INNOVATION CONTESTS WITH ENTRY AUCTION*

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Abstract

We consider procurement of an innovation from heterogeneous sellers. Innovations are random but depend on unobservable effort and private information. We compare two procurement mechanisms where potential sellers first bid in an auction for admission to an innovation contest. After the contest, an innovation is procured employing either a fixed prize or a first–price auction. We characterize Bayesian Nash equilibria such that both mechanisms are payoff–equivalent and induce the same efforts and innovations. In these equilibria, signaling in the entry auction does not occur since contestants play a simple strategy that does not depend on rivals' private information.

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1 Introduction

Consider a procurement problem where a buyer needs an innovative good that can potentially be provided by many sellers and where the good's quality depends on the seller's innovative ability. Although we focus on innovations, the setting applies to general goods where the good's value for the buyer is uncertain before it has been produced and where that value is correlated with the producer's type.

An innovation of any quality serves the procurer's needs, but the procurer's profit is increasing in innovation quality. Quality is random, but stochastically increasing in the seller's ability and R&D effort. Ability (type) is private information. Neither ability nor effort are observable. Innovation quality is observable between the seller and the procurer, but not verifiable and thus not contractible.

We consider two prominent procurement methods that are employed in real-world procurement settings:[1] innovation contests where an innovation is bought either employing a first-price (first-score) auction or a fixed prize.[2]

In a fixed-prize contest, a prize is paid in return for the best of all innovations that are delivered at some due date. In the first-score auction, each innovator submits an innovation and a financial bid from which the procurer computes a score. The highest score wins and the winner is paid his financial bid.

In a world with heterogeneous contestants, a well-known adverse selection problem arises:[3] Selecting the wrong contestants may dampen competition, reduce effort incentives, and produce an unsatisfactory result.

In principle, a fixed entry fee might solve that problem, the idea being that only the strongest contestants are willing to pay since their expected profit is large enough. However, setting the right entry fee requires a considerable amount of information while getting it wrong either leads to too many or too few contestants again implying an unsatisfactory result.

Following Fullerton and McAfee (1999), we consider an entry auction as a means of solving the adverse selection problem by setting an endogeneous entry fee. There, the buyer only needs to think about how many contestants to admit.

In particular, we combine each of the two mechanisms mentioned above with an entry auction where the highest-bidding participants pay an entry fee (according to the auction rules) and then enter the contest stage, where, in one mechanism, they compete for a fixed prize, and in the other, they compete in a scoring auction.

If the auction revenue does not accrue to the buyer, the entry auction can be interpreted in a way that makes those mechanisms similar to what we observe

[1] Scotchmer (2004) provides many current and historical examples.

[2] The first-score auction is a two-dimensional equivalent of a first-price auction. We use these terms synonymously. A bid has a financial and a quality dimension that are combined to a score. The highest score wins the auction and the winner receives his financial bid in return for his good. See Che (1993) for an analysis of this format in a procurement setting.

[3] See, e.g., the discussion in Fullerton and McAfee (1999).

in real procurement applications: the procurer announces a shortlisting procedure and then selects a few of the supposedly most able sellers to compete in a contest where the winner will be rewarded with a prize (or a contract). The shortlisted sellers face a cost of writing a detailed proposal or building a prototype (their "bid").

Fullerton and McAfee (1999) highlight the advantages of using an entry auction, i.e., the auction not only selects the right contestants, it also restricts entry which is generally optimal. Restricting entry avoids duplication of cost and it increases the effort incentives for the selected contestants. Moreover, at the procurement stage, it reduces the cost of evaluating proposals, i.e., of selecting the winner.

The focus and model of Fullerton and McAfee (1999) is close to ours. However, they model the contestants' heterogeneity as a different (constant) marginal effort cost. In the present model, the contestants' types affect the distribution of their innovations while marginal effort cost is constant and equal. We get different results in two important respects: First, in Fullerton and McAfee (1999) the optimal number of contestants is generally two, which is due to an economies-of-scale effect: Provided that you already have the best innovators, adding another one only increases fixed cost while that innovator's (constant) marginal cost is larger than that of all other contestants. In the present model, also larger numbers can be optimal since there is a tradeoff: adding contestants adds fixed cost but it also adds ability and thus improves the distribution of the winning innovation (for a given total effort). If increasing innovation quality is sufficiently profitable then the buyer employs more innovators. In our setting, ability can be interpreted as the value of a different approach or a second opinion. Second, Fullerton and McAfee (1999) show that standard auctions generally cannot be used as entry auctions, while an all-pay auction works if one adds a (negligible) interim prize for all auction winners. In the equilibria that we focus on in this paper, standard auctions work since auction winner's expected contest profit is a strictly increasing function of own type only.[4]

We characterize symmetric equilibria of the two mechanisms with the following appealing properties: a) all sellers participate in the entry auction, b) the entry auction selects the most able sellers, c) although abilities are private information, signaling in the entry auction does not occur;[5] d) at the contest stage, both mechanisms induce the same equilibrium efforts, the same expected innovations, the same buyer's profit (pointwise), and, e) in both mechanisms, sellers expect the same profit and that also holds if the fixed prize is not chosen optimally but is sufficiently large.

Given the huge literature on contests and on innovation (see, e.g., Konrad (2009) on contests and Scotchmer (2004) on the economics of innovation), we only mention work that is closely related to the present paper. Fullerton and McAfee

[4] In particular, it is independent of the marginal bidder's type. See the discussion in (Fullerton and McAfee, 1999, p.586ff).

[5] We allow for signaling but focus on equilibria where it is not an issue.

(1999) also analyze entry auctions for selecting participants for a fixed-prize contest. They focus on the (in)efficiency of standard auctions, while we compare fixed-prize procurement with the use of scoring auctions, both combined with an entry auction. Fullerton et al. (2002) is an experimental study that builds on the model of Taylor (1995). The contest winner is awarded through a first-price auction. Taylor (1995) looks at a contest as an optimal stopping problem, where identical innovators pay a fixed entry fee and then make a number of independent innovation draws where after each draw they decide whether to draw again. Che and Gale (2003) look at the optimal design of R&D contests assuming a deterministic innovation technology. They find that a first-score auction outperforms a fixed prize. Schöttner (2008) asks why we observe both fixed-prize contests and scoring auctions and presents a model where the fixed prize can outperform the auction. Both assume that entry fees are not feasible. Che (1993) studies the use of scoring auctions in procurement problems. Ding and Wolfstetter (2009) also analyze the performance of contests with fixed prize and first-score auction but they study the adverse selection problem that arises if the procurer cannot commit herself to never negotiating with inventors who circumvent the mechanism.

In this paper, we study particular equilibria induced by the play of two procurement mechanisms. We do not explicitly look for other, less plausible, equilibria but we will briefly discuss them. The paper proceeds as follows. In section 2 we introduce the model. In sections 3 and 4 we analyze the two mechanisms. We identify particular symmetric equilibrium candidates that we obtain by solving the continuation games backwards. In section 5 we show that these equilibria can indeed exist and we state the main result. In section 6 we discuss welfare issues. Section 7 provides a discussion and section 8 concludes. The appendix contains some of the proofs as well as some results on order statistics.

2 The model

There is a set of risk-neutral sellers, $I := \{1,\dots,N\}$, $N \geq 3$, and a risk-neutral buyer. Seller $i \in I$ has a privately known ability (or type). Ability is an i.i.d. random variable, A_i, with realizations a_i, c.d.f. H, continuous positive density and support $[\underline{a},\overline{a}]$, $\underline{a} \geq 0$. Sellers produce innovations by exerting nonobservable effort $e_i > 0$ at cost $C(e_i) = ce_i + \gamma$, where $c, \gamma > 0$. Zero effort is costless and does not produce an innovation. Seller i's innovation is the random variable Y_i with realizations y_i and is independently drawn from the c.d.f. $G^{a_i+e_i}$, where G is a c.d.f. with continuous positive density and support $[\underline{y},\overline{y}]$ where $\underline{y} \geq 0$.

We consider the following game.[6]

- **Stage 0**: Nature draws abilities, a_i, and each seller privately learns his own

[6]We assume random tie-breaking, e.g., when several innovators have the same innovation quality in the fixed-prize contest or when two innovators have the same bid (or score) in an auction. However, in the equilibria we consider, ties have zero probability.

ability. The buyer announces a procurement mechanism. It specifies the rules of an entry auction (stage 2), a number $n \in [2, N-1]$ of sellers who will be admitted to the contest, and a procurement method (stage 3).[7]

- **Stage 1**: Sellers simultaneously submit financial bids $b_i \geq 0$, bidding for entry to an innovation contest. Winners and losers make payments according to the auction rules. Bids are published.

