Horizontal mergers with synergies: first-price vs. profit-share auction*

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We consider takeover bidding in a Cournot oligopoly when firms have private information concerning the synergy effect of merging with a takeover target. Two auction rules are considered: standard first-price and profit-share auctions, supplemented by entry fees. Since non-merged firms benefit from a merger if the synergies are low, bidders are subject to a positive externality. Nevertheless, pooling does not occur; and the profit-share auction is strictly more profitable than the first-price auction, regardless of whether firms observe the synergy parameter or only the winning bid before they play the oligopoly game.

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1 Introduction

In the present paper we consider horizontal mergers, assuming that a takeover target is auctioned among competing firms, and firms have private information concerning their synergy benefits of a merger. Our analysis has several distinct features:

- bidders are competitors in a downstream Cournot market game and synergies take the form of cost reductions,
- bidders have private information concerning the synergy effect of merging their firm with the takeover target,
- before firms play the oligopoly game they observe either the merged firm's synergy parameter or the winning bid,
- bidders may influence their rivals' beliefs through their bid,
- the merger target is auctioned to the highest bidder, either in a standard first-price (cash auction) or a profit-share auction.

The presence of synergies assures that mergers are potentially profitable for the coalition of merged and merging firm, and the presence of private information makes auctions an appealing mechanism for matching the takeover target with another firm.

The fact that bidders are competitors in a downstream oligopoly implies that the takeover bidding is an auction game that is subject to externalities. And since non-merged firms benefit from a merger if synergies are low, bidders are subject to a positive externality with positive probability. Whereas if synergies are sufficiently high, bidders are subject to negative externality.

We consider two specifications of the model: before the oligopoly game is played, firms can either observe the merged firm's synergy parameter or only the winning bid. When firms can only observe bids (in particular the winning bid), the bidding games involve a signalling element, which exerts an upward pressure on equilibrium bids.

The fact that ownership stakes in the merged firm make post-merger profits verifiable to all co-owners makes it feasible to adopt a profit-sharing auction in lieu of a standard "cash auction" like a first-price auction. In a profit-share auction the payment by the winner of the auction to the owners of the takeover target is contingent on the merger profit which is correlated with the synergy parameter realized by the merger.

The paper is related to the ongoing debate on horizontal merger. A starting point of that literature is the "merger paradox" which observes that "small" mergers are not profitable if firms compete in a Cournot market game with substitutes and mergers do not involve synergy benefits (see Salant et al., 1983).

However, small mergers become profitable for the coalition of merged firms if synergies are sufficiently high (see Farrell and Shapiro, 1990) or firms produce differentiated goods in a Bertrand market game (Deneckere and Davidson, 1985), or, to some extent, if market demand is sufficiently concave (see Faulý-Oller, 1997).

Mergers can also be profitable if firms are uncertain about their post-merger synergy benefit (Choné and Linnemer, 2008, Amir et al., 2009). Indeed, mergers can be profitable even if, in expectation, there are no synergy benefits, provided the variance of the unknown synergy benefit is sufficiently high (see Hamada, 2010).

The use of auctions in horizontal mergers was considered for example by Jehiel and Moldovanu (2000) for whom takeover bidding in a Cournot oligopoly is a prime example of an auction that is subject to positive externalities, if synergies are sufficiently low. Auctions with positive externalities are viewed as interesting outliers where pooling occurs if bidders are subject to a minimum bid requirement.

Brusco et al. (2007) and Gärtner and Schmutzler (2009) consider mergers when firms are subject to double private information, because the takeover target does not know the synergy benefit brought about by a partner, and prospective partners do not know each other's pre-merger unit costs. While the former adopt an optimal mechanism design perspective, the latter focus on bargaining issues and aspire to resolve the puzzle why many horizontal mergers happen to flop, as observed in the empirical literature (Ravenscraft and Scherer, 1989, Moeller et al., 2005). Both incorporate a rich information structure; however, neither includes a full analysis of the interrelationship with the downstream oligopoly game.

Similar to Jehiel and Moldovanu (2000), the present paper adopts an auction perspective and assumes that firms have private information concerning their synergy parameter while firms' pre-merger unit costs are common knowledge. However, unlike Jehiel and Moldovanu (2000), we consider profit-share auctions in addition to standard cash auctions,[1] allow for nonlinear demand, more than three firms, and assume that firms may observe only an imperfect signal of the merged firm's synergy parameter before the oligopoly game is played, as it is the case when firms observe only bids, in particular the winning bid.

Our main results are as follows: we show that the bidding games have a separating equilibrium even though firms may be subject to a positive externality; and we show that a profit-share auction is more profitable than a first-price auction, regardless of whether firms observe the merged firm's synergy parameter or only an imperfect signal of it.

The plan of the paper is as follows. Section 2 introduces the framework and assumptions. Section 3 considers the benchmark model in which firms perfectly observe the synergy parameter of the merged firm before they play the oligopoly game. This assumption is then replaced in Section 4 where bidders observe only the winning bid, which introduces a signalling issue. The paper concludes with a discussion. Some proofs are in the Appendix.

2 Model

Consider a Cournot oligopoly composed of $N+1 \geq 3$ firms among which one, say firm $N+1$, is willing to be merged with either one of the firms $\{1, 2, \ldots, N\}$. The owners of the takeover

[1] Contingent-payment auctions like profit-share auctions were introduced by Hansen (1985). Crémer (1987) pointed out that if the post-auction valuation is verifiable, the auctioneer can, in principle, extract the full surplus. Samuelson (1987) discusses limitations of full surplus extraction.

target auction their firm either in a standard first-price or in a profit-share auction, supplemented by an entry or participation fee. Entry fees may be necessary to assure that the takeover target does not suffer losses in some states.

In a profit-share auction bids are shares in the equilibrium profit of the merged firm that bidders offer conditional on being merged with firm $N+1$. The takeover target selects the bidder who offers the highest share as winner. Profit-share auction are feasible in the takeover context because the parties that become co-owners of the merged firm can naturally verify the post-merger profit of that firm.

