash equilibrium each player i invests x_i^{NI} such that

$$x_i^{\text{NI}} \in \arg\max_{x_i} E\Big[u_i\Big(x_i, \mathbf{x}_{-i}^{\text{NI}}, C_i\Big)\Big] \quad \forall i,$$

where $\mathbf{x}_{-i}^{\text{NI}} := \Big(x_1^{\text{NI}}, \ldots, x_{i-1}^{\text{NI}}, x_{i+1}^{\text{NI}}, \ldots, x_n^{\text{NI}}\Big)$.[6] Note that because of $E\Big[u_i\Big(x_i, \mathbf{x}_{-i}^{\text{NI}}, C_i\Big)\Big] = u_i\Big(x_i, \mathbf{x}_{-i}^{\text{NI}}, E[C_i]\Big)$ we have $x_i^{\text{NI}} = x_i^{\text{CI}}(E[\mathbf{C}])$. Thus, the no information contest is equivalent to the complete information contest where each player i's costs are commonly known to amount to $E[C_i]$.

Finally, suppose the contest takes place at the interim stage such that each player is only informed about his own costs. When analyzing this *private information contest* we impose the following assumption on the distribution of cost parameters.

A1: The random variables $C_1, C_2, \ldots, C_n$ are independent. Each C_i is continuously distributed according to F_i. F_i has a bounded density f_i with support $[\underline{c}_i, \overline{c}_i]$.

Additionally, some of our results will require one of the following assumptions.

A2: $E[C_1] = E[C_2] = \cdots = E[C_n]$.

A3: $F_1 = F_2 = \cdots = F_n =: F$.

These assumptions impose different degrees of symmetry. Whereas A2 only requires type distributions to have the same mean, under A3 all types are drawn from the same distribution. Let $\xi_i(c_i)$ denote the level of effort that player i chooses if his privately known cost parameter is c_i. Under assumption A1, a pure-strategy Bayesian

[6]Throughout, we denote by $E[\cdot]$ the unconditional expectation with respect to all (upper case) random variables C_i contained in the argument.

Nash equilibrium for the private information contest specifies an equilibrium strategy $\xi_i : [\underline{c}_i, \overline{c}_i] \to \mathbb{R}_+$ for each player i such that

$$\xi_i(c_i) \in \arg\max_{x_i} E\Big[u_i\Big(x_i, \boldsymbol{\xi}_{-i}(\mathbf{C}_{-i}), c_i\Big)\Big] \quad \forall i, c_i \in [\underline{c}_i, \overline{c}_i], \tag{3}$$

where $\boldsymbol{\xi}_{-i}(\mathbf{C}_{-i}) := (\xi_1(C_1), \ldots, \xi_{i-1}(C_{i-1}), \xi_{i+1}(C_{i+1}), \ldots \xi_n(C_n))$.

Being closely related to each other, we will first take up the task of analyzing equilibria of the complete and the no information contest. After that, we will return to the private information case.

3 Complete Information and No Information

Consider the complete information contest: all players know the realization of $\mathbf{c}$ at the time of their effort decision. Realized costs typically vary among players. Such asymmetries in costs can be transformed into asymmetries in valuations for the prize or asymmetric contest success functions. For asymmetric lottery contests with $\sigma = 0$, equilibrium strategies have been identified by Hillman and Riley (1989) and uniqueness of the pure-strategy equilibrium follows from the more general analysis by Szidarovszky and Okuguchi (1997). Complete information contests with $\sigma > 0$ are, however, not covered by those results. In the following, we will adopt the *share function* approach proposed by Cornes and Hartley (2005) and extend their uniqueness proof to $\sigma > 0$. Moreover, we will determine the equilibrium strategies for $\sigma \geq 0$.

For the following it is useful to perform a change of variables by setting $y_i := x_i + \sigma$ for all i. Define $Y := \sum_{i=1}^n y_i$ and $Y_{-i} := Y - y_i$. A contest where player i chooses effort $x_i \in [0, \infty)$ obtaining utility $u_i(\mathbf{x}, c_i)$ is equivalent to a contest where player i chooses $y_i \in [\sigma, \infty)$ obtaining utility

$$v_i(y_i, Y_{-i}, c_i) := \begin{cases} \dfrac{y_i}{Y_{-i} + y_i} - c_i\,(y_i - \sigma) & \text{if } Y_{-i} + y_i > 0, \\[2ex] \dfrac{1}{n} - c_i\,(y_i - \sigma) & \text{otherwise.} \end{cases}$$

Observe that if $Y_{-i} > 0$, $v_i(y_i, Y_{-i}, c_i)$ is strictly concave in y_i. Hence, the following first order condition describes the global maximum of $v_i(y_i, Y_{-i}, c_i)$ with respect to y_i:

$$\frac{Y_{-i}}{\left(Y_{-i} + y_i\right)^2} - c_i \leq 0, \quad \text{with equality if } y_i > \sigma. \tag{4}$$

Accordingly, player i's best response to $Y_{-i} > 0$ is[7]

$$y_i(Y_{-i}) := \max\left\{\sqrt{\frac{Y_{-i}}{c_i}} - Y_{-i}, \sigma\right\}.$$

In a pure-strategy Nash equilibrium $y_1^*, \ldots, y_n^*$ we must have $y_i^* = y_i(Y_{-i}^*)$ for all i.

Following Cornes and Hartley (2005), we define the replacement function $r_i(Y)$ as being contestant i's best response to $Y_{-i} = Y - r_i(Y)$. Making use of (4) we have

$$r_i(Y) = \max\left\{Y - c_i Y^2, \sigma\right\}.$$

From this we obtain player i's share function $s_i(Y) := \frac{r_i(Y)}{Y}$ as

$$s_i(Y) = \max\{\alpha_i(Y), \beta(Y)\}$$

where

$$\alpha_i(Y) := 1 - c_i Y \quad \text{and} \quad \beta(Y) := \frac{\sigma}{Y}.$$

Let Y^* denote a value of Y that corresponds to a pure-strategy Nash equilibrium. Such an equilibrium requires that the sum of all share functions equals unity. Hence, Y^* is defined as a solution to

$$S(Y^*) := \sum_{i=1}^{n} s_i(Y^*) = 1$$

where $S(Y)$ is referred to as the aggregate share function.

Proposition 1. *The complete information contest has a unique Nash equilibrium in pure strategies. Suppose $c_1 \leq c_2 \leq \cdots \leq c_n$ and define $Y(0) := n\sigma$ as well as*

$$Y(m) := \frac{(m-1) + \sqrt{(m-1)^2 + 4(n-m)\sigma \sum_{i=1}^{m} c_i}}{2\sum_{i=1}^{m} c_i} \quad \textit{for } m \in \{1, 2, \ldots, n\}. \tag{5}$$

Moreover, let

$$m^* := \arg \max_{m \in \{0,1,\ldots,n\}} Y(m).$$

[7]As $y_i \in [\sigma, \infty)$ for all i, $Y_{-i} > 0$ is always true if $\sigma > 0$. If $\sigma = 0$, the best response to $Y_{-i} = 0$ does not exist: for any $\tilde{y}_i > 0$, player i can reduce his expenses and still win the contest with probability 1 by choosing a $y_i \in (0, \tilde{y}_i)$ instead. With $\sigma = 0$, there can therefore be no Nash equilibria where less than two players choose strictly positive efforts. Consequently, there results no loss in generality from assuming $Y_{-i} > 0$ in the following.

In the unique pure-strategy equilibrium, each player i chooses effort

$$x_i^{\mathrm{CI}}(\mathbf{c}) = \begin{cases} Y(m^*)(1 - c_i Y(m^*)) - \sigma & \textit{for } i \leq m^*, \\ 0 & \textit{for } i > m^*. \end{cases} \tag{6}$$

Proof. First, suppose $\sigma = 0$. As $Y \to 0$, $s_i(Y) \to 1$ such that $S(Y) > 1$ for sufficiently small Y. For sufficiently large Y, we have $S(Y) = 0$. Furthermore, $S(Y)$ is continuous for all $Y > 0$ and strictly decreasing if $S(Y) > 0$. Hence, there is a unique $Y^* \in (0, \infty)$ that solves $S(Y^*) = 1$. Now, consider the case $\sigma > 0$. Since all individual y_i are restricted to the interval $[\sigma, \infty)$, we have $Y \in [n\sigma, \infty)$. For all $Y \in [n\sigma, \infty)$, $s_i(Y)$ and therefore also $S(Y)$ are continuous and strictly decreasing. Moreover, $S(n\sigma) \geq n\beta(n\sigma) = 1$ while $S(Y) = n\beta(Y) < 1$ for sufficiently large values of Y. Consequently, there exists a unique $Y^* \in [n\sigma, \infty)$ that satisfies $S(Y^*) = 1$. This establishes the first part of the proposition.