- **Stage 2**: Contestants (the winners of stage 1) simultaneously choose unobservable effort, $e_i \geq 0$, and draw innovations, y_i.

- **Stage 3**: The buyer procures an innovation from among the contestants, either employing a fixed prize or a scoring auction (as specified at stage 0). In a scoring auction, all contestants who have drawn an innovation submit financial bids $\beta_i \geq 0$ and the highest-scoring contestant is paid his financial bid.

For simplicity, y_i is the buyer's net profit generated by employing seller i's innovation. We also call y_i the quality of i's innovation. Quality is not contractible, but observable between the buyer and the seller. The innovation is assumed to be worthless for the seller and the buyer can only employ one innovation.
The production technology is such that ability and effort are perfect substitutes in the sense that their marginal rate of substitution is constant. For innovation quality, only the sum $k_i := a_i + e_i$ is relevant and a larger k_i implies a stochastically better innovation in the sense of first-order stochastic dominance. The sum k_i can also be interpreted as the (noninteger) number of independent innovation draws from c.d.f. G.[8]
Any R&D activity implies a fixed cost γ while effort has a constant marginal cost. A seller' ability, a_i, can be interpreted as expertise or prior knowledge. Technically, it is a number of free draws from cdf G while additional draws are costly, or a minimum contest effort to which the player is comitted.
Throughout the paper, random variables are denoted by upper-case letters and the corresponding realizations by the respective lower-case letters. The superscripts F and S (e.g., in k^S) indicate mechanisms, *not powers*.
We make use of order statistics and denote them as follows: The mth highest of M independent draws from c.d.f. H (ability) is $A_{(m:M)}$ and its c.d.f. is $H_{(m:M)}$. The mth-best of M innovations is $Y_{(m:M)}$ with c.d.f. $G_{(m:M)}$.[9] For example, $H_{(1:N-1)}$ is the c.d.f. of the highest ability, $A_{(1:N-1)}$, among seller i's $N-1$ rivals; $G_{(2:n)}$ is the c.d.f. of the second-best innovation, $Y_{(2:n)}$, generated among n sellers; and $a_{(1:N)}$ is the highest type realization among all N sellers.

[7] In case of fixed-prize procurement, the prize P is also fixed at stage 0.

[8] If one makes k_i independent draws from c.d.f. G, then the highest order statistic (the best innovation draw) is distributed with c.d.f. G^{k_i}.

[9] Note that the exact form of $G_{(m:M)}$ depends on contest efforts and abilities and, thus, is potentially different for different mechanisms and equilibria.

3 Procurement with fixed-prize contest

We look at the first mechanism and concentrate on characterizing a particular symmetric pure-strategy equilibrium candidate while, for the moment, ignoring the issue of existence. We come back to that issue in section 5.

Consider procurement mechanism F (for "fixed" prize). At stage 0, the procurer announces a fixed prize P and a number $n \in [2, N-1]$ for an entry auction (stage 1) where the n highest-bidding participants win and pay the highest losing bid as an entry fee.[10] Bids are published. The n auction winners ("contestants") compete for the prize P that is awarded in return for the best innovation generated among them. All other sellers are excluded from the contest.[11]

Consider stage 3, the procurement stage, where cost and effort are sunk. The best (out of n) innovations is awarded the prize P.[12] Contestant i has produced innovation y_i and i's expected profit is (where the superscript $F3$ refers to the mechanism and the stage of the game)

$$\pi_i^{F3}(y_i) = PG^{\sum_{j\neq i} k_j}(y_i), \tag{1}$$

where $G^{\sum_{j\neq i} k_j}(y_i)$ is the probability that y_i is the best innovation.

At stage 2, contestant i chooses effort e_i and expects profit

$$\begin{aligned} \pi_i^{F2}(a_i, e_i) &= E\left[\pi_i^{F3}(Y_i)\right] - ce_i - \gamma & (2) \\ &= \frac{k_i}{k_i + \sum_{j\neq i} k_j} P - ce_i - \gamma, & (3) \end{aligned}$$

where

$$E\left[\pi_i^{F3}(Y_i)\right] = \int_{\underline{y}}^{\overline{y}} PG^{\sum_{j\neq i} k_j}(y_i) dG^{k_i}(y_i) = \frac{k_i}{k_i + \sum_{j\neq i} k_j} P. \tag{4}$$

The profit $\pi_i^{F2}(a_i, e_i)$ is strictly concave and, if P is sufficiently large, positive. The interior solution is characterized by the first-order conditions

$$\frac{\sum_{j\neq i} k_j}{\left(k_i + \sum_{j\neq i} k_j\right)^2} = \frac{c}{P}, \quad i = 1, \ldots n. \tag{5}$$

The RHS of (5) is constant, and the unique solution is $k_1 = k_2 = \cdots = k_n$. Substituting back into (5), we obtain the (candidate) equilibrium effort

$$e_i^F = \frac{(n-1)P}{n^2 c} - a_i \tag{6}$$

[10] Naturally, competition at the contest stage requires $n \geq 2$ while the bidding equilibrium of the entry auction requires $n \leq N-1$.

[11] Losers of the entry auction do not pay anything.

[12] Note, that it is in the buyer's interest to choose the best innovation: a different innovation would be procured at the same price but have a lower quality.

that is independent of rivals' private information (a_j, $j \neq i$). If $e_i^F > 0$ for all i then (6) characterizes the *unique* pure-strategy equilibrium. Intuitively, a sufficiently large prize ensures positive efforts, which is confirmed by (6). Note that $e_i^F + a_i$ is the same *constant* for each i. Denote

$$k^F := \frac{(n-1)P}{n^2 c} = e_i^F + a_i. \tag{7}$$

Inserting (6) into (3), i's equilibrium expected contest profit, $\pi^{F2}(a_i)$, is

$$\pi^{F2}(a_i) = \frac{P}{n} - c(k^F - a_i) - \gamma. \tag{8}$$

Summing up, in the candidate, all contestants choose positive effort independent of rivals' private information, expecting a profit that is entirely a function of own type. It follows that all contestants draw innovations from the same c.d.f., (9), and have the same probability of winning, $1/n$, regardless of ability.

$$G^{a_i + e_i^F} = G^{k^F} = G^{\frac{(n-1)P}{n^2 c}}. \tag{9}$$

At stage 1, sellers bid for entry. The maximum willingness to pay is equal to one's equilibrium expected contest profit conditional on entry. This profit, (8), is strictly increasing in ability. It only depends on a seller's own ability and is thus a pure private value, while, accordingly, rivals' equilibrium expected contest profits are i.i.d. random variables with the same distribution, H. Thus, we have symmetric independent private values, which implies that there is no signaling issue in the entry game. It is sufficient that everyone believes that the equilibrium "k^F" is played at the contest stage.

Thus, in particular, the discriminatory and the uniform-price auction formats are efficient and revenue-equivalent.[13] We analyze the uniform-price format because it is simpler, not because we recommend it or think it is the most appropriate format. There, bidders have the weakly dominant strategy to bid their expected contest profits conditional on entry,

$$\beta^F(a_i) = \pi^{F2}(a_i). \tag{10}$$

This strategy guarantees a non-negative expected profit since if i wins, the price (entry fee) he pays is not above his expected contest profit. Consider bidding more than $\beta^F(a_i)$. If i was previously a winner, he is still a winner with the same profit. If he was previously a loser, he is either still a loser or becomes a winner, in which case the previous price was at or above i's profit, and it is not lower now. Thus, i is not better off. A similar argument applies for bids below $\beta^F(a_i)$.

[13] In our setting, these formats are standard sealed-bid multi-unit auctions with single-unit demand. See, e.g., (Krishna, 2002, chs. 13, 14) for an analysis of these mechanisms in the symmetric independent private values framework.