If a merger occurs, the merged firm enjoys a synergy benefit in the form of a lower unit cost. Firms that are not part of the merger have the same unit cost c, whereas the merged firm has the unit cost $c-\theta$. Large mergers of more than two firms are not on the agenda or not approved by the Antitrust Authority.

Prior to the auction, firms $\{1,\ldots,N\}$ have private information concerning their synergy parameter θ. From the point of view of other firms, firms' synergy parameters are *iid* random variables, drawn from the log-concave distribution $F:[0,c]\to[0,1]$, with positive density, F', everywhere. We denote the c.d.f. of the largest synergy parameter of a sample of $N-1$ firms by $G(\theta):=F(\theta)^{N-1}$ and note that log-concavity of F implies log-concavity of G.

After the bidding game has been played firms play a Cournot oligopoly game. Two models are distinguished: In the first model, the synergy parameter of the merged firm becomes known to all firms before the oligopoly game is played. In the second model firms only observe the winning bid from which they draw inferences concerning the synergy parameter of the merged firm.

In the first model the downstream oligopoly game is one of complete information, which is fully determined by the cost parameter c and the synergy parameter of the merged firm, θ. In the second model the oligopoly game is one of incomplete information. There, firms update their beliefs concerning the synergy parameter of the merged firm, after they observe the winning bid. In turn bids may be used to influence the beliefs of rival bidders, which introduces a signalling aspect into the bidding game.

In the following we denote the equilibrium profit of the merged firm by $\pi_m(\theta)$, the equilibrium profit of the firms that have not been merged by $\pi_n(\theta)$, and the (default) equilibrium profits if no merger has taken place by π_0. Both π_m and π_n are functions of the synergy parameter of the firm that has been merged with firm $N+1$. Obviously, $\forall\theta:\pi_m(\theta)>\pi_0$, and $\forall\theta>0:\pi_m(\theta)>\pi_n(\theta)$, whereas for some $\hat{\theta}\in(0,c)$,

$$\pi_n(\theta)\gtreqless\pi_0 \iff \theta\lesseqgtr\hat{\theta} \quad \text{(positive/negative externality).} \tag{1}$$

In other words, for low θ the firm that has *not* been merged benefits from reduced competition due to the merger. However, if the synergy is sufficiently large, that *positive externality* turns into a *negative externality*, because then the disadvantage of facing a competitor whose cost has been reduced outweighs the benefit of reduced competition due to the merger.

We assume that c is sufficiently high to assure existence of $\hat{\theta}<c$, and sufficiently low to assure that mergers do not propel monopoly at all possible synergies. We also assume that firms are

risk neutral and inverse market demand P is a decreasing and concave function of aggregate output.[2]

Of course, "small" mergers are not profitable for the merger coalition if synergies are absent: $\pi_m(0) < 2\pi_0$ ("merger paradox").

3 Takeover bidding without signalling

Following Jehiel and Moldovanu (2000), we first consider a highly stylized model in which the synergy parameter becomes common knowledge after the auction and before the oligopoly game is played. There, the profits of the merged firm and the non-merged firms are fully described by the functions $\pi_m(\theta), \pi_n(\theta)$, which are exclusively functions of the merged firm's synergy parameter θ. Of course, if no firm bids, all firms earn the default equilibrium payoff π_0.

Bid functions are denoted by the Roman letters b (first-price) and s (profit-share auction).

3.1 First-price auction

The bidder who makes the highest bid wins the auction, the winner pays his bid, and all those who choose to bid must pay the entry or participation fee R.

As a working hypothesis suppose b is strictly increasing, and the entry fee induces a cutoff value of θ, denoted by r, such that $b(r) = 0$, and a bidder bids only if his synergy parameter is $\theta \geq r$ and otherwise abstains from bidding.

Consider a marginal bidder with $\theta = r$. That bidder must be indifferent between bidding and not bidding:

$$G(r)\,(\pi_m(r) - b(r)) + \int_r^c \pi_n(z)dG(z) - R = G(r)\pi_0 + \int_r^c \pi_n(z)dG(z).$$

Since $\pi_m(\theta)$ is strictly increasing and $b(r) = 0$ it follows that for all $R \in [0, \pi_m(c) - \pi_0)$, the entry fee R induces a unique critical valuation r, which is implicitly defined as the solution of the equation

$$R = G(r)\,(\pi_m(r) - \pi_0)\,. \tag{2}$$

Proposition 1 (First-price auction)**.** *The equilibrium strategy of the first-price auction is*

$$b(\theta) = \int_r^\theta \frac{G'(x)}{G(\theta)}\,(\pi_m(x) - \pi_n(x))\,dx. \tag{3}$$

Proof. By the assumed monotonicity of b, the equilibrium bidding problem for a bidder with $\theta, y \geq r$ can be stated in the form:

$$\theta = \arg\max_{y \geq r} G(y)\,(\pi_m(\theta) - b(y)) + \int_y^c \pi_n(z)dG(z) - R. \tag{4}$$

[2] This assures existence of a unique pure strategy equilibrium of the oligopoly game (see Szidarovszky and Yakowitz, 1977).

Therefore, b has to solve the differential equation,

$$(G(\theta)b(\theta))' = G'(\theta)\,(\pi_m(\theta) - \pi_n(\theta))\,. \tag{5}$$

Integrating and using the initial condition $b(r) = 0$ yields (3). To confirm the assumed strict monotonicity of b, note that $\pi_m(\theta) - \pi_n(\theta)$ is positive and strictly increasing for all θ. Using these facts and applying integration by parts gives:

$$\begin{aligned} b'(\theta) &= \frac{G'(\theta)}{G(\theta)}\left(\pi_m(\theta) - \pi_n(\theta) - \frac{1}{G(\theta)}\int_r^\theta G'(x)\,(\pi_m(x) - \pi_n(x))\,dx\right) \\ &= \frac{G'(\theta)}{G(\theta)^2}\left(G(r)\,(\pi_m(r) - \pi_n(r)) + \int_r^\theta \partial_x\,(\pi_m(x) - \pi_n(x))\,G(x)dx\right) > 0. \end{aligned} \tag{6}$$

Finally, we need to confirm that bidding is more profitable than not bidding if and only if $\theta \geq r$. That proof is in Appendix A.2. □

In order to pin down the role of the externality implied by mergers, let $\hat{b}$ denote the hypothetical equilibrium bid function that would apply if the loser of the auction were not affected by the merger, i.e, if $\pi_n(\theta)$ were equal to π_0. Then,[3]

$$\hat{b}(\theta) - b(\theta) \gtreqless E\left(\pi_n(\tilde{\theta}) - \pi_0 \mid \tilde{\theta} \leq \theta\right) \gtreqless 0. \tag{7}$$

In other words, bidding becomes less aggressive when the conditional expected value of the externality is positive (which occurs if θ is sufficiently "small") and more aggressive when that conditional expected value is negative. Of course, a positive externality makes it less attractive to win the auction, which makes bidders less eager to win, and *vice versa*. Therefore, this relationship is intuitively plausible.