For each individual share function there is a $\hat{Y}_i \in [n\sigma, \infty)$ such that $s_i(Y) = \alpha_i(Y)$ for $Y < \hat{Y}_i$ and $s_i(Y) = \beta(Y)$ for $Y \geq \hat{Y}_i$. Recall that contestant i's effort is $x_i = s_i(Y)Y - \sigma$. Accordingly, player i chooses a strictly positive effort if $Y < \hat{Y}_i$. In this case, we refer to i as an active player. For $Y \geq \hat{Y}_i$, player i chooses zero effort. Assuming $c_1 \leq c_2 \leq \cdots \leq c_n$ implies $\hat{Y}_1 \geq \hat{Y}_2 \geq \cdots \geq \hat{Y}_n$. Hence, if $Y \in \left(\hat{Y}_m, \hat{Y}_{m+1}\right)$ for some $m \in \{1, 2, \ldots, n-1\}$, then all $i \leq m$ are active while all $i > m$ choose zero effort. Similarly, if $Y < \hat{Y}_n$, all players are active, and if $Y \geq \hat{Y}_1$, all players choose zero effort.

Suppose the unique Nash equilibrium of the game is such that exactly m^* players are active with $m^* \in \{0, 1, \ldots, n\}$. The corresponding Y^* solves

$$S(Y^*) = \sum_{i=1}^{m^*} \alpha_i(Y^*) + (n - m^*)\beta(Y^*) = 1.$$

The solution to this equation is $Y^* = Y(m^*)$ where the function $Y(\cdot)$ is defined in (5). Now, consider an $\tilde{m} \neq m^*$. $Y(\tilde{m})$ is the solution to

$$\sum_{i=1}^{\tilde{m}} \alpha_i(Y(\tilde{m})) + (n - \tilde{m})\beta(Y(\tilde{m})) = 1.$$

Because $\tilde{m}$ does not correspond to an equilibrium, we have either $\alpha_i(Y(\tilde{m})) < \beta(Y(\tilde{m}))$ for some $i \leq \tilde{m}$, or $\alpha_i(Y(\tilde{m})) > \beta(Y(\tilde{m}))$ for some $i > \tilde{m}$. This implies $S(Y(\tilde{m})) > 1$ and, since $S(Y)$ is strictly decreasing, $Y(\tilde{m}) < Y(m^*)$. Consequently, $m^* = \arg\max_m Y(m)$. Given m^*, we obtain (6) from $x_i^{\mathrm{CI}}(\mathbf{c}) = s_i(Y(m^*))Y(m^*) - \sigma$. □

Amegashie (2006) derives equilibrium efforts for the case where all players are active, i.e., choose strictly positive efforts. In addition, he provides a condition under which all players are inactive (exert zero effort) and mentions the possibility of equilibria where only some of the players are active. The number of active players, of course, depends on the realization of costs **c**. Proposition 1 shows that the number of active players is uniquely determined as the number m that maximizes the function $Y(m)$. In equilibrium, the m^* players with the lowest costs are active, exerting efforts according to (6).

In the standard lottery contest (where $\sigma = 0$), we have $Y(0) = Y(1) = 0$ whereas $Y(m) > 0$ for $m > 1$, implying that there are always at least two players active in equilibrium. Introducing the parameter $\sigma > 0$ opens up the possibility of equilibria where only one player is active or where all players choose zero effort.

For $\sigma = 0$, Proposition 1 exactly corresponds to the known equilibrium of the standard lottery contest with asymmetric contestants. Our formulation of the equilibrium, however, differs from the one that is usually presented in the literature.[8] Determining the number of active players by maximizing the function $Y(m)$ turns out to be very useful when comparing different informational settings to each other. In particular, because $Y(m^*) - n\sigma$ is the total amount of effort invested in equilibrium, $Y(m) - n\sigma$ can serve as a lower bound for aggregate effort for any m.

For the case that all contestants have the same marginal cost c, it can be shown that $Y(0) < (>)\, Y(n)$ implies $Y(m) < (>)\, Y(n)$ for all $m < n$. Hence, in a symmetric contest either $m^* = n$ or $m^* = 0$.

Corollary 1. *Suppose $c_i = c$ for all i. Then, for each i, $x_i^{\mathrm{CI}}(\mathbf{c}) = x^{\mathrm{CI}}(c)$ with*

$$x^{\mathrm{CI}}(c) := \max\left\{\frac{n-1}{n^2 c} - \sigma, 0\right\}.$$

As we have noted before, the no information contest is equivalent to the complete information contest where marginal costs of each player i amount to $E[C_i]$. Therefore, Proposition 1 applies to the no information contest as well.

Corollary 2. *The no information contest has a unique Nash equilibrium in pure strategies. Player i's equilibrium effort is given by $x_i^{\mathrm{NI}} = x_i^{\mathrm{CI}}(E[\mathbf{C}])$.*

Having determined the equilibrium for both the complete information contest as well as the no information contest, let us now turn to the private information contest.

[8] For a recent treatment see, e.g., Corchón (2007).

4 Private Information

Suppose contestants engage in the private information contest, simultaneously deciding on their effort at the interim stage. Let assumption A1 be satisfied. If each player $j \neq i$ employs a strategy $\xi_j(c_j)$, the expected payoff for player i who has privately known cost c_i and exerts effort x_i is

$$E[u_i(x_i, \xi_{-i}(\mathbf{C}_{-i}), c_i)] = E[p_i(x_i, \xi_{-i}(\mathbf{C}_{-i}))] - c_i x_i.$$

Note that $E[u_i(x_i, \xi_{-i}(\mathbf{C}_{-i}), c_i)] \leq 1 - c_i x_i$ and that by choosing $x_i = 0$ player i can guarantee himself a nonnegative payoff. Therefore, effort levels $x_i > \frac{1}{c_i}$ are clearly dominated for type c_i of player i. Accordingly, we can restrict each player i's effort choice to the interval $[0, \frac{1}{\underline{c}_i}]$. In the following, we will apply general results from the literature on Bayesian games to our contest in order to study existence and uniqueness of a Bayesian Nash equilibrium.

Proposition 2. *Assume A1 and* $\sigma > 0$. *For the private information contest, there exists a Bayesian Nash equilibrium in nonincreasing pure strategies. If* $\sigma > \frac{n-1}{n^2} \max_{i,c} f_i(c)$, *the Bayesian Nash equilibrium of the private information contest is unique.*

Proof. See Appendix A.1. □

For proving the existence result in Proposition 2 we have applied Athey (2001). Assuming each player's action space to be finite, Athey shows, using a fixed point theorem, that a pure-strategy equilibrium exists if a specific single crossing condition is satisfied. Moreover, under the condition that a player's payoff is everywhere continuous in the actions of all players, she proves that there is a sequence of such equilibria for finite-action games that converges to an equilibrium for the game where players choose from a continuum of actions. For the private information contest the single crossing condition generally holds, whereas continuity of payoffs is ensured by $\sigma > 0$.

The uniqueness result in Proposition 2 follows from Mason and Valentinyi (2010). They show that if the effect of a player's own type on the expected payoff difference between two of his actions dominates the effect of his opponents' actions, the best response correspondence is a contraction which implies the existence of a unique equilibrium. In the private information contest, increasing the noise in determining the winner reduces the effect of his opponents' efforts on a player's payoff, leaving the effect of that player's costs unchanged. Hence, a sufficient condition for the equilibrium of the private information contest to be unique is that σ is large enough.

Note that Proposition 2 does not directly ensure existence of a pure-strategy Bayesian Nash equilibrium for the standard lottery contest where $\sigma = 0$. Yet, as our existence result holds for any $\sigma > 0$, a pure-strategy Bayesian Nash equilibrium exists for a contest that approximates the standard lottery contest arbitrarily closely (with an arbitrarily small σ). Moreover, if we impose assumption A3 in addition to A1, the existence proof offered by Ryvkin (2010) covers the $\sigma = 0$ case.

We now turn to establishing some properties of pure-strategy Bayesian Nash equilibria for $\sigma \geq 0$. As we have stated before, in any pure-strategy Bayesian Nash equilibrium, the corresponding equilibrium strategies $\xi_1, \ldots, \xi_n$ solve

$$\xi_i(c_i) = \arg\max_{x_i \geq 0} E\Big[p_i(x_i, \boldsymbol{\xi}_{-i}(\mathbf{C}_{-i}))\Big] - c_i x_i \qquad \forall i \text{ and } c_i \in [\underline{c}_i, \overline{c}_i]. \tag{7}$$

Note that $\sigma = 0$ represents a special case because of the discontinuity in $p_i(\mathbf{x})$ if $x_i = 0$ for all i. However, in any pure-strategy equilibrium for $\sigma = 0$ the event that all players choose zero effort must happen with probability zero. To see this, suppose by way of contradiction that there is an equilibrium where each player chooses zero effort with strictly positive probability, i.e., $\Pr[\xi_i(C_i) = 0] > 0$ for all i. Consider a type c_i of player i for which $\xi_i(c_i) = 0$ and suppose this type deviates by investing effort $\varepsilon > 0$ instead of zero. The corresponding gain in interim expected payoff for player i amounts to

$$E\Big[p_i(\varepsilon, \boldsymbol{\xi}_{-i}(\mathbf{C}_{-i})) - p_i(0, \boldsymbol{\xi}_{-i}(\mathbf{C}_{-i}))\Big] - c_i\varepsilon \;\geq\; (1 - \tfrac{1}{n}) \prod_{j \neq i} \Pr[\xi_j(C_j) = 0] - c_i\varepsilon$$

which is strictly positive for any ε small enough. Hence, there can be no equilibrium where $\Pr[\xi_i(C_i) = 0] > 0$ for all i. In any pure-strategy equilibrium for $\sigma = 0$ we must have $\xi_i(c_i) > 0$ for all $c_i \in [\underline{c}_i, \overline{c}_i]$ for at least one player i. We will in the following simplify the exposition by ignoring the discontinuity in p_i when studying properties of pure-strategy equilibria.