The bid $\beta^F(a_i)$ is positive and strictly increasing. Thus the auction selects the n most able sellers. Suppose all N sellers participate, then seller i's profit is[14]

$$\begin{aligned}
\pi^{F1}(a_i) &= \Pr\{a_i > A_{(n:N-1)}\}\left(\pi^{F2}(a_i) - E\left[\beta^F(A_{(n:N-1)})\middle| a_i > A_{(n:N-1)}\right]\right) \\
&= H_{(n:N-1)}(a_i)\pi^{F2}(a_i) - \int_{\underline{a}}^{a_i} \pi^{F2}(a)dH_{(n:N-1)}(a) \\
&= \int_{\underline{a}}^{a_i} \frac{\partial \pi^{F2}(a)}{\partial a} H_{(n:N-1)}(a)da \\
&= c\int_{\underline{a}}^{a_i} H_{(n:N-1)}(a)da \geq 0, \qquad (11)
\end{aligned}$$

which confirms that all N sellers participate.
At stage 0, the procurer chooses P and n; the optimal prize is denoted by P^F. By (9), the expected value of the best innovation is

$$E\left[Y^F_{(1:n)}\right] = \int_{\underline{y}}^{\overline{y}} y\,dG^{nk^F}(y) = \overline{y} - \int_{\underline{y}}^{\overline{y}} G^{nk^F}(y)dy. \qquad (12)$$

In order to obtain that innovation, the procurer pays P but also collects auction revenue, equal to n times the expected contest profit of the seller with the $n+1$st highest ability. The procurer's profit is

$$\Pi^F(n,P) = E\left[Y^F_{(1:n)}\right] - P + nE\left[\beta^F\left(A_{(n+1:N)}\right)\right]. \qquad (13)$$

Inserting (8), (10) and (12), we have

$$\begin{aligned}
\Pi^F(n,P) &= E\left[Y^F_{(1:n)}\right] - P + n\left(\frac{P}{n} - ck^F + cE\left[A_{(n+1:N)}\right] - \gamma\right). \\
&= \overline{y} - \int_{\underline{y}}^{\overline{y}} G^{nk^F}(y)dy - nck^F + ncE\left[A_{(n+1:N)}\right] - n\gamma \qquad (14)
\end{aligned}$$

Consider (14). We have, intentionally, written the buyer's expected profit as a function of k^F. By (7), any prize P has a unique corresponding k^F that characterizes the equilibrium efforts. Thus, choosing the prize P is equivalent to deciding which equilibrium efforts to implement. Then the buyer just sets the corresponding prize P. Denote (14) by $\Pi^F(n,k^F)$. The first-order condition with respect to k^F is (after dividing both sides by n)

$$-\int_{\underline{y}}^{\overline{y}} G^{nk^F}(y)\ln(G(y))dy = c. \qquad (15)$$

[14]We drop the subscript i from the profit function since it is the same function for all types.

Also note that (14) is strictly concave in k^F,

$$\frac{\partial^2 \Pi^F(n,k^F)}{(\partial k^F)^2} = -n^2 \int_{\underline{y}}^{\overline{y}} G^{nk^F}(y) \left(\ln(G(y))\right)^2 dy < 0. \tag{16}$$

If the model parameters are such that $k^F > \max\{a_1, \dots, a_n\}$ then there is a symmetric equilibrium (candidate) at the contest stage where every contestant has positive effort and $e_i^F = k^F - a_i$ for all contestants $i = 1, \dots, n$.[15] Thus, for any n, the buyer chooses k^F according to (15) and this determines the optimal fixed prize P^F, via (7). We do not discuss the optimal n since it is not needed for our results.

4 Procurement with first-price auction

Again, we characterize a certain symmetric pure-strategy equilibrium and for the moment ignore the issue of existence.

Here, the buyer at stage 0 announces mechanism S ("scoring"), i.e., a number $n \in [2, N-1]$ of contestants and a first-score auction for stage 3. Mechanisms S and F only differ at stage 3, where a scoring auction is used instead of a fixed prize. Whereas the fixed prize is paid to the *best* innovator, in the scoring auction all innovators compete on price and quality. The auction provides an endogenous reward, while a fixed prize is a strategic variable chosen by the buyer.

At stage 1, sellers bid in a uniform-price entry auction (as in the previous section). Bids are published. The n highest bids win, the winners pay the $n+1$st highest bid as an entry fee and enter the contest.[16] At stage 2, they simultaneously choose efforts and draw innovations. Finally, at stage 3, the procurer conducts a first-score auction. There, each bidder i submits an innovation, y_i, and a financial bid, β_i, from which a score, s_i, is computed. The highest score wins and the winner is paid his financial bid in return for his innovation. The procurer applies the ideal scoring rule $s_i = y_i - \beta_i$.[17]

Lemma 1. *In the symmetric equilibrium of the first-score auction, contestant i with innovation y_i has a score of $s(y_i)$ and a financial bid of $\beta(y_i)$, where*

$$s(y_i) = E\left[Y_{(1:n-1)} \middle| Y_{(1:n-1)} < y_i \right] \tag{17}$$

$$= y_i - \beta(y_i), \tag{18}$$

$$\beta(y_i) = \int_{\underline{y}}^{y_i} \frac{G_{(1:n-1)}(s)}{G_{(1:n-1)}(y_i)} ds. \tag{19}$$

[15] A sufficient condition is $k^F > \overline{a}$.

[16] Losers do not pay anything and do not enter the contest.

[17] This scoring rule is ideal in the sense that it is credible: it reflects the true profit of the procurer. Thus the procurer has an incentive to select the most profitable innovation (taking into account the financial bid), which, in equilibrium, is equal to the best innovation. Since the price paid to the winner his independent of innovation quality, the innovation need not be verifiable.

The corresponding expected payoff of contestant i is

$$\pi_i^{S3}(y_i) = \int_{\underline{y}}^{y_i} G_{(1:n-1)}(y)dy \geq 0. \tag{20}$$

The proof is in the appendix.

By (17), the equilibrium *score* is equal to the well-known symmetric equilibrium *bid* of a first-price auction in the independent private values framework where bidders bid for objects with private values y_i. The financial bid $\beta(y_i)$ is equal to the amount of *bid shading* in that standard auction.[18]

At stage 2, the contest stage, contestant i chooses effort e_i at cost $ce_i + \gamma$ and has an expected profit of (inserting (20) and recalling that $G_{(1:n-1)}(y) = G^{\sum_{j\neq i} k_j}(y)$)[19]

$$\begin{aligned}
\pi_i^{S2}(a_i, e_i) &= E[\pi_i^{S3}(Y_i)] - ce_i - \gamma \qquad (21)\\
&= \int_{\underline{y}}^{\overline{y}} \pi_i^{S3}(y_i) dG^{k_i}(y_i) - ce_i - \gamma \\
&= \int_{\underline{y}}^{\overline{y}} G^{\sum_{j\neq i} k_j}(y) \int_{y}^{\overline{y}} dG^{k_i}(y_i) dy - ce_i - \gamma \\
&= \int_{\underline{y}}^{\overline{y}} G^{\sum_{j\neq i} k_j}(y) \left(1 - G^{k_i}(y)\right) dy - ce_i - \gamma. \qquad (22)
\end{aligned}$$

The first derivative w.r.t. effort e_i delivers the first-order condition[20]

$$-\int_{\underline{y}}^{\overline{y}} G^{k_i + \sum_{j\neq i} k_j}(y) \ln(G(y)) dy = c. \tag{23}$$

Denote the sum $k_i + \sum_{j\neq i} k_j$ that solves (23) implicitly by $k^S := (1/n)(k_i + \sum_{j\neq i} k_j)$,

$$-\int_{\underline{y}}^{\overline{y}} G^{nk^S}(y) \ln(G(y)) dy = c. \tag{24}$$

If the model parameters are such that $k^S > \max\{a_1, \ldots, a_n\}$ then there is a symmetric equilibrium (candidate) where every contestant has positive effort and $e_i^S = k^S - a_i$ for all contestants $i = 1, \ldots, n$. Observe, however, that only total effort is determined by (23) while the allocation of that effort among the contestants is arbitrary. Thus, there is a continuum of equilibrium candidates.

[18] See, e.g., (Krishna, 2002, p.17).

[19] The third line is obtained after interchanging the order of integration.

[20] The second derivate is negative, also see (16). Thus,the first-order condition characterizes the maximizer.

Recalling (22), seller i's equilibrium profit at stage 2, $\pi^{S2}(a_i)$, is

$$\pi^{S2}(a_i) = \int_{\underline{y}}^{\overline{y}} G^{(n-1)k^S}(y)\left(1 - G^{k^S}(y)\right) dy - c\left(k^S - a_i\right) - \gamma. \tag{25}$$

Again, a seller's expected contest profit in equilibrium is a pure private value, i.e., a function of own ability only, and, again, these symmetric effort strategies, $e_i^S = k^S - a_i$ imply that all contestants draw from the same c.d.f., G^{k^S}, and thus have the same probability of winning, $1/n$.

