

Licensing Process Innovations when Losers' Bids Determine Royalty Rates[☆]

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Abstract

We consider a licensing mechanism for process innovations that awards a limited number of fixed-fee licenses to those firms that report the highest cost reductions, combined with royalty licenses to others. Firms' messages are dual signals: the message of those who win a fixed-fee license signals their cost reduction to rival firms, whereas losers' message influence the royalty rate set by the innovator. We derive conditions for existence of a truth-telling equilibrium, explain why a sufficiently high reserve price is essential for such an equilibrium, and show that the innovator generally benefits from the proposed mechanism.

Keywords: Patents, licensing, auctions, royalty, innovation, R&D, mechanism design.

2000 MSC: D21, D43, D44, D45.

1. Introduction

This paper revisits the licensing of a non-drastic process innovation by an outside innovator to a Cournot oligopoly. The cost reductions induced by that innovation are of the private values type and are firms' private information. The innovator employs a direct revelation mechanism to award licenses. The main feature is that we replace the standard license auction by a superior mechanism that awards a limited number of fixed-fee licenses to those firms that report the highest cost reductions and royalty licenses to the remaining firms. There, the innovator has two sources of revenue: the equilibrium transfers paid by those who win a fixed-fee license, and the royalty income paid by those who lose.

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This licensing mechanism gives rise to a dual signaling problem: If a firm wins a fixed-fee license, its message signals the own cost reduction to rival firms, whereas if it loses, its message signals the own cost reduction also to the innovator who sets the royalty rate equal to the reported cost reduction. Firms take into account that they can influence others' beliefs with their messages. Specifically, a firm gains a strategic advantage in the oligopoly game with an "inflated message" that signals a higher than true cost reduction, provided it happens to win. In turn, a firm can fool the innovator to set the royalty rate below the true cost reduction with a "deflated message", provided it happens to lose.

Of course, no such "misleading" signaling occurs on the equilibrium path of a truth-telling equilibrium. If such an equilibrium exists, the marginal benefit of signaling a cost reduction that deviates from the true cost reduction must be matched by a corresponding marginal cost in such a way that both kinds of signaling are deterred in all states of the world.

If messages can only influence the beliefs of rival firms, misleading signals are easily deterred by choosing an appropriate steepness of the transfer rule that prescribes payments to be made by winners. As a result, the possibility to influence the beliefs of rival firms with a winning message, simply exerts an upward pressure on winners' equilibrium transfers.

Similarly, the possibility to influence the beliefs of the innovator with the losing message exerts a downward pressure on winners' equilibrium transfers. However, one cannot deter firms from signaling to the innovator a lower than true cost reduction by adjusting the steepness of winners' equilibrium transfer rule alone. In addition, the innovator must set a sufficiently high reserve price or the support of the distribution must be sufficiently bounded away from zero. Otherwise, firms with a low cost reduction would always signal a zero cost reduction in order to lose and obtain the innovation for free.

In the face of the dual signaling problem, a truth-telling equilibrium exists only in combination with a sufficiently high reserve price. Of course, a seller can typically increase his revenue by adding a reserve price requirement. However, in the present framework, the role of the reserve price is more fundamental, because without it, no such equilibrium exists.

We analyze two specifications of the model that differ exclusively in the information available to firms in the downstream oligopoly game. In the first highly stylized specification, referred to as *model I*, firms' cost reductions become common knowledge among firms after licensing took place and before the oligopoly game is played. However, the innovator remains uninformed about cost reductions, and he can only update his prior beliefs after observing firms' messages. The innovator uses this information to set royalty rates for losers, and firms use their knowledge about the information available to the innovator to predict the royalty rate to be paid by those who lose.

In the second specification, referred to as *model II*, firms remain uninformed about their rivals' cost reductions after licensing. Like the innovator they can, however, update their prior beliefs from observed messages. Firms use this updated information to assess the costs of their rivals, to predict the royalty rates the losers have to pay and to predict their rivals' beliefs about the royalty rate they themselves have to pay if they lose.

Altogether, *model II* is more plausible. Nevertheless, *model I* is useful as a benchmark for comparison with the literature, and it prepares nicely the more

complex analysis of *model II*.

There is an extensive literature on patent licensing in oligopoly by an outside innovator, among which the following contributions are closely related to the present paper.

In their seminal contributions, Kamien (1992); Kamien and Tauman (1984, 1986); Katz and Shapiro (1985, 1986) show that auctioning a limited number of licenses is strictly more profitable for the innovator than other mechanisms, such as pure royalty contracts, fixed-fee licensing, and two-part tariffs.¹ The limitation of the classical literature is that it assumes complete information both in the auction and in the subsequent oligopoly game.