Figure 1 illustrates this relationship for the example of linear demand, $c = 0.49$, $r = 0.001$, and $F(\theta) = \theta/c$ (uniform distribution). There, the vertical, dotted line separates the range of positive externalities ($\pi_n(\theta) > \pi_0$) from negative externalities ($\pi_n(\theta) < \pi_0$).

We mention that if we would employ a minimum bid in lieu of an entry fee requirement, the equilibrium bid function would exhibit pooling at the reserve price. Jehiel and Moldovanu already observed that "entry fees and reserve prices are not equivalent in the positive externality case" (Jehiel and Moldovanu, 2000, p. 782).

3.2 Profit-share auction

Now consider the profit-share auction with entry fee R. There, bidders must pay the entry fee, regardless of winning or losing, the bidder who offers the highest share wins the auction, the winner has to grant the promised share $s(\theta)$ of the profit of the merged firm, and losers pay nothing.

[3] As one can confirm easily, that hypothetical equilibrium bid function is $\hat{b}(\theta) = \int_r^\theta (\pi_m(x) - \pi_0)\, G'(x)/G(\theta) dx$.

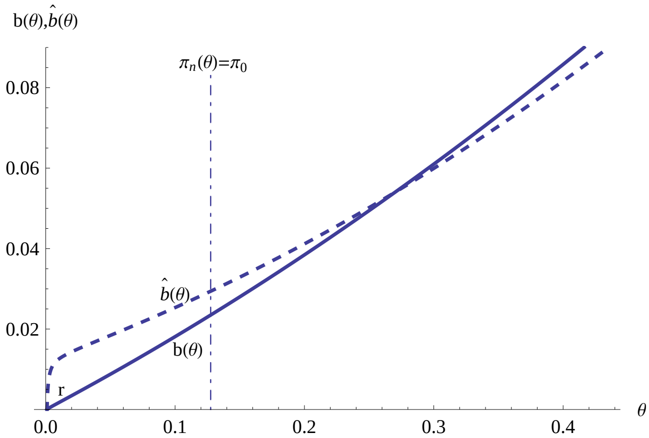

Figure 1: Equilibrium first-price auction with (solid) and without (dashed) externality

As one can easily confirm, R induces the same critical valuation r as the first-price auction, see (2).

Using the same solution procedure as the above, the equilibrium bidding problem of a bidder with $\theta, y \geq r$ can be stated in the form of the equilibrium requirement:

$$\theta = \arg\max_{y \geq r} G(y)\pi_m(\theta)(1 - s(y)) + \int_y^c \pi_n(z)dG(z) - R. \tag{8}$$

Therefore, the equilibrium strategy has to solve the first order differential equation

$$(G(\theta)s(\theta))' = G'(\theta)\frac{\pi_m(\theta) - \pi_n(\theta)}{\pi_m(\theta)}. \tag{9}$$

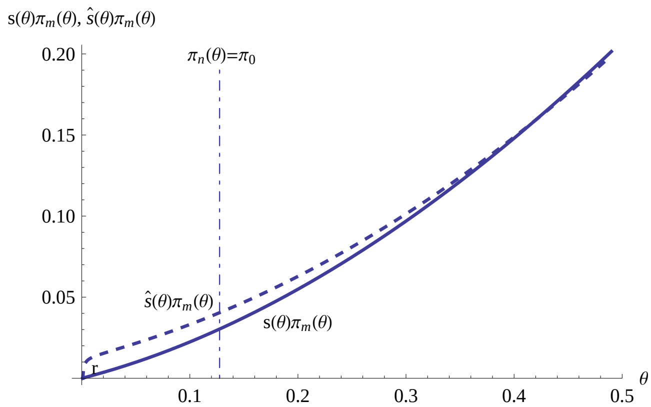

Figure 2: Equilibrium profit-share auction with (solid) and without (dashed) externality

Proposition 2 (Profit-share auction)**.** *The equilibrium strategy of the profit-share auction is*

$$s(\theta) = \int_r^\theta \frac{G'(x)}{G(\theta)} \frac{\pi_m(x) - \pi_n(x)}{\pi_m(x)} dx. \tag{10}$$

Proof. Integrating the differential equation (9), using the initial condition $s(r) = 0$, gives the equilibrium strategy (10). To confirm that s is strictly increasing, as assumed, note that $(\pi_m(\theta)-\pi_n(\theta))/\pi_m(\theta)$ is positive and strictly increasing. Using these facts and applying integration by parts gives:

$$\begin{aligned} s'(\theta) &= \frac{G'(\theta)}{G(\theta)}\left(\frac{\pi_m(\theta)-\pi_n(\theta)}{\pi_m(\theta)} - \frac{1}{G(\theta)}\int_r^\theta G'(x)\frac{\pi_m(x)-\pi_n(x)}{\pi_m(x)}dx\right) \\ &= \frac{G'(\theta)}{G(\theta)^2}\left(G(r)\frac{\pi_m(r)-\pi_n(r)}{\pi_m(r)} + \int_r^\theta \partial_x\left(\frac{\pi_m(x)-\pi_n(x)}{\pi_m(x)}\right)G(x)dx\right) > 0. \end{aligned}$$

Finally, we need to confirm the assumed cutoff participation strategy. The proof is in Appendix A.3. □

In Figure 2 we plot the payments, $s(\theta)\pi_m(\theta)$, that are implicitly offered in equilibrium by bidders provided $\theta \geq r$. We also plot the hypothetical payments based on the share function, $\hat{s}(\theta)$, that would apply if externalities were absent. These plots assume the same linear example that underlies Figure 1. Again, the presence of externalities exerts a downward pressure on equilibrium bids, except for high synergy parameters.