Consider stage 1, the entry auction stage. Similar to mechanism F, a bidder's weakly dominant strategy is

$$\beta^{S1}(a_i) = \pi^{S2}(a_i). \tag{26}$$

If all sellers participate, seller i's expected profit is[21]

$$\begin{aligned}\pi^{S1}(a_i) &= \Pr\{a_i > A_{(n:N-1)}\}\left(\pi^{S2}(a_i) - E[\beta^{S1}(A_{(n:N-1)})|a_i > A_{(n:N-1)}]\right) \\ &= c\int_{\underline{a}}^{a_i} H_{(n:N-1)}(a)da \geq 0.\end{aligned} \tag{27}$$

Thus, in our symmetric equilibrium, all N sellers participate.

Lemma 2. *The procurer's expected equilibrium profit (stage 1) is*

$$\begin{aligned}\Pi^S(n) &= E\left[s\left(Y_{(1:n)}^S\right)\right] + nE\left[\beta^{S1}\left(A_{(n+1:N)}\right)\right] \\ &= E\left[Y_{(2:n)}^S\right] + nE\left[\beta^{S1}\left(A_{(n+1:N)}\right)\right].\end{aligned} \tag{28}$$

It can be written as

$$\Pi^S(n) = \overline{y} - \int_{\underline{y}}^{\overline{y}} G^{nk^S}(y)dy - nck^S + ncE\left[A_{(n+1:N)}\right] - n\gamma. \tag{29}$$

The proof is in the appendix.

Consider (28). The expected winner's score in equilibrium is equal to the value of the expected second best innovation. The procurer's profit is that value plus the entry auction revenue.

5 Main Result

In the following we will repeatedly refer to "the equilibria of mechanisms F and S, characterized by k^F and k^S". By this we mean the equilibria of the games induced by the two mechanisms where the mechanisms are chosen optimally (i.e.,

[21] The computation is similar to that of (11).

with optimal fixed prize P^F and number of contestants n) given the symmetric continuation game equilibria characterized so far.

So far we ignored the issue of equilibrium existence. In general, we cannot solve the games analytically without further assumptions. We can, however, prove that there are feasible parameters for which these equilibria exist. To that end, we can use simple distributions, like, e.g., $G(x) = x^2$ or $G(x) = x$, and then straightforwardly compute the explicit solution of the games.

Naturally, these equilibria cannot exist for all parameters. The expected innovation must be sufficiently profitable to make it worthwhile to engage several innovators in competition and reimburse their cost. Moreover, employing an entry auction is only worthwhile if the advantage of selecting the best innovators covers the additional cost in the form of information rents.

Lemma 3. *There are model parameters such that the symmetric equilibria of mechanisms F and S, characterized by k^F and k^S, exist.*

The proof is in the appendix.

Collecting the results so far we now proceed to the main result.[22]

Proposition 1. *Consider the symmetric equilibria of mechanisms F and S, characterized by k^F and k^S. In these equilibria, both mechanisms implement*

1. *participation of all sellers,*
2. *an entry auction that selects the same number of the mosts able sellers,*
3. *the same equilibrium efforts,*
4. *the same expected innovations,*
5. *the same buyer's profit (pointwise), and,*
6. *the same expected seller's profit (as a function of ability).*

Proof of Proposition 1. First, since the sellers' expected profits, (11) and (27), are nonnegative, all sellers participate. Since the bid functions in the entry auction, (10) and (26), are strictly increasing in type, the entry auctions select the most able sellers.

Second, note that (15) and (24) have the same solution, $nk^F = nk^S$. Given that the equilibria exist, i.e., $k^F = k^S > \max\{a_1, \ldots, a_n\}$, and given some n, all contestants have the same sum of effort and ability, $k^F = k^S = e_i + a_i$. Thus, for the same n, efforts are the same. Since the most able sellers are in the contest, and $k^F = k^S$, all sellers draw innovations from the same c.d.f., $G^{n^F} = G^{n^S}$, which also implies that the best innovation is the same.

Since $k^F = k^S$, we have (14)=(29) which implies that the optimal n is also the same. The buyer's profit has two components: the innovation (which is the same

[22]The term "pointwise" in Proposition 1 means "for any realization of abilities".

under both mechanisms) and payments to and from the buyer. These payments have constant components (the prize P^F and the constant parts of the entry auction bids, see (8) and (25). Then there is part that depends on the realization of types, again see (8) and (25). But that part is the same in both mechanisms. Thus, for any realization of abilities the buyer's profits are the same.
Since (11)=(27), and the same n is optimal, sellers expect the same profit in both mechanisms. □

Remark 1. *Recall how* (11) *and* (27) *have been derived. For given n, sellers expect the same equilibrium profits in both mechanisms regardless of the choice of P, as long as P is sufficiently large to ensure equilibrium existence. Thus, an increase in the prize P will increase efforts (see* (6)*) but it does not affect the sellers' profits.*

6 Welfare

Here we look at welfare properties of the two mechanisms in the symmetric equilibria, characterized by k^F and k^S. Recall that the n most able sellers innovate. Thus, for given n, we look at the welfare generated by the n most able players. Consider arbitrary realizations of abilities that w.l.o.g. are ordered $a_1 > \cdots > a_N$. For given n, expected welfare, $W(n, e_1, \ldots, e_n)$, is the difference between the expected value of the best innovation and total social cost.

$$\begin{aligned} W(n, e_1, \ldots, e_n) &= E\left[Y_{(1:n)}\right] - c\sum_{i=1}^{n} e_i - n\gamma, \\ \text{where } E\left[Y_{(1:n)}\right] &= \int_{\underline{y}}^{\overline{y}} y\, dG(y)^{\sum_{i=1}^{n} a_i + e_i} = \overline{y} - \int_{\underline{y}}^{\overline{y}} G(y)^{\sum_{i=1}^{n} a_i + e_i} dy. \end{aligned} \tag{30}$$

From (30), it is obvious that for given n only total effort and total ability matter, while the allocation among the n sellers is inconsequential. Thus, we can replace the choice variables $e_1, \ldots, e_n$ and the abilities by $\hat{e}_n := \sum_{i=1}^{n} e_i$ and $\hat{a}_n := \sum_{i=1}^{n} a_i$ in (30). We get

$$W(n, \hat{e}_n) = \overline{y} - \int_{\underline{y}}^{\overline{y}} G(y)^{\hat{a}_n + \hat{e}_n} dy - c\hat{e}_n - n\gamma \tag{31}$$

The first-order condition with respect to $\hat{e}_n$ (interior solution) is

$$-\int_{\underline{y}}^{\overline{y}} G(y)^{\hat{a}_n + \hat{e}_n} \ln(G(y))\, dy = c \tag{32}$$

Denote the sum $\hat{a}_n + \hat{e}_n$ that solves (32) implicitly by $k^W := (1/n)(\hat{a}_n + \hat{e}_n)$.

Lemma 4. *In the equilibria of mechanisms F and S, characterized by k^F and k^S,*

1. *The c.d.f. of innovations is welfare-optimal,*
2. *Consider the buyer-optimal (i.e. equilibrium) number of contestants. For that number, the equilibrium efforts are welfare-maximizing.*

Proof of Lemma 4. First, denote the welfare-optimal n by n^W and observe the similarity of (32) with (15) and (24). In all three, the total sum of efforts and abilities that solves the equations is the same, $nk^F = nk^S = n^W k^W$. Therefore, in all three, the c.d.f. of the best innovation is the same, $G^{nk^F} = G^{nk^S} = G^{n^W k^W}$. Second, recall that only total effort, $\hat{e}_n$, is relevant. Also recall the discussions of (15) and (24) in the respective sections, where we said that as long as $k^F = k^S > \max\{a_1, \ldots, a_n\}$, there is positive effort for all contestants. The same holds here: For given n, we can assign the welfare-optimal total effort, implicitly defined by k^W, in the same way as in the contest equilibria of mechanisms F and S. □

We can now write welfare as

$$W(n, k^W) = \overline{y} - \int_{\underline{y}}^{\overline{y}} G(y)^{nk^W} dy - c\left(nk^W - \sum_{i=1}^{n} a_i\right) - n\gamma. \tag{33}$$

We have not looked at the welfare-maximizing number of contestants yet. Recall that the mechanism is announced at stage 0, simultaneously with nature's draw of abilities. In particular, the number n of "active" innovators is fixed at that stage. Note that fixing n has two consequences: the social fixed R&D cost of $n\gamma$ is incurred regardless of subsequent effort choices.[23] It is appropriate to define a welfare benchmark that also fixes n before abilities are realized. Therefore, we look for the number n that maximizes expected welfare where the expectation is about abilities and we insert the optimal efforts (characterized by k^W above).