Returning to the general case where $\sigma \geq 0$, we can rewrite (7) as

$$\xi_i(c_i) = \arg\max_{x_i \geq 0} U_i(x_i, c_i) \qquad \forall i \text{ and } c_i \in [\underline{c}_i, \overline{c}_i]$$

where[9]

$$U_i(x_i, c_i) := E\left[\frac{x_i+\sigma}{\sum_{j\neq i}\xi_j(C_j)+x_i+n\sigma}\right]-c_i x_i.$$

Since $U_i(x_i, c_i)$ is strictly concave in x_i, the first order condition $\frac{\partial U_i(x_i,c_i)}{\partial x_i}\leq 0$, with equality if $x_i > 0$, defines the best response x_i for type c_i of player i. As in equilibrium player i chooses $x_i = \xi_i(c_i)$, we obtain, for each i, the equilibrium condition

$$E\left[\frac{\sum_{j\neq i}\xi_j(C_j)+(n-1)\sigma}{\left(\sum_{j\neq i}\xi_j(C_j)+\xi_i(c_i)+n\sigma\right)^2}\right]\leq c_i, \text{ with equality for } c_i \text{ where } \xi_i(c_i)>0. \tag{8}$$

In general, there is no closed form solution to this system of equations. We can, however, still infer some properties of equilibrium efforts from condition (8), as we will do in the following lemma.

Lemma 1. *Assume A1. In any pure-strategy Bayesian Nash equilibrium of the private information contest, player i's equilibrium strategy $\xi_i(c)$ has the following properties. There exists a $\tilde{c}_i \in [\underline{c}_i, \overline{c}_i]$ such that $\xi_i(c) = 0$ for $c > \tilde{c}_i$ while $\xi_i(c)$ is positive and strictly decreasing for $c < \tilde{c}_i$. If $\sigma > 0$, $\tilde{c}_i \leq \max\left\{\frac{n-1}{n^2\sigma}, \underline{c}_i\right\}$. If $\sigma = 0$, $\tilde{c}_i = \overline{c}_i$ for at least one $i \in \{1, 2, \dots n\}$. Moreover,*

$$\xi_i(c) \leq \frac{1}{4c} - \sigma \quad \textit{for } c < \tilde{c}_i.$$

The sum of ex ante expected equilibrium efforts satisfies

$$\sum_{i=1}^{n} E[\xi_i(C_i)] \geq \frac{n-1}{\sum_{i=1}^{n} E[C_i]} - n\sigma.$$

Proof. See Appendix A.2. □

Fey (2008) considers a private information contest where $n = 2$, $\sigma = 0$, and costs are drawn from the uniform distribution on $[0.01, 1.01]$. Looking for a symmetric equilibrium, he numerically approximates the equilibrium strategies and finds that, for each c, $\xi_1(c) = \xi_2(c)$ is smaller than the equilibrium effort in the complete infor-

[9] For $\sigma = 0$, as shown above, there must be in equilibrium at least one player k that exerts strictly positive effort for all types. Expected payoffs of all $i \neq k$ are hence given by $U_i(x_i, c_i)$. If $\Pr[\xi_i(C_i) = 0] > 0$ for all $i \neq k$, using $U_k(0, c_k)$ for player k's expected payoff when choosing $x_k = 0$ is not correct. Yet, as we have argued above, $x_k = 0$ does not maximize k's expected payoff.

mation contest where both players are commonly known to have cost c. Lemma 1 shows that $\xi_i(c) \leq \max\left\{\frac{1}{4c} - \sigma, 0\right\}$ where, according to Corollary 1, the RHS is exactly the equilibrium effort of the symmetric complete information contest with $n = 2$. Hence, we have shown that Fey's finding generally holds for any pure-strategy equilibrium of the two-player private information contest where cost distributions satisfy A1 and $\sigma \geq 0$. In independent research, Ryvkin (2010) has shown this result additionally assuming A3 and $\sigma = 0$, but allowing for an in other respects more general contest success function.[10]

For the remainder of this section, we will simplify the model by imposing assumption A3, i.e., by assuming that costs are all drawn from same distribution F. We exclusively focus in this case on symmetric equilibria where all players choose their effort according to the same equilibrium strategy $\xi(c)$.[11] Equilibrium condition (8) thus simplifies to a single equation:

$$E\left[\frac{\sum_{i=1}^{n-1}\xi(C_i)+(n-1)\sigma}{\left(\sum_{i=1}^{n-1}\xi(C_i)+\xi(c)+n\sigma\right)^2}\right] \leq c, \quad \text{with equality for } c \text{ where } \xi(c) > 0.^{12} \tag{9}$$

Note that if $\sigma = 0$, $\xi(c) > 0$ for all c. This follows from the same argument we used above to show that $\xi_i(c_i) > 0$ for all c_i for at least one i.

Although the equilibrium condition is simplified when assuming A3 and focusing on symmetric equilibria, there is in general no closed form solution for $\xi(c)$. Given a specific assumption concerning F, however, numerical methods can be applied to (9) so as to compute an approximation to the symmetric equilibrium strategy $\xi(c)$. Appendix B contains some notes on the methods we employed to find the numerical results presented in this paper.

For the case where costs are uniformly distributed on $[0.5, 1.5]$, numerical approx-

[10]However, numerical results of my own as well as independent numerical examples by Ryvkin (2010) suggest that $\xi_i(c)$ does not have this property if $n > 2$.

[11]In Appendix B of Kadan (2002) a variant of Theorem 1 by Athey (2001) is proved, stating that if types are all drawn from the same distribution, a symmetric pure-strategy equilibrium exists for finite-action games. As Theorem 2 by Athey (2001) continues to hold, the existence of a symmetric equilibrium for games with a continuum of actions follows. In turn, our Proposition 2 could be modified so as to yield existence of a symmetric equilibrium. Moreover, note that if the equilibrium is unique, it has to be symmetric. Ryvkin (2010) studies existence of a symmetric equilibrium for a class of contests that includes the lottery contest with $\sigma = 0$.

[12]$\sum_{i=1}^{n-1}\xi(C_i)$ is chosen arbitrarily and could be replaced by any sum over $n-1$ distinct $i \in \{1, 2, \ldots, n\}$.

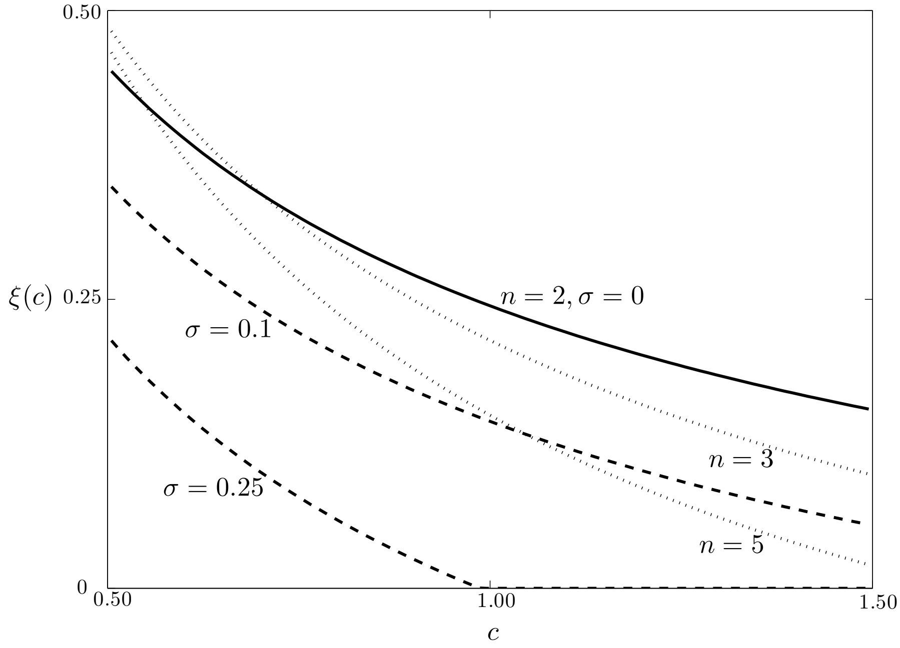

Figure 1: Symmetric equilibrium strategy $\xi(c)$ for uniformly distributed costs.

imations to the equilibrium strategy are shown in Figure 1. The solid line represents $\xi(c)$ when $n = 2$ and $\sigma = 0$, the dotted and dashed lines display the effect of increasing n and σ, respectively. Increasing the number of players in general reduces the influence a single player's effort has on winning probabilities, as for any symmetric strategy the sum of efforts of player i's opponents increases. Yet the sum of opponents' efforts is not simply scaled upwards, its distribution changes as well: as n increases, the variance of average efforts decreases. When more players take part in the contest, the symmetric equilibrium strategy generally requires players with high costs to reduce their efforts. As n is increased from 2 to 3, players with very low costs exert more effort. If n increases further, all types reduce their efforts, yet players with low costs do so by less than players with high costs. A higher σ increases the noise in determining the winner, making players' efforts less effective in changing winning probabilities. Consequently, equilibrium efforts for all types decrease in response to an increase in σ.