3.3 Superiority of the profit-share auction

Proposition 3. *The profit-share auction is more profitable for the owners of the merger target than the first-price auction, for all R.*

Proof. Let θ be the highest of the sample of N synergy parameters. Then, the difference in equilibrium profits of firm $N+1$ in the profit-share and the first-price auction is equal to:

$$\begin{aligned} \Delta_U(\theta) :&= s(\theta)\pi_m(\theta) - b(\theta) \\ &= \pi_m(\theta)\int_r^\theta \frac{G'(x)}{G(\theta)}\frac{\pi_m(x)-\pi_n(x)}{\pi_m(x)}dx - b(\theta) \\ &> \int_r^\theta \frac{G'(x)}{G(\theta)}(\pi_m(x)-\pi_n(x))\,dx - b(\theta) \quad \text{(since } \partial_\theta\pi_m > 0) \\ &\equiv 0 \quad \text{(by (3))}. \end{aligned}$$

Therefore, the expected profit in the profit-share auction, $U_s(r) = \int_r^c \pi_m(\theta)s(\theta)dF_{(1)}(\theta) + \mu(r)$, is higher than that of the first-price auction, $U_c(r) = \int_r^c b(\theta)dF_{(1)}(\theta) + \mu(r)$, for all $r \in [0, c)$.[4] □

In Figure 3 we plot U_c, U_s, π_0 as functions of the critical valuations r induced by the entry fee R for the example of linear demand, $c = 0.49$, and $F(\theta) = \theta/c$ (uniform distribution). Evidently, the maximum of U_s far exceeds that of U_c and U_c far exceeds π_0. This indicates that the profit-share auction is considerably more profitable than the first-price auction and than the *status quo* prior

[4] Here $F_{(1)}(\theta)$ denotes the c.d.f. of the order statistic of the highest synergy parameter, and $\mu(r)$ the expected value of collected entry fees.

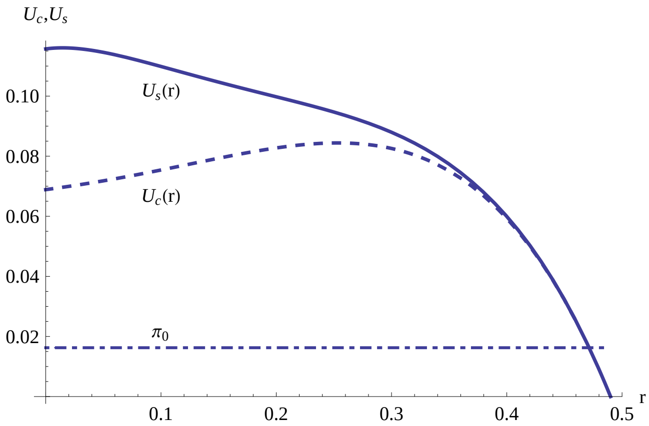

Figure 3: Expected profit: profit-share (solid) vs. first-price (dashed) auction

to the merger. The figure also suggests that the entry fee plays no significant role in the share auction, unlike in the first-price auction.

We mention that one may interpret the superiority of the profit-share auction as an example of the "linkage principle". According to that well-known principle, linking the price to a variable that is correlated with bidders' private information lowers bidders' information rent (Milgrom, 1987).

4 Takeover bidding with signalling

An implausible feature of the above model is that the synergy parameter of the merged firm becomes known before the oligopoly game is played. We now switch to the more plausible model in which firms only observe the winning bid, and then update their beliefs concerning the synergy parameter of the merged firm before playing the oligopoly game.

This modification introduces a signalling aspect into the bidding game. Firms are no longer exclusively concerned with winning or losing the auction, but also with how their bid impacts rivals' beliefs. In particular, firms may wish to inflate their bids in order to signal high synergy, with the intention to gain a strategic advantage in the subsequent oligopoly game.[5]

In order to visibly distinguish between the two models, equilibrium bid functions are now denoted by the Greek letters β (first-price) and σ (profit-share auction).

We employ the following solution procedure: As a working hypothesis suppose the bidding game has a symmetric, strictly monotone increasing equilibrium that allows the losers of the auction to draw a perfect inference from the observed winning bid to the underlying synergy parameter of the merged firm. We consider one bidder, say bidder 1 with synergy parameter θ, who assumes that his rivals play the strictly increasing equilibrium strategy β, resp. σ but considers to make a deviating bid.

[5] Signalling in auctions with downstream interaction has been analyzed in the context of patent licensing by Das Varma (2003), Goeree (2003), and Fan et al. (2009).

Without loss of generality all relevant deviating bids are captured by bids from the interval $[\beta(r), \beta(c)]$, resp. $[\sigma(r), \sigma(c)]$, because bidding outside that interval is obviously dominated. In other words, bidding according to the equilibrium strategy β, resp. σ *as if* the synergy parameter were equal to $y \in (r, c)$ captures all relevant deviating bids.

We first characterize all oligopoly subgames that may occur if bidder 1 unilaterally deviates from the equilibrium bid while everyone believes that all rival firms play the equilibrium bidding strategy β resp. σ.

4.1 Downstream oligopoly "subgames"

Suppose $y \geq \theta$; then two classes of oligopoly subgames must be distinguished:[6]

Case a): $y > \theta_j, \forall j \neq 1$ In this case firm 1 wins the auction. All other firms believe that the synergy parameter of the merged firm is equal to y. Therefore, they believe to play an N player oligopoly game that is characterized by the profile of unit costs $(c - y, c, \ldots, c)$. Denote the equilibrium strategy of players $2, \ldots, N$ by $q_n(y)$ and their equilibrium profit by $\pi_n(y)$. (The full characterization of the equilibrium of this game which players $(2, 3, \ldots, N)$ believe to play, is contained in Appendix 6.)