Definition 1 (Welfare Benchmark)**.** *The welfare benchmark is the maximum expected welfare obtainable by the social planner given that the number of innovators, n, is chosen when abilities are still unknown. The social planner employs the n most able innovators, and does not face incentive or other informational constraints. The benchmark is given by*

$$\max_{n\in[1,N]} E_{A_{(1:N)},\ldots,A_{(n:N)}}\left[W(n, k^W)\right]. \tag{34}$$

Observe that the benchmark allows for $n = 1$ and $n = N$ which is not feasible under mechanisms F and S. Expected welfare for given n can be computed as

$$E\left[W(n, k^W)\right] = \overline{y} - \int_{\underline{y}}^{\overline{y}} G(y)^{nk^W} dy - cnk^W + c\sum_{i=1}^{n} E\left[A_{(i:N)}\right] - n\gamma. \tag{35}$$

[23] We discuss this feature in section 7. Observe that it does not make sense to "fix" some n and later, after abilities become known, decide to make use of a lower number $n' < n$ of innovators in order to save the fixed cost γ if that is more profitable. Then we could as well say that we "fix" $n = N$ (all innovators) and later decide how many to employ. But then "fixing n" is meaningless.

By Lemma 4 and comparison of (35) with (14) and (29), we see that the only difference is $nE\left[A_{(n+1:N)}\right]$ vs. $\sum_{i=1}^{n} E[A_{(i:N)}]$. The latter is larger, but since both terms are functions of n and depend on the distribution of abilities, it is hard to evaluate their effect on the optimal n.[24]
Again, for the uniform distribution example (see the proof of Lemma 3) one can straightforwardly compute that the welfare benchmark sets a larger n.[25]
In contrast to the benchmark, the profit-maximizing buyer pays informational rents. This difference is expressed in the terms $nE\left[A_{(n+1:N)}\right]$ and $\sum_{i=1}^{n} E[A_{(i:N)}]$. The term $\sum_{i=1}^{n} E[A_{(i:N)}]$ in the benchmark signifies the saving of variable effort cost due to ability. The term $nE\left[A_{(n+1:N)}\right]$ in the profit-maximizing buyer's problems (see (14) and (29)) can be written as

$$nE\left[A_{(n+1:N)}\right] = \sum_{i=1}^{n} E[A_{(i:N)}] - \sum_{i=1}^{n} E\left[A_{(i:N)} - A_{(n+1:N)}\right] \tag{36}$$

The first term is the same as in the benchmark and the second term signifies the loss due to informational rents, where each summand is the rent paid to one contestant. The highest rent is paid to the most able contestant: $E\left[A_{(1:N)} - A_{(n+1:N)}\right]$.
Moreover, there is a cost tradeoff that has to be observed by both, the benchmark and the profit-maximizing buyer: adding an "active" innovator (i.e. moving from n to $n+1$) adds the fixed cost γ while, in our equilibria, it reduces total effort (and thus variable cost), since the expected innovation is the same. More formally, suppose that, as argued above, the optimal n at the benchmark, denoted by n^B satisfies $n^B > n^F = n^S$. This implies $\hat{a}_{n^B} > \hat{a}_{n^F} = \hat{a}_{n^S}$; and since $n^F k^F = n^S k^S = n^B k^W$, we have $k^B < k^F = k^S$. Therefore, $\hat{e}_{n^B} < \hat{e}_{n^F} = \hat{e}_{n^S}$, i.e., total effort is lower under the welfare benchmark. Recall that, by Lemma 4, the same innovation is produced.

7 Discussion

In this section we discuss various aspects of the model and our results.

Differences between F and S Intuitively, an auction (stage 3) is more competitive than a fixed-prize mechanism. In the latter, the best innovator wins for sure, while in the former, less successful innovators can compete via lower financial bids. By that intuition, the buyer should procure the innovation at a price below the fixed prize of mechanism F. In our equilibria, however, the bids in the entry auction take that into account and equalize sellers' profits across both mechanisms.
For our model, it is not easy to pin down this difference in general. We can however employ our uniform distribution example (as in the proof of Lemma

[24] One would also have to analyze the curvature of (35) and (14), resp. (29).

[25] Ignoring the integer constraint on n we have for F and S, $n^* = \frac{1}{2}\left(N - (N+1)\frac{\gamma}{c}\right)$, while at the benchmark, $n^* = \frac{1}{2} + N - (N+1)\frac{\gamma}{c}$, which is larger.

3). There, one can straightforwardly compute that the buyer's expected stage-3 profit is larger in S, $\Pi^{S3}(n) > \Pi^{F3}(n, P^F)$. Since the same innovations are produced and procured in our equilibria, this implies that the procurement price in mechanism S is lower. For any feasible n (ignoring the integer constraint),

$$\Pi^{F3}(n, P^F) = E\left[Y^F_{(1:n)}\right] - P^F = 1 - \sqrt{c} - \frac{n(\sqrt{c} - c)}{n-1}, \tag{37}$$

$$\Pi^{S3}(n) = E\left[Y^S_{(2:n)}\right] = \frac{(n-1)(1-\sqrt{c})^2}{n-1+\sqrt{c}}. \tag{38}$$

Thus, for a given realization of abilities, the winners of the entry auction pay larger entry fees in mechanism F. Then they proceed to the contest and invest the same cost across both mechanisms while, in the end, $n-1$ of them (the final losers) will not win a prize and thus have a different profit than in mechanism S where they would have paid lower entry fees.

For the buyer, there is a corresponding difference in the "composition" of her profit, i.e., entry fees vs. procurement price at stage 3, but the innovation is the same and the sum of transfers is the same. Thus, the buyer's payoff is not affected by the choice of the mechanism.

This makes clear why in Proposition 1 sellers' *expected* profit is equal in both mechanisms while the buyer's profit is *pointwise* the same. Thus, even a risk-averse buyer would be indifferent between the mechanisms. Our result goes beyond "revenue equivalence"[26] which is a statement about *expected* profit.

The common structure of F and S Our results are due to the special R&D technology where type and effort are additive. Moreover, we assumed constant and equal marginal effort cost. In both mechanisms, these ingredients provide different first-order conditions of equilibrium effort choice but what they have in common is that they are entirely a function of all players' *sum* of type and effort, $k_i = a_i + e_i$, such that the same sum for all contestants, $k = k_1 = \cdots = k_n$, is a solution. Thus, in any equilibrium, only that sum is relevant while its composition is inconsequential. Contestants do not need to infer rivals' types since they only care about rivals' "contribution" to the contest, k, but not about their profits.

One might guess that a fixed prize and a first-score auction are not the only procurement methods that produce the result of Proposition 1. In fact, all we need is an equilibrium at stage 3 that implements a "winner-take-all" prize structure, i.e., where only the best innovation is rewarded. Given this, it does not matter what the reward exactly looks like. The equilibrium reward is, generally, a function of all innovations. It may be constant (as in F), or, it may depend on a contestant's own innovation only, as in S. Alternatively, it might be a payment contingent on the second-best innovation, e.g., a second-score auction, or it might be conditional on the whole vector of innovations, e.g., when the winner is paid

[26]as used in standard auction theory, see, e.g., Krishna (2002)

the average score. Observe, however, that procurement methods where the reward is conditional on other player's innovations, e.g., a second-score auction, are not feasible under our assumption that innovation quality is not verifiable. In order to induce effort, one has to reward innovation quality. Payments that are only type-specific cannot induce effort. In our equilibria, the bids in the entry auction are such that differences in the profitability of stage 3 in both mechanisms are "competed away" in the entry auction. In order to elicit private information, the buyer pays information rents. The rents are larger for more able sellers. They depend on the difference between a contestant's ability and that of the most able seller who is *not* admitted to the contest (see the discussion in 6). Finally, selection of the highest types is achieved by using an auction (with an increasing equilibrium bid function).