Later Jehiel and Moldovanu (2000) introduce incomplete information at the bidding stage, combined with complete information in the oligopoly game, (which is the information structure to which we already referred as *model I*). They show that patent licensing under incomplete information is a prime example of an auction with negative externalities, where bidders' payoffs depend not only on their own types, but also on the type of the player who wins the auction.

In an auction with negative externalities, the reserve price plays a much less prominent role, because there the seller has a stronger incentive to induce participation. As Jehiel and Moldovanu (2000, p. 777) put it succinctly: "when the seller sells more often, the buyers are more afraid that the good will fall into the hands of the competitor, and they bid more aggressively".

More recently, Das Varma (2003) and Goeree (2003) reconsider that model under the more plausible assumption that firms do not know each other's cost reductions after the auction and before the oligopoly game is played, which corresponds to the information structure in our *model II*. This introduces the possibility to signal the own cost reduction to rival firms.

Das Varma (2003) shows that a separating equilibrium exists under Cournot competition with linear demand, but generally fails to exist under Bertrand competition when goods are substitutes. Goeree (2003) assumes reduced form payoff functions of the oligopoly "subgames" that are generally satisfied for Cournot but not for Bertrand market games with substitutes. He compares three standard auction formats – first-, second-price, and English auctions and shows that they are revenue equivalent.

In a preceding paper, Giebe and Wolfstetter (2008) introduce an optional royalty scheme into license auctions assuming complete information, a common value non-drastic innovation, and a potentially large number of firms. There, the innovator charges the losers of the auction a royalty rate equal to the cost reduction induced by the innovation. As a result, the royalty scheme has no effect on equilibrium bids because the losers' payoff functions in the oligopoly game are not affected. The innovator's expected profit is increased, provided no loser is crowded out of the market. The challenge of that paper was to show that the optimal number of auctioned licenses is such that no such exits are induced and royalties are always paid.

¹Recently, Sen (2005) shows that if one takes into account that the number of licenses must be an integer, pure royalty contracts can be superior to license auctions. However, this result is again reversed if one generalizes the format of license auctions (see Giebe and Wolfstetter, 2008). See also Sen and Tauman (2007) who analyze an auction of royalty contracts.

Unlike Giebe and Wolfstetter (2008), the present paper assumes that firms' cost reductions due to the innovation are their private information, unknown both to their rivals and to the innovator. In that case adding the royalty scheme works in a different way: first, it gives rise to a complex dual signaling problem where the winning and the losing messages signal information to rival firms and to the innovator; second, it leads to lower equilibrium transfers to be paid by winners. Therefore, the innovator faces a trade-off between income earned from the equilibrium transfer paid by winners and the royalty income from losers. The main focus of the present paper is to show that the gain in royalty income exceeds the loss in revenue earned from winners.

Unlike the classical literature on patent licensing, but similar to Jehiel and Moldovanu (2000), Das Varma (2003), and Goeree (2003), the present paper assumes that the cost reductions induced by the innovation are firm-specific.

Our main findings can be summarized as follows: 1) A mechanism with strictly increasing transfer rule for the winner is truthfully implementable in Bayesian Nash equilibrium under fairly standard conditions concerning the probability distribution of firms' cost reductions, provided the innovator sets a sufficiently high reserve price. 2) The reserve price plays a crucial role in assuring existence of a truth-telling equilibrium; without it, no such equilibrium exists, unlike in the standard license auction where the reserve price plays a minor role. 3) Adding the royalty contract for losers exerts a downward pressure on winners' equilibrium transfers, which contributes to lower the innovator's expected revenue. 4) However, the additional royalty income weighs more than that loss in revenue, unless the probability distribution of cost reductions exhibits an extreme concentration on low values. Therefore, the proposed mechanism is generally more profitable. 5) Adding the royalty scheme lowers winners' equilibrium transfers more at low than at high cost reductions. Therefore, adding the royalty scheme is particularly profitable when the probability distribution exhibits a concentration on high cost reductions, because it entails a relatively small loss in winners' equilibrium transfers combined with a high royalty income.

The paper is organized as follows: In Section 2 we present the model and introduce basic assumptions. In Section 3 we analyze model I in which we also show in detail why the reserve price plays a crucial role in assuring that the first-order conditions concerning the winners' equilibrium transfers yield global maxima. In Section 4 we analyze the more plausible model II which nicely complements and extends the analysis of model I. We find more general results for model II, and show that adding the royalty scheme is more profitable in model II. In Section 5 we confirm robustness by showing that our analysis can be extended to more than two firms. In Section 6 we discuss our results and explain why our restriction on royalty rates is compelling due to antitrust concerns. Some of the proofs are contained in the Appendix.