5 Expected Efforts and Rent Dissipation

Having analyzed equilibrium efforts for all three contests, we are now ready to study the impact of uncertainty and private information on the behavior of contestants. Comparing the ex ante expected sum of efforts in the complete information contest to the sum of efforts in the no information contest we find that the former is at least as high as the latter.

Proposition 3. *Ex ante expected aggregate effort is lower in the no information contest than in the complete information contest, i.e.,*

$$\sum_{i=1}^{n} x_i^{\mathrm{NI}} = \sum_{i=1}^{n} x_i^{\mathrm{CI}}(E[\mathbf{C}]) \leq \sum_{i=1}^{n} E[x_i^{\mathrm{CI}}(\mathbf{C})].$$

Proof. See Appendix A.3. □

According to Proposition 3, if players are ignorant of all cost realizations, they exert less effort than when all cost parameters are commonly known. Additionally imposing assumptions A1 and A2, it turns out that effort under ignorance of all cost realizations is also lower than when each player is privately informed about his own cost parameter.

Proposition 4. *Assume A1 and A2. Then, ex ante expected aggregate effort is lower in the no information contest than in the private information contest, i.e.,*

$$\sum_{i=1}^{n} x_i^{\mathrm{NI}} \leq \sum_{i=1}^{n} E[\xi_i(C_i)]. \tag{10}$$

Proof. Under assumption A2, Corollaries 1 and 2 imply

$$\sum_{i=1}^{n} x_i^{\mathrm{NI}} = \max\left\{ \frac{n-1}{\sum_{i=1}^{n} E[C_i]} - n\sigma, 0 \right\}.$$

From Lemma 1 immediately follows (10). □

Propositions 3 and 4 show that, under assumptions A1 and A2, the no information contest yields the smallest aggregate effort. An organizer of a contest who is interested in maximizing ex ante expected aggregate effort thus prefers to let the contest take place at the interim or ex post stage.

A \ B	(i) difference in effort 1	2	4	8	(ii) difference in expenses 1	2	4	8	
1	-0.61	0.27	0.58	0.36	-0.32	0.38	0.50	0.26	$\times 10^{-3}$
2	-0.54	-0.30	0.05	0.19	-0.44	-0.16	0.12	0.17	
4	-0.17	-0.18	-0.11	0.01	-0.17	-0.16	-0.06	0.03	
8	-0.03	-0.05	-0.05	-0.03	-0.04	-0.05	-0.05	-0.02	

Table 1: (i) $E[x_i^{\text{CI}}(\mathbf{C})] - E[\xi_i(C_i)]$ and **(ii)** $E[C_i x_i^{\text{CI}}(\mathbf{C})] - E[C_i \xi_i(C_i)]$ if $n = 2$, $\sigma = 0$, and F is the beta distribution on $[0.5, 1.5]$ with parameters A and B.

Comparing expected efforts in the complete information contest and the private information contest to each other is more difficult. Malueg and Yates (2004) consider the standard Tullock contest between two players. A player's type is either high or low, each with (unconditional) probability $\frac{1}{2}$. Malueg and Yates (2004) find a player's interim expected effort in the complete information contest to exactly match his effort in the private information contest. However, this result is not robust. Analyzing a corresponding variant of our model where costs are independently drawn from $\{c_L, c_H\}$ with $c_H > c_L$ and q denoting the probability for c_L, we find that interim expected efforts are higher in the complete information contest than in the private information contest if $q \in (\frac{1}{2}, 1)$ whereas the opposite is true if $q \in (0, \frac{1}{2})$.

Suppose assumptions A1 and A3 hold, i.e., costs are all drawn from the same continuous distribution F. In order to numerically study equilibrium efforts we now consider a specific distribution F that is consistent with A1. Let costs be drawn from the (generalized) beta distribution on $[0.5, 1.5]$ with parameters $A, B \geq 1$. The corresponding probability density function amounts to

$$f(c) = \frac{(c - 0.5)^{A-1} (1.5 - c)^{B-1}}{\int_0^1 z^{A-1} (1 - z)^{B-1} \, \mathrm{d}z} \qquad \text{if } c \in [0.5, 1.5]$$

and $f(c) = 0$ otherwise. Changing the free parameters A, B of the beta distribution allows for obtaining a variety of differently shaped densities. Assuming $n = 2$ and $\sigma = 0$, Panel (i) of Table 1 reports the difference between the ex ante expected effort in the complete information contest and that in the symmetric equilibrium of the private information contest for different combinations of A and B. Note that $A = B = 1$ corresponds to the uniform distribution. Moving from the binary uniform distribution considered by Malueg and Yates (2004) to a continuous uniform distribution, we hence observe that the complete information contest leads to lower ex ante expected efforts than the private information contest. In fact, according to Table 1 private information efforts are higher for all parameter choices where $A \geq B$. If B is sufficiently larger than A, however, complete information efforts exceed private information efforts. For $A = B$ the density f is symmetric about the mean; if $A > (<) B$, f is negatively (positively) skewed. Hence, our numerical results are well in accord with the intuition provided by the binary case. If the distribution places more weight on high costs, the private information contest yields higher expected efforts than the complete information contest.

From our numerical examples we can conclude that in the two-player lottery contest there is no general ranking between private and complete information in terms of effort. At first sight, this result seems to contradict Ryvkin (2010) who finds that in two-player contests efforts are always lower under private information than under complete information. The reason for this contradiction is that Ryvkin's conclusion is based on a different comparison: he compares the private information effort of a player with cost c to the equilibrium effort in a complete information contest where all players are identical and have cost c. Hence, in contrast to us as well as Malueg and Yates (2004), when moving from private to complete information Ryvkin not only changes the informational setting, but in addition also imposes a symmetry assumption on the complete information contest. This additional assumption turns out to be essential for Ryvkin's general ranking.

Presenting numerical examples where we hold F fixed, we will argue in the following that the ranking between private and complete information also depends on σ and n. Under the assumption that all costs are uniformly distributed on $[0.5, 1.5]$, Table 2 exhibits a player's ex ante expected effort in each of the three types of contests for various combinations of n and σ. As predicted by Propositions 3 and 4, effort is generally lowest in the no information contest. For $\sigma = 0$ expected effort is always highest under private information. Whereas a single player's effort in all three contests is decreasing in n, the sum of expected efforts is increasing. For $\sigma > 0$ we observe that the complete rather than the private information contest induces the highest expected efforts if the number of players is sufficiently large.

A measure that is often studied in models of rent-seeking contests is the ratio between total expenses and the value of the prize. This ratio determines to what extent the rent the winner obtains is dissipated through contestants' investment of resources. In our model, players compete for a rent of value 1. Therefore, rent dis-

		$n=2$	$n=3$	$n=5$	$n=7$	$n=10$
	complete info.	0.2616	0.2297	0.1706	0.1368	0.1068
$\sigma=0$	private info.	0.2622	0.2383	0.1786	0.1419	0.1100
	no info.	0.2500	0.2222	0.1600	0.1224	0.0900
	complete info.	0.1616	0.1352	0.0876	0.0584	0.0311
$\sigma=0.1$	private info.	0.1622	0.1383	0.0870	0.0565	0.0291
	no info.	0.1500	0.1222	0.0600	0.0224	0
	complete info.	0.0440	0.0373	0.0095	0	0
$\sigma=0.25$	private info.	0.0440	0.0369	0.0092	0	0
	no info.	0	0	0	0	0

Table 2: Ex ante expected effort by player i for uniform F.

sipation is simply defined as $\sum_{i=1}^{n} c_i x_i$. In the remainder of this section we try to shed some light on how uncertainty and private information affect rent dissipation. Similar to Propositions 3 and 4, the following two propositions compare expected rent dissipation in the no information contest to that in the private and the complete information contest. Note, however, that the results on rent dissipation require stronger assumptions than the corresponding results on efforts.