Tullock contests Note that (3) can be written as

$$\pi_i(k_i, a_i) = \frac{k_i}{k_i + \sum_{j \neq i} k_j} P - c k_i - (c a_i + \gamma). \tag{39}$$

This is similar to a symmetric Tullock lottery contest[27] with choice variable k_i (where we ignore the different "fixed cost" $c a_i + \gamma$ since it does not affect the interior solution). Thus, the equilibrium characterized by k^F corresponds to the symmetric equilibrium in that Tullock contest. It has been shown by Schweinzer and Segev (2008) that the winner-take-all structure is optimal if this symmetric equilibrium exists and if the objective is to maximize total effort. That objective, in our case, corresponds to maximizing $n k^F$, and, thus, the expected value of the best innovation.
Similar to the model of Fullerton and McAfee (1999) one might see ours as a micro-foundation for the Tullock contest.[28]

The number of contestants Sellers' expected profits (see (11) and (27)) are *increasing* in the number of contestants. Sellers face a tradeoff. Of course, an additional rival decreases every contestant's chance of winning (it is $1/n$ in our equilibria) and it lowers individual efforts since the expected prize is decreasing (see (20)). This is due to the fact that more competition leaves more profit with the buyer (at stage 3, see the reasoning above).[29] On the positive side, however, adding another contestant decreases the entry fee. Recall that we consider an equilibrium where positive effort is worthwhile for everybody. The equilibrium

[27]The contest success function is $k_i^r / (\sum_{j=1}^n k_j^r)$ with $r = 1$. There, 'lottery' refers to the fact that the winning probability is the same as if everybody had bought k_j lottery tickets where each ticket has equal chance of winning, see Tullock (1980).

[28]See also Corchón and Dahm (2010), Fu and Lu (2007), Jia (2008) and Skaperdas (1996) for different foundations of contest success functions.

[29]This can also be seen in (15) and (24). There, $n k^F = n k^S$ is a constant. A larger n corresponds to a lower k and thus lower efforts.

entry fee characterizes the size of the information rents. Everybody pays according to the lowest ability in the contest. Thus, making participation worthwile for an additional less able type increases the profit for the stronger ones.

Other equilibria and signaling Our procurement problem, in principle, exhibits a signaling issue. Players may want to signal their ability at the entry stage in order to influence their potential contest rivals' effort choices (or induce them to exit after the auction). We focused on symmetric equilibria where strategies do not require predicting rivals' types.[30] Although these equilibria are appealing, we do not want to downplay the relevance of signaling.

However, in section 4 we found that, apart from "our" equilibria, mechanism S has a continuum of equilibria at the contest stage wher signaling is indeed an issue. Note that this includes many symmetric equilibria. There is no convincing argument why a certain one of those would be more relevant than others. We argue that the only equilibrium that clearly stands out is the equilibrium without signaling. And precisely that one happens to be the unique equilibrium in the fixed-prize mechanism.[31]

The relevance of an equilibrium without signaling can also be justified as follows. When a game has multiple equilibria, one has to decide which, if any, equilibrium is the "solution" of the game. In complex decision problems, players may have to revert to simple heuristic strategies. This may be due to time, cognitive or cost constraints, etc. In this sense, simple strategies, like the ones we derived (based on one's own information), might be the appropriate solution.

The literature sometimes assumes that the private information becomes common knowledge before the game stage that would be affected by signaling (e.g., Fullerton and McAfee (1999)). This assumption is justified, e.g., in settings where players know each other such that they are sufficiently well informed as soon as the identity of their rivals is revealed after the auction.

Our equilibria require that the contest stage is attractive for any auction winner, for all realizations of types. Otherwise, contestants need to infer their rivals' strength in order to determine their efforts. They also might want to drop out of the contest if they face tough competition. In that case, however, there might be a coordination problem since it need not necessarily be the weak players who exit. Moreover, strong players might have an incentive to exert just a very small effort if additional effort is not worthwhile.[32]

It is also in the buyer's interest to make the contest attractive. Otherwise, exits or mixed-strategy play might endanger the procurement success, i.e., then procurement cannot be guaranteed.

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[30] Thus, we do not need beliefs other than the belief that these equibria will be played.

[31] Recall that we are talking about equilibria where each contestant plays a positive effort.

[32] Then their best reply does not exist since we assume that zero effort does not produce an innovation.

In the fixed-prize mechanism one can obviously generate an interior solution (i.e., positive efforts) at the contest stage by choosing a sufficiently large prize. However, we saw that the optimal prize is chosen such that the outcome becomes similar to that of the scoring-auction mechanism where the buyer does not choose the reward. So how can one make the mechanisms more attractive? Several ways seem possible. The buyer could, e.g., pay fixed interim prizes that would effectively lower the fixed R&D cost[33] or pay negative entry fees, e.g., let sellers bid for the lowest amount of subsidy required.

Why not choose n after the entry auction? In real procurement settings, we often observe that procurers announce in advance how many sellers will be allowed to compete if there is a costly entry stage (e.g., where prototypes or models have to be developed). For sellers, this is important since they need to decide whether it is worthwhile to participate. Fixing the number of contestants in advance makes that decision considerably simpler.

Government procurers are often bound by fixed procurement rules, specifying, e.g., a (minimum) number of offers to elicit. This is meant to increase transparency and prevent corruption by government agents to whom the task of procurement is delegated. It is also a typical feature of multi-unit auctions that the number of objects to be sold is announced before the auction. In our case, an object means entry to the contest.

From a theoretical (or welfare) perspective, one might ask if it is optimal to let the buyer choose the number of contestants before the entry stage, since it prevents her from using information about abilities collected in the auction to optimally adjust the number of (costly) contestants.

First, observe that the entry auction already provides an automatic adjustment of the mechanism to the given realization of abilities, in the sense that the equilibrium entry fees are strictly increasing in ability (of the marginal bidder). Second, mechanisms where n is chosen after the entry auction introduce other complications, as we discuss next.

Suppose mechanisms F and S are modified such that the procurer announces n after the entry auction (and, in F, the prize P^F; alternatively, the buyer might announce a prize function $P^F(n)$ at stage 0; then the uncertainty is only about n). The weaknesses of this design are that sellers cannot express their willingness to pay for different n (which might lead to cautious bidding, and, in the end to lower buyer's profit) and that predicting the choice of n is complicated.[34]

Now consider the more appealing modification of F and S where bids are contingent on the subsequent choice of n (and in mechanism F, the buyer announces a prize function $P^F(n)$ at stage 0). There, seller i submits bids $\beta_i(n)$

[33] That feature is central to the proposed auction in Fullerton and McAfee (1999).

[34] For related reasons, combinatorial auctions are used for the sale of, e.g., radio spectrum, instead of "simple" multi-object auctions where bidders just make a bid for every item. In the presence of complementarities, it is important for bidders to express their preferences for combinations of items.

for each $n \in [2, N-1]$. The buyer selects the most profitable n and collects the entry fees of an entry auction with bids $\beta_1(n), \dots, \beta_N(n)$. This is equivalent to saying that the sellers take part in $N-2$ different auctions and then the buyer selects one of them (and the corresponding n and $P^F(n)$) to be payoff-relevant. A complication is that sellers need beliefs about how the buyer chooses n (i.e. infers abilities) if bids are inconsistent (e.g. suppose bidder i submits the highest bid for $n=2$ but the second-highest bid for $n=3$).

The dominant strategies we derived for the uniform-price entry auction might still be an intuitive candidate (i.e., bidding one's expected contest profit conditional on entry for each n), but there is a potential incentive to deviate: Suppose everybody bids as in our basic games (for each n). Suppose the buyer then chooses some $n = \tilde{n}$. Then, say, seller j's bid, $\beta_j(\tilde{n})$ is the $\tilde{n}+1$st highest bid for $\tilde{n}$ which implies that j sets the entry fee while not being selected as a contestant. If j deviates by reducing his bid $\beta_j(\tilde{n})$ then the entry fee for $n = \tilde{n}$ decreases and $\tilde{n}$ becomes less attractive for the buyer.[35] If this induces the buyer to choose a larger n then j enters the contest with a positive expected continuation profit.

Entry fees In our setting with heterogeneous types, it is vital to select the most able innovators. Similar to Fullerton and McAfee (1999), we adopted an entry auction. The alternative, a fixed entry fee, would be an additional strategic variable and thus requires more information. The auction provides an endogenous entry fee that is adjusted to the given realizations of abilities.