However, firm 1 privately knows that the merged firm's synergy parameter is equal to θ rather than the pretended y. Therefore, firm 1 plays its best response strategy

$$q_m(\theta, y) := \arg\max_q \pi(q, q_n(y), \ldots, q_n(y), \theta)$$

and earns the equilibrium payoff $\bar{\pi}_m(\theta, y) := \pi(q_m(\theta, y), q_n(y), \ldots, q_n(y), \theta)$.

Case b): $y < x := \max\{\theta_2, \ldots, \theta_N\}$ In this case, firm 1 loses the auction and the synergy parameter realized by the merger is equal to x. The subsequent oligopoly subgame is characterized by the profile of unit costs $(c, c, \ldots, c, c - x, c, \ldots, c)$ and the associated equilibrium profit of firm 1 is denoted by $\pi_n(x)$.

Note that $\bar{\pi}_m(\theta, y)|_{y=\theta} = \pi_m(\theta)$, and $\partial_y\, \bar{\pi}_m(\theta, y)|_{y=\theta} > 0$.

4.2 First-price auction with signalling

By a procedure similar to that used in the model without signalling, the equilibrium requirement concerning β can be stated as follows, for $\theta \geq r$

$$\theta = \arg\max_{y \geq r} G(y)\,(\bar{\pi}_m(\theta, y) - \beta(y)) + \int_y^c \pi_n(x) dG(x) - R. \tag{11}$$

The relationship between r and R is the same as in the model without signalling.

Therefore, β must solve the differential equation for all $\theta \geq r$:

$$(\beta(\theta)G(\theta))' = G'(\theta)\,(\bar{\pi}_m(\theta, \theta) - \pi_n(\theta)) + G(\theta)\partial_y\, \bar{\pi}_m(\theta, y)|_{y=\theta}\,. \tag{12}$$

And we find:

[6] The case of $y \leq \theta$ is similar, and requiring that no "downward" deviating bids should be profitable, yields the same differential equation (12), resp. (15).

Proposition 4 (First-price auction). *In the first-price auction, the potential for signalling induces more aggressive equilibrium bidding, for all $\theta > r$ and for all r:*

$$\beta(\theta) = b(\theta) + \int_r^\theta \frac{G(x)}{G(\theta)} \partial_y \bar{\pi}_m(x, y)|_{y=x} dx > b(\theta). \tag{13}$$

Proof. Using the fact that $\bar{\pi}_m(\theta, y)|_{y=\theta} = \pi_m(\theta)$ and the initial condition, $\beta(r) = 0$, it is easy to confirm that β solves the differential equation (12). The assertion that $\beta(\theta) > b(\theta), \forall\theta > r$ follows from the fact that $\partial_y \bar{\pi}_m(x, y)|_{y=x} > 0$. To complete the proof one needs to confirm the assumed strict monotonicity of β, which we confirm in Appendix A.4. □

The intuition for this result is straightforward. If rival bidders would play the strategy b (which is the equilibrium without signaling), every bidder would benefit from signalling strength by bidding as if the own synergy parameter where higher than it is. Of course, in equilibrium no such misleading signalling can occur. Therefore, the bid function must be adjusted in such a way that signalling strength is made sufficiently costly, which is achieved by raising bids pointwise; hence, $\beta(\theta) > b(\theta), \forall\theta > r$. In other words, the potential for signalling, exerts an upward pressure on equilibrium bids, to the benefit of the takeover target.

4.3 Profit-share auction with signalling

Denote the equilibrium bid function in the signalling model by σ. Similar to the above, the equilibrium requirement takes the form, for all $\theta \geq r$,

$$\theta = \arg\max_{y \geq r} G(y)\bar{\pi}_m(\theta, y)\,(1 - \sigma(y)) + \int_y^c \pi_n(z) dG(z) - R. \tag{14}$$

The relationship between r and R is the same as in the model without signalling.

Therefore, σ must solve the differential equation for all $\theta \geq r$:

$$\sigma'(\theta) + \alpha(\theta)\sigma(\theta) - (\alpha(\theta) - \gamma(\theta)) = 0 \tag{15}$$

$$\alpha(\theta) := \partial_\theta \ln G(\theta) + \partial_y \ln \bar{\pi}_m(\theta, y)|_{y=\theta} \tag{16}$$

$$\gamma(\theta) := \frac{G'(\theta)}{G(\theta)} \frac{\pi_n(\theta)}{\bar{\pi}_m(\theta, \theta)}. \tag{17}$$

And we find:

Proposition 5 (Profit-share auction). *The equilibrium strategy of the profit-share auction in the model with signalling is, for all $\theta \geq r$:*

$$\sigma(\theta) = \int_r^\theta (\alpha(x) - \gamma(x))\,\varphi(x, \theta) dx \tag{18}$$

$$\varphi(x, \theta) := \exp\left(-\int_x^\theta \alpha(z) dz\right). \tag{19}$$

Proof. It is straightforward to confirm that the asserted equilibrium bid function (18) solves the differential equation (15).

For a constructive proof, multiply the differential equation with the positive valued $\mu(\theta) := \exp\left(\int_r^\theta \alpha(z)dz\right)$. Then, one can rewrite the differential equation (15) as

$$(\mu(\theta)\sigma(\theta))' = \mu(\theta)\left(\alpha(\theta) - \gamma(\theta)\right).$$

Integrating and using the initial condition $\sigma(r) = 0$ yields (18).