Proposition 5. *Assume $\sigma = 0$ and A2. Then, ex ante expected rent dissipation is larger in the no information contest than in the complete information contest.*

Proof. Assuming $\sigma = 0$ and A2, Corollaries 1 and 2 imply that ex ante expected rent dissipation in the no information contest amounts to $\frac{n-1}{n}$. For the complete information contest Cornes and Hartley (2005) show that with m active players rent dissipation is bounded above by $\frac{m-1}{m}$ which is smaller than $\frac{n-1}{n}$ for all $m \leq n$. □

Proposition 6.

(i) Assume $n = 2$, $\sigma \leq \frac{1}{4\max\{\overline{c}_1,\overline{c}_2\}}$, A1, and A2. Then, ex ante expected rent dissipation is larger in the no information contest than in the private information contest.

(ii) Assume $\sigma = 0$, A1, and A3. Then, ex ante expected rent dissipation is larger in the no information contest than in any symmetric equilibrium of the private information contest.

Proof. See Appendix A.4. □

For the standard lottery contest, Proposition 5 shows that under assumption A2 no information leads to higher rent dissipation than complete information. According to Proposition 6, when we additionally impose A1 and A3, rent dissipation in the no information contest is also higher than in the private information contest. Moreover, for two-player contests this result continues to hold when replacing A3 by A2 and considering a strictly positive σ that is not too big.

In general, under assumptions A1 and A3 and provided that a symmetric equilibrium is played in the private information contest, all players expect ex ante to win the prize of value 1 with the same probability, regardless of the type of contest they engage in. Consequently, they prefer the contest with the lowest expected rent dissipation. Propositions 5 and 6 imply that in the standard lottery contest from an ex ante perspective contestants prefer to compete under complete or private information rather than under no information.

Propositions 3 and 4 have shown that ex ante expected effort is lowest in the no information contest. From this, one would perhaps expect that also rent dissipation is lowest in the no information contest. Yet, Propositions 5 and 6 show that exactly the opposite is true for the standard lottery contest. The explanation for this surprising result lies in fact that aggregate effort is allocated differently among players in each informational setting. Under A2, players in a no information contest all choose the same amount of effort which is independent of the actual realization of costs. In contrast, under private or complete information players with low costs invest more effort than players with high costs, inducing, as it were, a more efficient production of aggregate effort. Hence, a given level of aggregate effort under private or complete information is associated with lower rent dissipation than under no information. Under the assumptions identified in Propositions 5 and 6, the efficiency gain under complete or private information outweighs the fact that aggregate effort is higher in those cases, resulting in lower rent dissipation than under no information.

Similar to the result on expected efforts, we find that there is no general ranking between private and complete information in terms of expected rent dissipation. Suppose A1 and A3 hold. For the example of the Beta distribution we considered above, Panel (ii) of Table 1 reports the difference between ex ante expected rent dissipation in the complete information contest and that in the symmetric equilibrium of the private information contest. In this example, the rankings in terms of effort and rent dissipation coincide.

Again assuming all costs to be uniformly distributed on [0.5, 1.5], Table 3 presents ex ante expected rent dissipation in all three contests for different values for n and

		$n=2$	$n=3$	$n=5$	$n=7$	$n=10$
	complete info.	0.4780	0.5919	0.6534	0.6833	0.7136
$\sigma=0$	private info.	0.4787	0.6249	0.7206	0.7484	0.7752
	no info.	0.5000	0.6667	0.8000	0.8571	0.9000
	complete info.	0.2780	0.3148	0.2951	0.2545	0.1797
$\sigma=0.1$	private info.	0.2787	0.3250	0.2944	0.2468	0.1679
	no info.	0.3000	0.3667	0.3000	0.1571	0
	complete info.	0.0571	0.0682	0.0259	0	0
$\sigma=0.25$	private info.	0.0570	0.0673	0.0248	0	0
	no info.	0	0	0	0	0

Table 3: Ex ante expected rent dissipation for uniform F.

σ. If $\sigma = 0$, the no information contest induces the largest dissipation, followed by the private information contest and the complete information contest. If $\sigma = 0.25$, this ranking is reversed. In addition, rent dissipation is generally decreasing in n for $\sigma = 0.25$ whereas it is increasing for $\sigma = 0$. With $\sigma = 0.1$, rent dissipation is lowest in the complete information contest and increasing in n if $n \leq 3$ whereas it is lowest in the no information contest and decreasing in n if the number of players is sufficiently large. As we observe for $n = 5$, it is also possible that rent dissipation is lowest in the private information contest.

Under assumptions A1 and A3, Corollaries 1 and 2 imply that ex ante expected rent dissipation in the no information contest amounts to $\max\left\{\frac{n-1}{n} - \sigma n E[C], 0\right\}$. If $\sigma = 0$, we obtain the classic result that the rent is fully dissipated as $n \to \infty$. In contrast, if $\sigma > 0$, dissipation is decreasing in n for all $n > (\sigma E[C])^{-\frac{1}{2}}$. For n or σ large enough, i.e., if $\sigma \geq \frac{n-1}{n^2 E[C]}$, there is no dissipation at all since the noise in determining the winner is so large that players invest zero effort. Table 3 suggests that rent dissipation under complete and private information responds qualitatively similar to changes in n and σ. Yet, when increasing n or σ, dissipation drops not as fast to zero as under no information, resulting in higher rent dissipation under complete and private information. For example, if $\sigma = \frac{n-1}{n^2 E[C]}$, aggregate effort and rent dissipation are zero under no information. Under complete and private information, however, still many realizations of costs are possible where at least some players learn that they have considerably lower costs than $E[C]$ and thus find it profitable to invest strictly positive effort. This is why Propositions 5 and 6 in general do not extend to $\sigma > 0$.

6 From Uncertain Costs to Uncertain Values

In the literature on contests among asymmetric players, those players are sometimes assumed to differ in their valuation for the prize rather than in their costs. While models with asymmetric costs and asymmetric valuations are in many ways equivalent, this is not always true when comparing different informational settings to each other. In the following, we carefully study this issue and examine to what extent the results obtained in preceding sections carry over to models with uncertain values.

Suppose $c_i = 1$ for all i, but each player i values the prize v_i rather than 1. For each i, valuation v_i is a realization of the random variable V_i that is independently distributed according to the continuous distribution function $\tilde{F}_i$ on $[\underline{v}_i, \overline{v}_i]$ with $0 < \underline{v}_i < \overline{v}_i < \infty$. Accordingly, player i's ex post payoff amounts to

$$\tilde{u}_i(\mathbf{x}, v_i) := p_i(\mathbf{x})v_i - x_i.$$

Let $\tilde{x}_i^{\mathrm{CI}}(\mathbf{v})$, $\tilde{\xi}_i(v_i)$, and $\tilde{x}_i^{\mathrm{NI}}$ denote player i's equilibrium strategies in the complete, private, and no information contest for this modified setup. The equilibrium strategies have to satisfy

$$\tilde{x}_i^{\mathrm{CI}}(\mathbf{v}) \in \arg\max_{x_i} \tilde{u}_i(x_i, \tilde{\mathbf{x}}_{-i}^{\mathrm{CI}}(\mathbf{v}), v_i) \quad \forall i, \tag{11}$$

$$\tilde{\xi}_i(v_i) \in \arg\max_{x_i} E[\tilde{u}_i(x_i, \tilde{\boldsymbol{\xi}}_{-i}(\mathbf{V}_{-i}), v_i)] \quad \forall i, v_i \in [\underline{v}_i, \overline{v}_i], \tag{12}$$

$$\tilde{x}_i^{\mathrm{NI}} \in \arg\max_{x_i} E[\tilde{u}_i(x_i, \tilde{\mathbf{x}}_{-i}^{\mathrm{NI}}, V_i)] \quad \forall i.$$

Assume A1 and let, for all i, V_i be a transformation of the random variable C_i such that

$$V_i = \frac{1}{C_i} \quad \text{and therefore} \quad \tilde{F}_i(v_i) = 1 - F_i(\tfrac{1}{v_i}). \tag{13}$$

With this transformation of variables the maximization problems in (11) and (12) coincide with those in (2) and (3) for the original model.[13] Consequently,

$$\tilde{x}_i^{\mathrm{CI}}(\mathbf{v}) = x_i^{\mathrm{CI}}(\tfrac{1}{v_1}, \dots, \tfrac{1}{v_n}) \quad \text{and} \quad \tilde{\xi}_i(v_i) = \xi_i(\tfrac{1}{v_i}).$$

All our results for the original model concerning the complete and private informa-

[13]Note that, for each player i, $\tilde{u}_i(x_i, \tilde{\mathbf{x}}_{-i}^{\mathrm{CI}}(\mathbf{v}), v_i) = v_i u_i(x_i, \tilde{\mathbf{x}}_{-i}^{\mathrm{CI}}(\mathbf{v}), \frac{1}{v_i})$ and $E[\tilde{u}_i(x_i, \tilde{\boldsymbol{\xi}}_{-i}(\mathbf{V}_{-i}), v_i)] = v_i E[u_i(x_i, \tilde{\boldsymbol{\xi}}_{-i}(\mathbf{V}_{-i}), \frac{1}{v_i})]$ where v_i is a positive constant.

tion contest and their comparison to each other therefore directly extend to the case with uncertain values.