One can interpret the entry auction in a way that makes our mechanisms more similar to real procurement settings: If we do not take the bids literally in the sense that they are payments to the buyer, we can interpret them as (sunk) cost of writing a proposal, or building a prototype. Then one might expect that the most able sellers have the best proposals or the most promising prototypes and can thus be identified. They bear this entry cost and then compete for a prize (or a contract). Of course the buyer might reimburse (part of) that bid preparation cost. This is something we observe in reality.

Also, we can abstract away from the particular auction format we used (the uniform-price auction). In our equlibria, we have seen that the entry auction is a competition for independent private values (the tournament profit conditional on entering the contest). Thus, one can consider any auction format that fits the above story and has an increasing equilibrium bid fuction, e.g., an all-pay auction, or a discriminatory auction.

For this modified setting, i.e., if there is no auction revenue, Proposition 1 in part still applies without further checks, e.g., sellers still expect the same profit in both mechanisms. However, the optimal prize in F might be different.

[35] The feasible range for that "deviation" depends on the next-lowest bid. If j undercuts the next-lowest rival then that rival sets the entry fee and j's bid is irrelevant.

8 Conclusion

We considered procurement of an innovation when innovations are random, non-verifiable, and depend on innovators' privately known ability and unobservable effort. In that setting, the questions of selecting the right innovators arises. Moreover, one should generally restrict entry in order to avoid duplication of cost and dampening of incentives.

We looked at two prominent procurement mechanisms: procurement using a fixed prize and a scoring auction. These mechanisms are observed in reality, not least because they are feasible if the good's quality is not verifiable.

We argued that in reality we also observe entry, or bid preparation, cost for sellers who decide to compete for contracts. Following Fullerton and McAfee (1999), we model this by way of an entry auction. Although we analyzed it as a regular auction that generates revenue, we argued that some of our results still hold if bids represent sunk cost and there is no revenue.

In the analysis of the combined mechanisms we focused on appealing equilibria with simple strategies where signaling in the entry auction is not an issue. In these equilibria the mechanisms are payoff-equivalent and implement the same efforts and innovations. In fixed-prize procurement, the buyer needs to set the right prize and we saw that only the *optimal* prize can match the profit of the scoring auction mechanism. This corresponds to large parts of the literature that recommend the use of auctions for their simplicity and low informational requirements.

An intuitive explanation for the equivalence result might be that it is caused by the sellers' competition with each other at the entry stage. A more generous reward, in the sense of a more profitable continuation game, leads to more aggressive bidding. This might also explain why sellers' expected profits are not only independent of the choice of the mechanism but also independent of the choice of the fixed prize: different profit opportunities are "competed away" between the sellers.

As a practical conclusion, it seems intuitive that having a competitive entry stage makes the choice of the subsequent procurement method a bit more arbitrary since stronger competition requires more generous rewards (in the sense of expected profits) which dampens the differences in profitability of different procurement methods. Thus, the focus of attention can be shifted in favor of other relevant issues when choosing a procurement method, like, e.g., informational requirements, multiplicity of equlibria, or collusion.

9 Appendix

9.1 Standard results on order statistics

Results we repeatedly use are

$$H_{(i:N)}(a) = \sum_{j=0}^{i-1} \binom{N}{j} H(a)^{N-j} (1 - H(a))^j , \tag{40}$$

$$G_{(i:n)}(y) = \sum_{j=0}^{i-1} \binom{n}{j} \left(G^k(y)\right)^{n-j} \left(1 - G(y)^k\right)^j , \tag{41}$$

$$G_{(1:n-1)}(y) = G^{(n-1)k}(y), \tag{42}$$

$$G_{(2:n)}(y) = nG^{(n-1)k}(y) - (n-1)G^{nk}(y), \tag{43}$$

$$\left(G_{(2:n)}(y)\right)' = n(n-1)kG'(y)\left(G^{(n-1)k-1}(y) - G^{nk-1}(y)\right). \tag{44}$$

For the uniform distribution on $[0,1]$, we have

$$E\left[A_{(i:N)}\right] = \frac{N+1-i}{N+1}, \tag{45}$$

$$E\left[\sum_{i=1}^{n} A_{(i:N)}\right] = \frac{n(2N-n+1)}{2(N+1)}, \tag{46}$$

$$G_{(2:n)}(y) = ny^{k(n-1)} - (n-1)y^{kn}, \tag{47}$$

$$G'_{(2:n)}(y) = kn(n-1)\left(y^{k(n-1)-1} - y^{kn-1}\right). \tag{48}$$

9.2 Proofs

Proof of Lemma 1. Recall that $G_{(1:n-1)}(y_i) = G^{\sum_{j\neq i} k_j}(y_i)$ is the c.d.f. of the best innovation generated among player i's $n-1$ rivals. Its exact form depends on the equilibrium at the contest stage, where the k_j are chosen. We solve the auction supposing that everybody believes that all contestants have drawn their innovations from the same c.d.f. G^k. In our equilibria, these beliefs are confirmed.
The candidate, (17), is strictly increasing. This implies that the equilibrium is efficient and that the equilibrium probability of winning is $\Pr\{y_i > Y_{(1:n-1)}\} := G_{(1:n-1)}(y_i)$. Thus, the expected payoff is

$$\pi_i^{S3}(y_i) = G_{(1:n-1)}(y_i)\beta(y_i) = \int_{\underline{y}}^{y_i} G_{(1:n-1)}(y)dy \geq 0. \tag{49}$$

Finally, we demonstrate that deviating bids are not profitable. Given bidder i's innovation y_i, any deviating financial bid $z \in [0, y_i]$ results in a nonnegative score that is equal to the equilibrium score generated by innovation $\tilde{y}$, implicitly defined by $s(\tilde{y}) = \tilde{y} - \beta(\tilde{y}) = y_i - z$.[36] Then expected profit is

$$\pi_i^{S3}(z, y_i) = G_{(1:n-1)}(\tilde{y})z \tag{50}$$

$$= G_{(1:n-1)}(\tilde{y})\left(y_i - \tilde{y} + \int_{\underline{y}}^{\tilde{y}} \frac{G_{(1:n-1)}(s)}{G_{(1:n-1)}(\tilde{y})} ds\right) \tag{51}$$

$$= G_{(1:n-1)}(\tilde{y})\left(y_i - \tilde{y}\right) + \int_{\underline{y}}^{\tilde{y}} G_{(1:n-1)}(s) ds. \tag{52}$$

Therefore,

$$\begin{aligned} \pi_i^{S3}(y_i, y_i) - \pi_i^{S3}(z, y_i) &= \int_{\tilde{y}}^{y_i} G_{(1:n-1)}(s) ds - G_{(1:n-1)}(\tilde{y})\left(y_i - \tilde{y}\right) \\ &= \int_{\tilde{y}}^{y_i} G_{(1:n-1)}(s) - G_{(1:n-1)}(\tilde{y}) ds > 0, \end{aligned} \tag{53}$$

which holds for all $\tilde{y} \neq y_i$. □

Proof of Lemma 2. The first line in (28) is obvious. We show that $E[s(Y_{(1:n)})] = E[Y_{(2:n)}]$. At stage 1 we know that all contestants draw from the same cdf G^k where $k = k^S$. Thus, the c.d.f.s of the best and second-best innovations are $G_{(1:n)} = G^{nk}$ and $G_{(2:n)} = G^{nk} + nG^{(n-1)k}(1 - G^k)$. Employing (17), we get

$$E\left[s\left(Y_{(1:n)}\right)\right] = \int_{\underline{y}}^{\overline{y}} \frac{1}{G_{(1:n-1)}(y)} \int_{\underline{y}}^{y} s\, dG_{(1:n-1)}(s) dG_{(1:n)}(y) \tag{54}$$

$$= \int_{\underline{y}}^{\overline{y}} s \int_{s}^{\overline{y}} \frac{1}{G_{(1:n-1)}(y)} dG_{(1:n)}(y) dG_{(1:n-1)}(s) \tag{55}$$

$$= \int_{\underline{y}}^{\overline{y}} s \int_{s}^{\overline{y}} \frac{nkG^{nk-1}(y)G'(y)}{G^{(n-1)k}(y)} dy (n-1)kG^{(n-1)k-1}(s)G'(s) ds \tag{56}$$

$$= \int_{\underline{y}}^{\overline{y}} sn\left(1 - G^k(s)\right)(n-1)kG^{(n-1)k-1}(s)G'(s) ds \tag{57}$$

$$= \int_{\underline{y}}^{\overline{y}} s\, dG_{(2:n)}(s) = E\left[Y_{(2:n)}\right]. \tag{58}$$