The assumed strict monotonicity of σ is confirmed in Appendix A.5. □

Finally, we show that the revenue ranking of the two auction formats extends to the signalling model:

Proposition 6. *The profit-share auction is more profitable than the first-price auction, for all R.*

Proof. Using the definitions of α and γ, rewrite the bid function β as:

$$\beta(\theta) = \frac{1}{G(\theta)} \int_r^\theta \left(\alpha(x) - \gamma(x)\right) G(x)\bar{\pi}_m(x,x)dx.$$

Let θ be the highest of the sample of N synergy parameters. Then, the difference between the equilibrium profits of the takeover target firm $N+1$ in the profit-share and the first-price auction is:

$$\begin{aligned}\Delta_U(\theta) &:= \sigma(\theta)\bar{\pi}_m(\theta,\theta) - \beta(\theta)\\ &= \int_r^\theta \left(\alpha(x) - \gamma(x)\right)\left(\varphi(x,\theta)\bar{\pi}_m(\theta,\theta) - \frac{G(x)}{G(\theta)}\bar{\pi}_m(x,x)\right)dx.\end{aligned}$$

We will show that $\varphi(x,\theta)\bar{\pi}_m(\theta,\theta) - \frac{G(x)}{G(\theta)}\bar{\pi}_m(x,x) > 0$, which together with the fact that $\alpha(x) - \gamma(x) > 0$ proves $\Delta_U(\theta) > 0, \forall\theta$.

A bit of rearranging gives:

$$\begin{aligned}&\varphi(x,\theta)\bar{\pi}_m(\theta,\theta) - \frac{G(x)}{G(\theta)}\bar{\pi}_m(x,x)\\ &= \exp\left(-\int_x^\theta \alpha(z)dz\right)\bar{\pi}_m(\theta,\theta) - \frac{G(x)}{G(\theta)}\bar{\pi}_m(x,x)\\ &= \exp\left(-\int_x^\theta \left(\partial_z \ln G(z) + \partial_y \ln \bar{\pi}_m(z,y)|_{y=z}\right)dz\right)\bar{\pi}_m(\theta,\theta) - \frac{G(x)}{G(\theta)}\bar{\pi}_m(x,x)\\ &= \frac{G(x)}{G(\theta)}\left(\frac{\bar{\pi}_m(\theta,\theta)}{\exp\left(\int_x^\theta \partial_y \ln \bar{\pi}_m(z,y)|_{y=z})dz\right)} - \bar{\pi}_m(x,x)\right).\end{aligned}$$

The latter is positive if $\exp\left(\int_x^\theta \partial_y \ln(\bar{\pi}_m(z,y)|_{y=z})dz\right) < \frac{\bar{\pi}_m(\theta,\theta)}{\bar{\pi}_m(x,x)}$, or equivalently if

$$\int_x^\theta \partial_y \ln \bar{\pi}_m(z,y)|_{y=z}\, dz < \ln \frac{\bar{\pi}_m(\theta,\theta)}{\bar{\pi}_m(x,x)}$$
$$\equiv \int_x^\theta \partial_z \ln \bar{\pi}_m(z,y)|_{y=z}\, dz.$$

Evidently, the marginal impact of a truthfully revealed cost reduction on the merged firm's profit is greater than that of an equally sized purely pretended cost reduction; therefore,

$$\partial_z\, \bar{\pi}_m(z,y)|_{y=z} > \partial_y\, \bar{\pi}_m(z,y)|_{y=z}\,, \tag{20}$$

(for a formal proof see Appendix A.6). Hence, $\Delta_U(\theta) > 0, \forall\theta$, as asserted. □

5 Discussion

One limitation of the present paper is that we ignore that profit-sharing arrangements may adversely affect incentives for reorganizing a firm.[7] In particular, if the takeover target is subject to organizational slack, bidders may be willing to pay a premium for acquiring full residual claimant status, and thus avoid diluted incentives, which in turn tilts the balance in favor of cash auctions.

A Appendix

A.1 Linear example

Here we sketch the linear example that underlies the plots in Figures 1-3.

There we set $N = 2$ and assume linear demand $P(Q) := \max\{1-Q, 0\}$, $Q := q_1 + q_2$, which gives $\pi_0 = {}^{(1-c)^2}/_{16}$, $\quad \pi_m(\theta) = {}^{(1-c+2\theta)^2}/_{9}$, $\quad \pi_n(\theta) = {}^{(1-c-\theta)^2}/_{9}$. Hence, for all $\theta : \pi_m(\theta) > \pi_0$, and for all $\theta > 0 : \pi_m(\theta) > \pi_n(\theta)$, and

$$\pi_n(\theta) \gtreqless \pi_0 \iff \theta \lesseqgtr \hat{\theta} := {}^{(1-c)}/_{4} \quad \text{(positive/negative externality).}$$

To compute the bid functions, b, s and the expected profits U_c, U_s plotted in Figs. 1, 3, we also assume $F(\theta) = {}^{\theta}/_{c}$ (uniform distribution). The computations are in a *Mathematica* file available upon request from the authors.

[7] One may argue that profit sharing adversely affects incentives also in running the merged firm, not just in reorganizing the takeover target. However, if ownership is widely dispersed to begin with, it is not clear why the sharing of ownership in the merged firm should significantly aggravate the agency problem.

A.2 Supplement to the proof of Proposition 1

We show that participation in the first-price auction is more profitable than non-participation if and only if $\theta \geq r$. Denote the expected payoff from bidding by Π_p, from non-bidding by Π_n, and $\Delta := \Pi_p - \Pi_n$.

1) Let $\theta > r$, then

$$\Delta = \left(G(\theta)\left(\pi_m(\theta) - b(\theta)\right) + \int_\theta^c \pi_n(z) dG(z) - R \right) - \left(G(r)\pi_0 + \int_r^c \pi_n(z) dG(z) \right).$$

Evidently, $\Delta(r) = 0$, by definition of r, and Π_n is independent of θ. Therefore, using (5), $\partial_\theta \Delta = \partial_\theta \Pi_p = G(\theta)\partial_\theta \pi_m(\theta) > 0$. Hence, $\Delta > 0, \forall \theta > r$.