Now consider the no information contest. Similar to the original model, we have $\tilde{x}_i^{\text{NI}} = \tilde{x}_i^{\text{CI}}(E[\mathbf{V}])$. Hence, under the transformation of variables in (13), Jensen's inequality implies

$$\tilde{x}_i^{\text{NI}} = x_i^{\text{CI}}(\tfrac{1}{E[V_1]}, \dots, \tfrac{1}{E[V_n]}) \geq x_i^{\text{CI}}(E[\tfrac{1}{V_1}], \dots, E[\tfrac{1}{V_n}]) = x_i^{\text{NI}}. \tag{14}$$

If (13) holds, efforts under cost uncertainty are smaller than when values are uncertain.[14] As a result, Propositions 3 and 4 stating that expected efforts in the no information contest are smaller than in the other two contests do not extend to the model with uncertain valuations.

Studying a numerical example of the standard lottery contest where values for two players are drawn from two different discrete distributions with the same mean, Hurley and Shogren (1998a) find that a player's ex ante expected effort in the no information contest exceeds that in the private information contest. This is exactly the opposite of what Proposition 4 states. Making use of results derived in preceding sections, we establish the following.

Proposition 7. *Assume A1 and let $V_i = \frac{1}{C_i}\,\forall i$. Suppose $n = 2$, $E[V_1] = E[V_2]$, and $\underline{v}_i \geq 4\sigma$ for $i = 1, 2$. Then, ex ante expected efforts in the no information contest are higher than in the private information contest:*

$$\tilde{x}_i^{\text{NI}} \geq E[\tilde{\xi}_i(V_i)] \qquad \textit{for } i = 1, 2.$$

Proof. Let $\tilde{\mu} := E[V_1] = E[V_2]$. From Lemma 1 follows, with $\underline{v}_i \geq 4\sigma$,

$$\tilde{\xi}_i(v_i) = \xi_i(\tfrac{1}{v_i}) \leq \frac{1}{4} v_i - \sigma$$

implying

$$E[\tilde{\xi}_i(V_i)] \leq \frac{1}{4}\tilde{\mu} - \sigma.$$

[14] Suppose the prize is measured in dollars and effort in hours, such that $c_i = \frac{1}{v_i}$ is the price of one hour in dollars. As long as player i knows this price, his optimization problem is unchanged when expressing payoffs in terms of hours rather than dollars. However, if the price c_i is random, i's payoff measured in dollars follows a different distribution than if measured in hours. That is why optimal effort choice in the no information contest changes when moving from the original to the modified setup.

According to Corollaries 1 and 2,

$$\tilde{x}_i^{\mathrm{NI}} = x_i^{\mathrm{CI}}\left(\tfrac{1}{E[V_1]}, \tfrac{1}{E[V_2]}\right) = \frac{1}{4}\tilde{\mu} - \sigma. \qquad \square$$

Proposition 7 generalizes the numerical result by Hurley and Shogren (1998a) to any standard two-player lottery contest with values drawn from continuous distributions with equal means. Moreover, provided that the additional noise σ is not too large, the result continues to hold for $\sigma > 0$. Interestingly, Morath and Münster (2010) find the same ranking of expected efforts to generally hold for the two-player all-pay auction with uncertain values.[15]

Finally, let us turn to rent dissipation. The literature provides several different approaches for measuring rent dissipation in contests with asymmetric valuations. We will follow Nti (1999) who defines rent dissipation as the fraction of a player's valuation expended in the contest. In that sense, Propositions 5 and 6 extend to contests with uncertain values. Suppose (13) holds. Using (14), Propositions 5 and 6 then imply

$$\sum_{i=1}^{n} E\left[\frac{\tilde{x}_i^{\mathrm{NI}}}{V_i}\right] \geq \sum_{i=1}^{n} E\left[\frac{\tilde{x}_i^{\mathrm{CI}}(\mathbf{V})}{V_i}\right] \quad \text{and} \quad \sum_{i=1}^{n} E\left[\frac{\tilde{x}_i^{\mathrm{NI}}}{V_i}\right] \geq \sum_{i=1}^{n} E\left[\frac{\tilde{\xi}_i(V_i)}{V_i}\right].$$

Hence, under the assumptions stated in Propositions 5 and 6, ex ante expected rent dissipation, measured as the sum of each player's effort divided by his valuation, is larger in the no information contest than in both the complete and the private information contest with uncertain values.

7 Conclusion

In order to study the impact of uncertainty and private information on imperfectly discriminating contests, we compare three different informational settings to each other. The model we employ is the Tullock lottery contest, augmented by an additional noise parameter σ. By considering more than two players and types that are drawn from general continuous probability distributions, we extend the analysis of rent-seeking contests under incomplete information.

For both the no information and the complete information contest we find a

[15] Note that, for the same reason as in the rent-seeking contest, their result does not extend to the all-pay auction with uncertain costs of effort.

unique pure-strategy Nash equilibrium. For any $\sigma > 0$ we prove that the private information contest has a Bayesian Nash equilibrium in monotone pure strategies. In addition, we find the Bayesian Nash equilibrium to be unique if σ is large enough. Apart from analytically deriving properties of the equilibrium strategies, we also identify numerical methods suitable for computing approximations to those strategies. The simple application of Athey (2001) we present for proving equilibrium existence in the private information contest could readily be extended to a more general class of contest. For example, this class includes contest success functions of the form $g_i(x_i) \Big/ \sum_j g_j(x_j)$ where, for each i, $g_i(\cdot)$ is an increasing and strictly positive continuous function. Analyzing the corresponding equilibrium strategies is an interesting task for future research.

In general, ex ante expected aggregate effort is lowest in the no information contest, provided that cost distributions all have the same mean. Yet at the same time we find that rent dissipation in the no information contest is larger than in the other two contests if $\sigma = 0$ and all costs are drawn from the same distribution. In this case, assuming a symmetric equilibrium is played under private information, contestants as well as a contest organizer benefiting from players' efforts would ex ante prefer the private and the complete information contest over the no information contest. Hence, we would expect contestants to try to gather information before competing. Moreover, the organizer would have an incentive to encourage such behavior. Our analysis can therefore be seen as a first step for future work on acquisition and provision of information in imperfectly discriminating contests.

As our numerical examples illustrate, there is no general ranking between complete and private information, neither in terms of expected efforts nor in terms of rent dissipation. Which of the two contests yields higher efforts or rent dissipation depends on the distribution of types, the number of players, and the parameter σ.

Whereas our model with uncertain costs is in many respects equivalent to a model with uncertain valuations, there is one exception. When comparing the no information contest to the private or complete information contest, the two models can lead to different conclusions. This is an issue that is not restricted to our specific contest format. It should be kept in mind when comparing results in the literature that involve no information contests.

Appendix A: Proofs

A.1 Proof of Proposition 2

In order to apply results by Athey (2001) as well as Mason and Valentinyi (2010) to the private information contest, it is useful to perform the change of variables $t_i := -c_i$. With that and assuming $\sigma > 0$, contestant i's payoff is equivalent to

$$w_i(\mathbf{x}, t_i) := \frac{x_i + \sigma}{\sum_{j \neq i} x_j + x_i + n\sigma} + t_i x_i$$

where $x_i \in [0, \frac{1}{\underline{c}_i}]$ and $t_i \in [-\overline{c}_i, -\underline{c}_i]$. Under assumption A1 each t_i is drawn from the distribution $\hat{F}_i(t_i) := 1 - F_i(-t_i)$. Given a strategy $\gamma_j : [-\overline{c}_j, -\underline{c}_j] \to [0, \frac{1}{\underline{c}_j}]$ for each $j \neq i$, player i's interim expected payoff amounts to

$$W_i(x_i, t_i) := \int \dots \int_{\mathcal{T}_i} \frac{x_i + \sigma}{\sum_{j \neq i} \gamma_j(t_j) + x_i + n\sigma} \prod_{j \neq i} \mathrm{d}\hat{F}_j(t_j) + t_i x_i$$

where $\mathcal{T}_i := [-\overline{c}_1, -\underline{c}_1] \times \cdots \times [-\overline{c}_{i-1}, -\underline{c}_{i-1}] \times [-\overline{c}_{i+1}, -\underline{c}_{i+1}] \times \cdots \times [-\overline{c}_n, -\underline{c}_n]$.

From $\frac{\partial^2 W_i(x_i, t_i)}{\partial x_i \partial t_i} = 1$ for all i follows that the *Single crossing condition for games of incomplete information* in Athey (2001) is satisfied. Note that our model is consistent with Athey's assumption A1. Moreover, each player i's actions are restricted to the interval $[0, \frac{1}{\underline{c}_i}]$ and $w_i(\mathbf{x}, t_i)$ is continuous in $\mathbf{x}$ for all i as long as $\sigma > 0$. Hence, existence of an equilibrium in nondecreasing strategies $\gamma_i(t_i)$ follows from Corollary 2.1 in Athey (2001). Of course, this corresponds to nonincreasing strategies in the original game where types are described by c_i.