Finally, (29) can be computed straightforwardly, noting that

$$E\left[Y_{(2:n)}\right] = \overline{y} - \int_{\underline{y}}^{\overline{y}} G_{(2:n)}(y) dy, \tag{59}$$

[36] Thus, we rule out negative scores.

where $G_{(2:n)}(y)$ is given by (43), and, using (26) and (25),

$$E\left[\beta^{S1}\left(A_{(n+1:N)}\right)\right] = \int_{\underline{y}}^{\overline{y}} G^{(n-1)k^S}(y)\left(1-G^{k^S}(y)\right)dy - nck^S + ncE\left[A_{(n+1:N)}\right] - \gamma.$$

$\square$

Proof of Lemma 3. The proof is done by computing an example under the following set of parameters. Abilities and innovations are uniformly distributed on $[0,1]$. Thus, $[\underline{y},\overline{y}] = [\underline{a},\overline{a}] = [0,1]$ and $H(x) = G(x) = x$ for $x \in [0,1]$. Furthermore,

$$0 < \gamma < \frac{c}{N-2} \leq c < \frac{1}{(N+2)^2} < 1. \tag{60}$$

Start with the fixed-prize mechanism. The buyer's profit, (14), simplifies to

$$\Pi^F(n,k^F) = 1 - \frac{1}{nk^F+1} - nck^F + nc\frac{N-n}{N+1} - n\gamma, \tag{61}$$

$$\frac{\partial \Pi^F}{\partial k^F} = \frac{n}{(nk^F+1)^2} - nc. \tag{62}$$

We get the interior maximizer and, from that, the optimal prize P^F (using (7))

$$k^F = e_i^F + a_i = \frac{1}{n}\left(\frac{1}{\sqrt{c}} - 1\right), \tag{63}$$

$$P^F = \frac{nc}{n-1}\left(\frac{1}{\sqrt{c}} - 1\right) = \frac{n\left(\sqrt{c}-c\right)}{n-1}. \tag{64}$$

We have $\overline{a} = 1$ and thus effort e_i is positive for all ablities iff

$$e_i = \frac{1}{n}\left(\frac{1}{\sqrt{c}} - 1\right) > 1 \iff c < \frac{1}{(n+1)^2}, \tag{65}$$

which is satisfied since, by (60), $c < 1/(N+2)^2$ and $n \leq N-1$.
Now, insert k^F and P^F into $\pi^{F2}(a_i)$ (see (8)),

$$\pi^{F2}(a_i) = \frac{\sqrt{c}-c}{n(n-1)} + ca_i - \gamma. \tag{66}$$

This is positive if it is positive for the lowest ability, $\underline{a} = 0$. Thus check if

$$\frac{\sqrt{c}-c}{n(n-1)} + ca_i > \gamma. \tag{67}$$

By (60), $c < 1/(N+1)^2$. We get

$$c < \frac{1}{(N+1)^2} \Rightarrow c < \frac{1}{N^2} \iff \frac{(N-1)(N-2)}{N-2)} + 1 < \frac{1}{\sqrt{c}}. \tag{68}$$

Since $n \leq N-1$, we continue

$$\Rightarrow \frac{n(n-1)}{N-2}+1<\frac{1}{\sqrt{c}} \iff \frac{c}{N-2}<\frac{\sqrt{c}-c}{n(n-1)} \Rightarrow \gamma<\frac{\sqrt{c}-c}{n(n-1)}. \tag{69}$$

The last step follows from (60), where $\gamma < c/(N-2)$. The sellers' stage-1 profit is nonnegative by (11). It remains to show that the buyer's profit is positive. The buyer's profit at the optimal prize, $\Pi^F(n, P^F)$, is

$$\Pi^F(n,P^F)=\left(1-\sqrt{c}\right)^2+n\left(c\frac{N-n}{N+1}-\gamma\right). \tag{70}$$

Since $(N-n)/(N+1)>0$, the above is positive if $\left(1-\sqrt{c}\right)^2>n\gamma$. This is satisfied by our assumptions: By (60), $c<1/(N+2)^2$. This can be written as $c<(1-\sqrt{c})^2/(N+1)^2$. Since $\gamma<c$ (by (60)), $\gamma<(1-\sqrt{c})^2/(N+1)^2$. And since $n \leq N-1$ and $N \geq 3$, this implies $\gamma<(1-\sqrt{c})^2/n$ which proves the assertion.
Now turn to mechanism S. Equation (22) becomes

$$\pi_i^{S2}(a_i,e_i)=\frac{1}{1+\sum_{j\neq i}k_j}-\frac{1}{1+k_i+\sum_{j\neq i}k_j}-ce_i-\gamma. \tag{71}$$

It is strictly concave in e_i and, thus,

$$e_i^S=k^S-a_i=\frac{1}{n}\left(\frac{1}{\sqrt{c}}-1\right)-a_i. \tag{72}$$

Since $e_i^S=e_i^F$, $e_i^S>0$. Seller i's contest profit becomes

$$\pi^{S2}(a_i)=\frac{\sqrt{c}(1-\sqrt{c})^2}{n(n-1+\sqrt{c})}+ca_i-\gamma. \tag{73}$$

We show that it is positive. Since $\gamma<c/(N-2)$ by (60), we only need to show that

$$\frac{c}{N-2}<\frac{\sqrt{c}(1-\sqrt{c})^2}{n(n-1+\sqrt{c})}. \tag{74}$$

Recall that $n \leq N-1$. Thus we can replace n by $N-1$ in (74). We get

$$\frac{c}{N-2}<\frac{\sqrt{c}(1-\sqrt{c})^2}{(N-1)(N-2+\sqrt{c})} \iff \frac{\sqrt{c}(N-1)(N-2+\sqrt{c})}{N-2}<(1-\sqrt{c})^2. \tag{75}$$

Next, we use assumption $c<1/(N+2)^2$ (see (60)): We replace $\sqrt{c}$ by $1/(N+2)$ on both sides. This makes the LHS larger and the RHS smaller. We get an inequality in N that is satisfied for $N \geq 3$ (as we assume).
The sellers' stage-1 profits are nonnegative by (27). Also, it is obvious that (17) and (20) are nonnegative.

Now recall (28). The expected second-best innovation, $E\left[Y^S_{(2:n)}\right]$, is

$$\begin{aligned}\int_0^1 yG'_{(2:n)}(y)dy &= \int_0^1 (n-1)\left(\frac{1}{\sqrt{c}}-1\right)\left(y^{\frac{n-1}{n}\left(\frac{1}{\sqrt{c}}-1\right)}-y^{\left(\frac{1}{\sqrt{c}}-1\right)}\right)dy \\ &= \frac{(n-1)\left(1-\sqrt{c}\right)^2}{n-1+\sqrt{c}}.\end{aligned} \tag{76}$$

The expected entry fee is, using (73) and (45),

$$\begin{aligned}E\left[\pi^{S2}\left(A_{(n+1:N)}\right)\right] &= \frac{\sqrt{c}(1-\sqrt{c})^2}{n(n-1+\sqrt{c})}+cE\left[A_{(n+1:N)}\right]-\gamma \\ &= \frac{\sqrt{c}(1-\sqrt{c})^2}{n(n-1+\sqrt{c})}+c\frac{N-n}{N+1}-\gamma.\end{aligned} \tag{77}$$

Thus, (28) becomes

$$\begin{aligned}\Pi^S(n) &= \frac{(n-1)\left(1-\sqrt{c}\right)^2}{n-1+\sqrt{c}}+n\left(\frac{\sqrt{c}(1-\sqrt{c})^2}{n(n-1+\sqrt{c})}+c\frac{N-n}{N+1}-\gamma\right) \\ &= \left(1-\sqrt{c}\right)^2+n\left(c\frac{N-n}{N+1}-\gamma\right).\end{aligned} \tag{78}$$

Since (78) is equal to (70), the profit is here positive as well.
For both mechanisms, we did not specify the optimal n. However, we have shown that the profit is positive for any feasible n, i.e., $n \in [2, N-1]$, $n \in \mathbb{N}$. Thus, it is positive for the optimal n. □

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