2) Let $\theta < r$ and suppose a bidder participates and makes a bid $b(y)$, as if his synergy parameter were equal to $y \geq r > \theta$. Then, using (2), (3), and applying integration by parts,

$$\begin{aligned}
\Delta &= \left(G(y)\left(\pi_m(\theta) - b(y)\right) + \int_y^c \pi_n(z) dG(z) - R \right) - \left(G(r)\pi_0 + \int_r^c \pi_n(z) dG(z) \right) \\
&= G(y)\left(\pi_m(\theta) - b(y)\right) - \int_r^y \pi_n(z) dG(z) - G(r)\pi_m(r) \\
&= G(y)\left(\pi_m(\theta) - \pi_m(y)\right) + \int_r^y \partial_z \pi_m(z) G(z) dz \\
&= G(y)\pi_m(\theta) - G(r)\pi_m(r) - \int_r^y \pi_m(z) dG(z) \\
&< G(y)\pi_m(\theta) - G(r)\pi_m(r) - \pi_m(r)\left(G(y) - G(r)\right) \\
&= G(y)\left(\pi_m(\theta) - \pi_m(r)\right) < 0 \quad (\text{since } \theta < r).
\end{aligned}$$

A.3 Supplement to the proof of Proposition 2

Like in Appendix A.2 we show that participation in the profit-share auction is more profitable than non-participation if and only if $\theta \geq r$.

1) Let $\theta > r$, then

$$\Delta = \left(G(\theta)\pi_m(\theta)\left(1 - s(\theta)\right) + \int_\theta^c \pi_n(z) dG(z) - R \right) - \left(G(r)\pi_0 + \int_r^c \pi_n(z) dG(z) \right).$$

Evidently, $\Delta(r) = 0$, by definition of r, and Π_n is independent of θ. Therefore, using (9), $\partial_\theta \Delta = \partial_\theta \Pi_p = G(\theta)(1 - s(\theta))\partial_\theta \pi_m(\theta) > 0$. Hence, $\Delta > 0, \forall \theta > r$.

2) Let $\theta < r$ and suppose a bidder participates and makes a bid $s(y)$, as if his synergy parameter were equal to $y \geq r > \theta$. Then, using (2), (10), one has,

$$\begin{aligned}
\Delta &= \left(G(y)\pi_m(\theta)(1 - s(y)) + \int_y^c \pi_n(z) dG(z) - R \right) - \left(G(r)\pi_0 + \int_r^c \pi_n(z) dG(z) \right) \\
&= G(y)\pi_m(\theta) - G(r)\pi_m(r) - \int_r^y \left(\pi_n(z) + \pi_m(\theta)\left(1 - \frac{\pi_n(z)}{\pi_m(z)}\right)\right) dG(z) \\
&= G(r)\left(\pi_m(\theta) - \pi_m(r)\right) - \int_r^y \frac{\pi_n(z)\left(\pi_m(z) - \pi_m(\theta)\right)}{\pi_m(z)} dG(z) < 0 \quad (\text{by } y \geq r > \theta).
\end{aligned}$$

A.4 Supplement to the proof of Proposition 4

$$
\begin{aligned}
\beta'(\theta) &= b'(\theta) - \frac{G'(\theta)}{G(\theta)^2}\int_r^\theta \partial_y\, \bar{\pi}_m(z,y)|_{y=z}\, G(z)dz + \partial_y\, \bar{\pi}_m(\theta,y)|_{y=\theta} \\
&= \frac{G'(\theta)}{G(\theta)^2}\left(G(r)\left(\pi_m(r)-\pi_n(r)\right) + \int_r^\theta \frac{d}{dz}\left(\bar{\pi}_m(z,z)-\pi_n(z)\right)G(z)dz\right) \\
&\quad - \frac{G'(\theta)}{G(\theta)^2}\int_r^\theta \partial_y\, \bar{\pi}_m(z,y)|_{y=z}\, G(z)dz + \partial_y\, \bar{\pi}_m(\theta,y)|_{y=\theta} \quad \text{(by (6))} \\
&= \frac{G'(\theta)}{G(\theta)^2}\left(\int_r^\theta \left(\frac{d}{dz}\left(\bar{\pi}_m(z,z)-\pi_n(z)\right) - \partial_y\, \bar{\pi}_m(z,y)|_{y=z}\right)G(z)dz\right. \\
&\quad \left. + G(r)\left(\bar{\pi}_m(r,r)-\pi_n(r)\right)\right) + \partial_y\, \bar{\pi}_m(\theta,y)|_{y=\theta} \\
&= \frac{G'(\theta)}{G(\theta)^2}\left(\int_r^\theta \left(\partial_z\left(\bar{\pi}_m(z,y)|_{y=z} - \pi_n(z)\right)\right)G(z)dz + G(r)\left(\bar{\pi}_m(r,r)-\pi_n(r)\right)\right) \\
&\quad + \partial_y\, \bar{\pi}_m(\theta,y)|_{y=\theta} \\
&> 0.
\end{aligned}
$$

A.5 Supplement to the proof of Proposition 5

Note, $\varphi(x,\theta)$ is a c.d.f. with support (r,θ), since $\varphi(x,\theta)$ is nonnegative, increasing in x, and $\varphi(\theta,\theta) = 1$. Moreover, $\partial_\theta \varphi(x,\theta) = -\partial_x \varphi(x,\theta)\frac{\alpha(\theta)}{\varphi(x)}$. Therefore, after differentiating (18),

$$
\begin{aligned}
\sigma'(\theta) &= (\alpha(\theta)-\gamma(\theta)) + \int_r^\theta (\alpha(x)-\gamma(x))\,\partial_\theta \varphi(x,\theta)dx \\
&= (\alpha(\theta)-\gamma(\theta)) - \alpha(\theta)\int_r^\theta \left(1-\frac{\gamma(x)}{\alpha(x)}\right)\partial_x \varphi(x,\theta)dx \\
&= \alpha(\theta)\left(\left(1-\frac{\gamma(\theta)}{\alpha(\theta)}\right) - E_\varphi\left(1-\frac{\gamma(X)}{\alpha(X)}\right)\right).
\end{aligned}
$$

A bit of rearranging gives,

$$
\frac{\gamma(x)}{\alpha(x)} = \frac{\pi_n(x)}{\bar{\pi}_m(x,x) + \partial_y\, \bar{\pi}_m(x,y)|_{y=x}\,\frac{G(x)}{G'(x)}}.
$$

This is strictly monotone decreasing, since $\bar{\pi}_m$ is increasing in x and y, by equation (22), $\partial_y \bar{\pi}_m(x,y)$ is increasing in x, since $\partial_{yx}\bar{\pi}_m(x,y) = (N-1)q_n'(y)\left(P'(\cdot)+q_m(x,y)P''(\cdot)\right)\partial_x q_m(x,y) > 0$, π_n is decreasing in x, and G is log-concave. Hence, it follows immediately that $\sigma'(x) > 0$, as asserted.