For the uniqueness result we apply findings by Mason and Valentinyi (2010). First, we will show that their assumptions U1-U3 and D1-D2 hold in our model. Note that with the usual Euclidean metric we have $d_T(t_i, t_i') = |t_i - t_i'|$. For $\sigma > 0$ and $x_i \in [0, \frac{1}{\underline{c}_i}]$, $w_i(\mathbf{x}, t_i)$ is differentiable everywhere. Therefore, we can check U1 and U2 by looking at derivatives of $w_i(\mathbf{x}, t_i)$ as follows.[16] From $\frac{\partial^2 w_i(\mathbf{x}, t_i)}{\partial x_i \partial t_i} = 1$ for all i follows that U1 is satisfied with $\delta = 1$. U2 asks for $\left|\frac{\partial w_i(\mathbf{x}, t_i)}{\partial x_i}\right|$ to be bounded for all i. Note that the fraction on the RHS of

$$\frac{\partial w_i(\mathbf{x}, t_i)}{\partial x_i} = \frac{\sum_{j \neq i} x_j + (n-1)\sigma}{\left(\sum_{j \neq i} x_j + x_i + n\sigma\right)^2} + t_i$$

[16]This can be shown by appealing to the mean value theorem.

is clearly positive while it is maximized at $x_i = 0$ and $\sum_{j\neq i} x_j = 0$. Hence,

$$t_i < \frac{\partial w_i(\mathbf{x}, t_i)}{\partial x_i} \leq \frac{n-1}{n^2\sigma} + t_i \tag{15}$$

such that U2 is satisfied with $\omega = \max_i \omega_i$ where $\omega_i := \max\left\{\overline{c}_i, \frac{n-1}{n^2\sigma} - \underline{c}_i\right\}$. For U3 we have to find a $\kappa \in (0,\infty)$ such that, for all $x_i \geq x_i'$, $\mathbf{x}_{-i}$, $\mathbf{x}_{-i}'$, t_i, i,

$$\left|\left(w_i(x_i, \mathbf{x}_{-i}, t_i) - w_i(x_i', \mathbf{x}_{-i}, t_i)\right) - \left(w_i(x_i, \mathbf{x}_{-i}', t_i) - w_i(x_i', \mathbf{x}_{-i}', t_i)\right)\right| \leq \kappa \left(x_i - x_i'\right).$$

A sufficient condition for this is that

$$\max_{\mathbf{x}} \frac{\partial w_i(\mathbf{x}, t_i)}{\partial x_i} - \min_{\mathbf{x}} \frac{\partial w_i(\mathbf{x}, t_i)}{\partial x_i} \leq \kappa \qquad \forall i.$$

Using (15) we find that U3 holds with $\kappa = \frac{n-1}{n^2\sigma}$. As we have assumed that types are independently distributed, D1 is satisfied with $\iota = 0$ and D2 holds with $\nu = \max_{i,c} f_i(c)$. Finally, Theorem 4 in Mason and Valentinyi (2010) states that if $\delta > \iota\omega + \nu\kappa$, i.e., if $1 > \frac{n-1}{n^2\sigma} \max_{i,c} f_i(c)$, there is a unique Bayesian Nash equilibrium (which is in nondecreasing pure strategies). □

A.2 Proof of Lemma 1

Observe that the fraction on the LHS of (8) is strictly decreasing in $\xi_i(c_i)$. Hence, if $\xi_i(\hat{c}) > 0$ for some $\hat{c}$, then $\xi_i(c) > \xi_i(\hat{c})$ for all $c < \hat{c}$. Consequently, there must be a $\tilde{c}_i \in [\underline{c}_i, \overline{c}_i]$ such that $\xi_i(c) = 0$ for $c > \tilde{c}_i$ while $\xi_i(c)$ is positive and strictly decreasing for $c < \tilde{c}_i$.

Suppose $\sigma > 0$ and $c_i < \tilde{c}_i$. In this case (8) holds with equality. Note that the fraction on the LHS of (8) is maximized if $\sum_{j=1}^n \xi_j(c_j) = 0$, which implies $c_i \leq \frac{n-1}{n^2\sigma}$. Therefore, we must have $\tilde{c}_i \leq \frac{n-1}{n^2\sigma}$. Now, let $\sigma = 0$. As we have argued above, there must be at least one player choosing strictly positive effort for all types, i.e., $\tilde{c}_i = \overline{c}_i$ for at least one player i.

Assume $c_i < \tilde{c}_i$. Multiplying (8) on both sides with $\xi_i(c_i) + \sigma$ yields

$$E\left[\frac{\left(\sum_{j\neq i} \xi_j(C_j) + (n-1)\sigma\right)(\xi_i(c_i) + \sigma)}{\left(\sum_{j\neq i} \xi_j(C_j) + \xi_i(c_i) + n\sigma\right)^2}\right] = c_i(\xi_i(c_i) + \sigma).$$

Since

$$\frac{\left(\sum_{j\neq i}\xi_j(C_j)+(n-1)\sigma\right)(\xi_i(c_i)+\sigma)}{\left(\sum_{j\neq i}\xi_j(C_j)+\xi_i(c_i)+n\sigma\right)^2}$$
$$=\frac{\xi_i(c_i)+\sigma}{\sum_{j\neq i}\xi_j(C_j)+\xi_i(c_i)+n\sigma}\left(1-\frac{\xi_i(c_i)+\sigma}{\sum_{j\neq i}\xi_j(C_j)+\xi_i(c_i)+n\sigma}\right)\leq\frac{1}{4},$$

we obtain

$$\frac{1}{4}\geq c_i(\xi_i(c_i)+\sigma) \quad \text{or} \quad \xi_i(c_i)\leq\frac{1}{4c_i}-\sigma.$$

Replacing c_i by the random variable C_i, taking expectations on both sides of (8), and summing over all i, we obtain

$$E\left[\frac{(n-1)\sum_{i=1}^n\xi_i(C_i)+n(n-1)\sigma}{\left(\sum_{i=1}^n\xi_i(C_i)+n\sigma\right)^2}\right]\leq\sum_{i=1}^n E[C_i].$$

This can be rearranged to yield

$$E\left[\frac{1}{\sum_{i=1}^n\xi_i(C_i)+n\sigma}\right]\leq\frac{1}{n-1}\sum_{i=1}^n E[C_i].$$

Applying Jensen's inequality we find

$$\frac{1}{E\left[\sum_{i=1}^n\xi_i(C_i)\right]+n\sigma}\leq E\left[\frac{1}{\sum_{i=1}^n\xi_i(C_i)+n\sigma}\right]$$

and therefore

$$\sum_{i=1}^n E\left[\xi_i(C_i)\right]\geq\frac{n-1}{\sum_{i=1}^n E[C_i]}-n\sigma. \qquad \square$$

A.3 Proof of Proposition 3

Let $E[C_1]\leq E[C_2]\leq\cdots\leq E[C_n]$ and suppose expected costs are such that in the no information contest $m^*>0$ players choose a strictly positive effort. According to

Corollary 2,

$$\sum_{i=1}^{n} x_i^{\text{NI}} = Y(m^*) - n\sigma = \frac{(m^*-1) + \sqrt{(m^*-1)^2 + 4(n-m^*)\sigma \sum_{i=1}^{m^*} E[C_i]}}{2\sum_{i=1}^{m^*} E[C_i]} - n\sigma.$$

Now, consider the complete information contest. From Proposition 1,

$$\sum_{i=1}^{n} x_i^{\text{CI}}(\mathbf{c}) = \max_m Y(m) - n\sigma \geq Y(m^*) - n\sigma$$
$$\geq \frac{(m^*-1) + \sqrt{(m^*-1)^2 + 4(n-m^*)\sigma \sum_{i=1}^{m^*} z_i}}{2\sum_{i=1}^{m^*} z_i} - n\sigma$$

where $z_1, z_2, \ldots, z_n$ is a reordering of $c_1, c_2, \ldots, c_n$ such that $z_1 \leq z_2 \leq \cdots \leq z_n$. Taking expectations, we have

$$\sum_{i=1}^{n} E[x_i^{\text{CI}}(\mathbf{C})] \geq E\left[\frac{(m^*-1) + \sqrt{(m^*-1)^2 + 4(n-m^*)\sigma \sum_{i=1}^{m^*} Z_i}}{2\sum_{i=1}^{m^*} Z_i}\right] - n\sigma.$$

Note that the term we take the expectation of on the RHS is decreasing and convex in $\sum_{i=1}^{m^*} Z_i$. Jensen's inequality thus implies

$$\sum_{i=1}^{n} E[x_i^{\text{CI}}(\mathbf{C})] \geq \frac{(m^*-1) + \sqrt{(m^*-1)^2 + 4(n-m^*)\sigma \sum_{i=1}^{m^*} E[Z_i]}}{2\sum_{i=1}^{m^*} E[Z_i]} - n\sigma.$$

Since the expected sum of the first m order statistics cannot be larger than the sum of the m smallest means, i.e., $\sum_{i=1}^{m} E[Z_i] \leq \sum_{i=1}^{m} E[C_i]$ we finally obtain

$$\sum_{i=1}^{n} E[x_i^{\text{CI}}(\mathbf{C})] \geq \frac{(m^*-1) + \sqrt{(m^*-1)^2 + 4(n-m^*)\sigma \sum_{i=1}^{m^*} E[C_i]}}{2\sum_{i=1}^{m^*} E[C_i]} - n\sigma. \qquad \square$$

A.4 Proof of Proposition 6

(i) With $\sigma \leq \frac{1}{4\max\{\overline{c}_1, \overline{c}_2\}}$, Lemma 1 implies $c_i \xi_i(c_i) \leq \frac{1}{4} - c_i\sigma$ for all $c_i \in [\underline{c}_i, \overline{c}_i]$, $i = 1, 2$. Under A2, expected rent dissipation in the private information contest therefore satisfies

$$E[C_1\xi_1(C_1) + C_2\xi_2(C_2)] \leq \frac{1}{2} - 2\mu\sigma$$

where $\mu := E[C_1] = E[C_2]$. Due to Corollaries 1 and 2 combined with $\sigma < \frac{1}{4\mu}$, expected rent dissipation in the no information contest amounts to

$$E[C_1 x_1^{\mathrm{NI}} + C_2 x_2^{\mathrm{NI}}] = 2\mu x^{\mathrm{NI}} = \frac{1}{2} - 2\mu\sigma.$$

(ii) Assuming A3 we focus on symmetric equilibrium strategies ξ. $\sigma = 0$ implies $\xi(c_i) > 0$ for all c_i such that (9) holds with equality for all c_i. Multiplying (9) on both sides with $\xi(c_i)$ yields

$$E\left[\frac{\sum_{j\neq i}\xi(C_j)\xi(c_i)}{\left(\sum_{j\neq i}\xi(C_j)+\xi(c_i)\right)^2}\right] = c_i\xi(c_i).$$

Replacing c_i by the random variable C_i, taking expectations on both sides, and summing over all i, we obtain

$$E\left[\frac{\sum_{i=1}^n\sum_{j\neq i}\xi(C_j)\xi(C_i)}{\left(\sum_{i=1}^n\xi(C_i)\right)^2}\right] = \sum_{i=1}^n E[C_i\xi(C_i)]. \tag{16}$$

For the fraction on the LHS of (16) we have

$$\begin{aligned}\frac{\sum_{i=1}^n\sum_{j\neq i}\xi(C_j)\xi(C_i)}{\left(\sum_{i=1}^n\xi(C_i)\right)^2} &= \frac{\left(\sum_{i=1}^n\xi(C_i)\right)^2 - \sum_{i=1}^n\xi(C_i)^2}{\left(\sum_{i=1}^n\xi(C_i)\right)^2}\\ &= \frac{n-1}{n} - \frac{\sum_{i=1}^n\xi(C_i)^2 - \frac{1}{n}\left(\sum_{i=1}^n\xi(C_i)\right)^2}{\left(\sum_{i=1}^n\xi(C_i)\right)^2}\\ &= \frac{n-1}{n} - \frac{n\sum_{i=1}^n\left(\xi(C_i)-\frac{1}{n}\sum_{j=1}^n\xi(C_j)\right)^2}{\left(\sum_{i=1}^n\xi(C_i)\right)^2}.\end{aligned}$$

Hence (16) implies for expected rent dissipation in the private information contest

$$\sum_{i=1}^n E[C_i\xi(C_i)] < \frac{n-1}{n}.$$

Due to Corollaries 1 and 2 combined with $\sigma = 0$, expected rent dissipation in the no information contest amounts to

$$\textstyle\sum_{i=1}^{n} E[C_i x_i^{\mathrm{NI}}] = \dfrac{n-1}{n}. \qquad \square$$

Appendix B: Notes on Numerical Methods

For the private information contest, equilibrium strategies can in general not be obtained in closed form. In such a case, progress in studying contestants' behavior can be made by approximating equilibrium strategies numerically. In this appendix we provide a short discussion of the computational methods we applied to obtain the numerical results presented in Sections 4 and 5.

For all our numerical results we assume A1 and A3, i.e., all costs are independently drawn from the same continuous distribution F. In this case, a symmetric equilibrium strategy $\xi(c)$ has to fulfill condition (9) which can be restated as

$$H(\xi,c)\begin{cases} =0 & \text{if} \quad \xi(c)>0 \\ \leq 0 & \text{if} \quad \xi(c)=0 \end{cases} \tag{17}$$

where

$$H(\xi,c) := \int_{\underline{c}}^{\overline{c}} \cdots \int_{\underline{c}}^{\overline{c}} \frac{\sum_{j=1}^{n-1} \xi(k_j)+(n-1)\sigma}{\left(\sum_{j=1}^{n-1} \xi(k_j)+\xi(c)+n\sigma\right)^2} f(k_1)\mathrm{d}k_1 \ldots f(k_{n-1})\mathrm{d}k_{n-1} - c.$$

We approximate $\xi(c)$ numerically by a discrete function on a grid of points in $[\underline{c},\overline{c}]$. Denoting the size of the grid by g, we consider the set of points

$$\widehat{\mathbf{c}} = \left\{c^1, c^2, \ldots, c^g\right\} \qquad \text{where } c^i = \frac{2i-1}{2g}\left(\overline{c}-\underline{c}\right)+\underline{c}.$$

Our goal is now to find a set of function values $\widehat{\boldsymbol{\xi}} = \left\{\hat{\xi}^1, \hat{\xi}^2, \ldots, \hat{\xi}^g\right\}$ corresponding to $\widehat{\mathbf{c}}$ that represents a good approximation of the continuous function $\xi(c)$. With a discrete version of $H(\xi,c)$, denoted by $\hat{H}(\widehat{\boldsymbol{\xi}}, c^i)$, at hand, standard iterative algorithms can be applied to compute a $\widehat{\boldsymbol{\xi}}$ that fulfills a discrete approximation to condition (17).

How to compute $\hat{H}(\widehat{\boldsymbol{\xi}}, c^i)$? Note that $H(\xi,c)$ consists of an $n-1$-dimensional integral. The simplest method for approximating this integral on $\widehat{\mathbf{c}}$, repeatedly summing the areas of rectangles, requires a number of function evaluations that grows exponentially in n. For $n>3$ and a reasonable grid size (e.g., $g=100$), the computation of $\hat{H}(\widehat{\xi}, c^i)$ becomes so slow that finding a good approximation to $\xi(c)$ using iterative algorithms is impossible (even in the simplest case where $\sigma = 0$). A more efficient method to compute integrals in multiple dimensions is Monte Carlo integration. Applying this method, we evaluate the integrand at a uniformly distributed sequence of pseudorandom points in $\widehat{\mathbf{c}}^{n-1}$ and take the average. We can further improve our results by choosing points from a low-discrepancy sequence, such as the Sobol sequence, instead of pseudorandom points. This is sometimes called quasi-Monte Carlo integration and yields, for the same number of function evaluations, more accurate results.[17]

As we have argued in Section 4, for $\sigma = 0$, $\xi(c) > 0$ for all c. Accordingly, (17) simplifies to $H(\xi, c) = 0$ for all c. A numerical approximation to $\xi(c)$ is a $\widehat{\xi}$ that solves $\hat{H}(\widehat{\xi}, c^i) = 0$ for all $c^i \in \widehat{\mathbf{c}}$. We numerically solve this system of g equations with g unknowns using the trust-region dogleg algorithm as implemented in the function `fsolve` that is provided with the Matlab Optimization Toolbox.

The case where $\sigma > 0$ is computationally more expensive. To make it suitable for the algorithm we apply, we restate the discrete version of condition (17) as

$$\widehat{\xi} = \arg\min_{\overline{\xi}} \sum_{i=1}^{g} D(\overline{\xi}, c^i)^2 \quad \text{s.t.} \quad \hat{\xi}^i \geq 0 \;\; \forall i \tag{18}$$

where

$$D(\widehat{\xi}, c^i) := \begin{cases} 0 & \text{if } \hat{\xi}^i = 0 \text{ and } \hat{H}(\widehat{\xi}, c^i) \leq 0, \\ \hat{H}(\widehat{\xi}, c^i) & \text{otherwise.} \end{cases}$$

The minimization problem with inequality constraints (18) can be solved numerically using the active-set algorithm that is implemented in the function `fmincon` of the Matlab Optimization Toolbox.

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[17]For a comprehensive introduction to low-discrepancy sequences see, e.g., Galanti and Jung (1997) where the application of quasi-Monte Carlo methods is discussed in the context of option pricing.

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