A.6 Supplement to the proof of Proposition 6

Here we prove inequality (20). Note that (here z is the true and y the pretended cost reduction of firm 1)

$$
\bar{\pi}_m(z,y) := \left(P\left(q_m(z,y)+(N-1)q_n(y)\right) - c + z\right)q_m(z,y).
$$

By the envelope theorem,

$$\partial_z \bar{\pi}_m(z, y) = q_m(z, y) \tag{21}$$
$$\partial_y \bar{\pi}_m(z, y) = P'(\cdot)(N-1)q_n'(y)q_m(z, y). \tag{22}$$

Therefore,

$$\partial_z \bar{\pi}_m(z, y) - \partial_y \bar{\pi}_m(z, y) = \big(1 - P'(\cdot)(N-1)q_n'(y)\big)\, q_m(z, y). \tag{23}$$

By construction, if firm 1 wins the auction, the $N-1$ other firms believe that they play an oligopoly game with the profile of unit cost $(c-y, c, \ldots, c)$, which has the equilibrium solution:[8]

$$q_m^*(y) = \arg\max_q \left(P(q + (N-1)q_n(y)) - c + y\right) q,$$
$$q_n(y) = \arg\max_q \big(P(q_m^*(y) + (N-2)q_n(y) + q) - c\big)\, q.$$

The associated first–order conditions are:

$$P'(q_m^*(y) + (N-1)q_n(y))q_m^*(y) + P(q_m^*(y) + (N-1)q_n(y)) - c + y = 0$$
$$P'(q_m^*(y) + (N-1)q_n(y))q_n(y) + P(q_m^*(y) + (N-1)q_n(y)) - c = 0.$$

Differentiating these w.r.t. y, and solving the equation system for $q_n'(y)$ one finds:

$$q_n'(y) = \frac{P'(\cdot) + P''(\cdot)q_n(y)}{P'(\cdot)\big((N+1)P'(\cdot) + P''(\cdot)\big(q_m^*(y) + (N-1)q_n(y)\big)\big)}. \tag{24}$$

Finally, substituting (24) into (23), confirms (20):

$$\begin{aligned}\partial_z \bar{\pi}_m(z, y) - \bar{\partial}_y \pi_m(z, y) &= \big(1 - P'(\cdot)(N-1)q_n'(y)\big)\, q_m(z, y)\\ &= \frac{2P'(\cdot) + P''(\cdot)q_m^*(y)}{(N+1)P'(\cdot) + P''(\cdot)\big(q_m^*(y) + (N-1)q_n(y)\big)} q_m(z, y)\\ &> 0.\end{aligned}$$

References

Amir, R., Diamantoudi, E., Xue, L., 2009. Mergere performance under uncertain efficiency gains. International Journal of Industrial Organization 27, 264–273.

Brusco, S., Lopomo, G., Robinson, D., Viswanathan, S., 2007. Efficient mechanisms for mergers and acquisitions. International Economic Review 48, 995–1035.

Choné, P., Linnemer, L., 2008. Assessing horizontal mergers under uncertain efficiency gains. International Journal of Industrial Organization 25, 382–401.

[8] Note, $q_m^*(y)$ is the strategy that firms $(2, 3, \ldots N)$ believe firm 1 to play; it is not the strategy that firm 1 actually plays, since firm 1 has private information about its cost reduction.

Crémer, J., 1987. Auctions with contingent payments: Comment. American Economic Review 77, 746.

Das Varma, G., 2003. Bidding for a process innovation under alternative modes of competition. International Journal of Industrial Organization 21, 15–37.

Deneckere, R., Davidson, C., 1985. Incentives to form coalitions with Bertrand competition. RAND Journal of Economics 16, 473–486.

Fan, C., Jun, B., Wolfstetter, E., 2009. Licensing a common value innovation in oligopoly when signaling strength may backfire. Discussion Paper 292, SFB/TR 15, University of Bonn.

Farrell, J., Shapiro, C., 1990. Horizontal mergers: An equilibrium analysis. American Economic Review 80, 107–126.

Faulý-Oller, R., 1997. On merger profitability in a Cournot setting. Economics Letters 54, 75–79.

Goeree, J., 2003. Bidding for the future: Signaling in auctions with an aftermarket. Journal of Economic Theory 108, 345–364.

Gärtner, J., Schmutzler, A., 2009. Merger negotiations and ex-post regret. Journal of Economic Theory 144, 1636–1664.

Hamada, K., 2010. Uncertainty and horizontal merger. mimeo, Niigata University.

Hansen, R. G., 1985. Auctions with contingent payments. American Economic Review 75, 862–865.

Jehiel, P., Moldovanu, B., 2000. Auctions with downstream interaction among buyers. RAND Journal of Economics 31, 768–791.

Milgrom, P. R., 1987. Auction theory. In: Bewley, T. F. (Ed.), Advances in Economic Theory. Econometric Society Monographs No. 12, Cambridge University Press, Cambridge, pp. 1–32.

Moeller, S., Schlingemann, F., Stulz, R., 2005. Wealth destruction on a massive scale? a study of acquiring-firm returns in the recent merger wave. Journal of Finance 60, 757–782.

Ravenscraft, D. J., Scherer, F. M., 1989. The profitability of mergers. International Journal of Industrial Organization 7, 101–116.

Salant, W., Switzer, S., Reynolds, R. J., 1983. Losses from horizontal merger: The effects of an exogenous change in industry structure on Cournot–Nash equilibrium. Quarterly Journal of Economics 98, 185–199.

Samuelson, W., 1987. Auctions with contingent payments: Comment. American Economic Review 77, 740–745.

Szidarovszky, F., Yakowitz, S., 1977. A new proof of the existence and uniqueness of the Cournot equilibrium. International Economic Review 18, 787–789.