2. The Model

An outsider innovator licenses a non-drastring process innovation protected by a patent to a Cournot duopoly. Prior to the innovation, firms have the same constant unit cost c . The innovation reduces this cost by an amount $x_i \in X := [0, c]$ which is firms' private information.

Licensing mechanism. The innovator offers to license his innovation to both firms, using a direct revelation mechanism (k, t, R) with *allocation rule* k , *transfer rule* t , and *reserve price* R . There, one firm, called “winner”, is offered a fixed-fee license, and the other firm, called “loser”, is awarded a mandatory royalty contract. Licenses do not prescribe firms’ subsequent output choices.

The mechanism game is a simultaneous moves game; there, firms decide whether to participate and report their cost reductions. Based on firms’ messages the mechanism (k, t, R) is executed.

If both firms participate, the firm that reports the highest cost reduction wins and pays a fixed fee, and the other loses and pays output based royalties. If only one firm participates, that firm wins and pays the reserve price R , and the other does not get a license. If no firm participates, no one obtains a license.

The royalty rate per output unit is equal to the loser’s reported cost reduction. And, similar to a second-price auction, the transfer paid by the winner is only a function of the loser’s reported cost reduction, and never smaller than R .

Stated formally, let k_i denote i ’s probability of winning, $\beta(x_j)$ the transfer to be paid by the winner, q_{iL} the output produced by the loser in the subsequent duopoly game,² and $\mathbb{1}_A$ the indicator function of subset $A \subseteq X^2$. Then, the allocation and transfer rules are:

If both firms participate:

$$k_i(x_i, x_j) = \mathbb{1}_{x_i > x_j} \tag{1}$$

$$t_i(x_i, x_j) = \mathbb{1}_{x_i > x_j} \beta(x_j) + \mathbb{1}_{x_j > x_i} x_i q_{iL} \tag{2}$$

$$\beta(x_i) \geq R. \tag{3}$$

If only firm i participates:

$$k_i(x_i) = 1, \quad t_i(x_i) = R. \tag{4}$$

Without loss of generality, the innovator chooses β in such a way that truth-telling is a Bayesian Nash equilibrium of the game induced by the mechanism and the subsequent duopoly game.

The transfer rule is reminiscent of a second-price auction with a reserve price. However, unlike in a standard second-price license auction, the loser of the auction is awarded a mandatory royalty contract and pays a royalty rate equal to his cost reduction, indirectly reported to the innovator through the losing bid. Based on this interpretation one may refer to the licensing mechanism as a second-price auction supplemented by a royalty contract for the loser, and to β as the bid function.

We emphasize that, due to antitrust concerns, the royalty rate cannot exceed the cost reduction. Therefore, we assume that the highest feasible royalty rate is adopted. We further discuss this restriction in Section 6.

Duopoly “subgames”. After licensing took place, the innovator publishes all messages. Having observed the messages, the duopolists compete in a Cournot market game.

²Note, after licenses have been awarded, firms are free to choose outputs; in other words, the mechanism does not prescribe outputs.

We consider two models that differ in the information available in the duopoly game: In *model I*, firms know each other's true cost reductions after licenses have been awarded and before the duopoly game is played, and in *model II* the true cost reductions remain private information, and firms can only observe their rival's message. In both models, the innovator generally does not know firms' cost reductions, and only observes firms' messages.

Assumptions. Firms and the innovator are risk neutral and inverse market demand $P(Q)$ is a decreasing and concave function of aggregate output $Q := q_1 + q_2$.³

The firm-specific cost reductions induced by the innovation, denoted by x, y are *iid* random variables, drawn from the c.d.f. $F : [0, c] \rightarrow [0, 1]$, with positive density f everywhere. Both F and the reliability function $1 - F$ are assumed to be log-concave which rules out that F has parts that are highly convex or highly concave.⁴ The log-concavity of $1 - F$ is equivalent to the well-known hazard rate monotonicity.

At some points in the paper we also consider the case when the support of the distribution is bounded away from zero. For this purpose we will consider the family of truncations of $F : [0, c] \rightarrow [0, 1]$ from below, denoted by $H : [d, c] \rightarrow [0, 1]$, $d \geq 0$:

$$H(x) = \frac{F(x) - F(d)}{1 - F(d)}. \quad (5)$$

In this case, the innovation gives rise to some cost reduction, $x \geq d$, with probability one. We mention that truncations preserve log-concavity of the c.d.f. and of the reliability functions.

We consider only non-drastic innovations whose exclusive use does not propel a monopoly. If $P(Q) = 1 - Q$, innovations are non-drastic if and only if $c \in [0, 1/2)$.

We denote the two highest order statistics of the sample of cost reductions by $X_{(1)}, X_{(2)}$, and the associated p.d.f. of $X_{(2)}$ by $g_2(y) = 2(1 - F(y))f(y)$ and the joint p.d.f. of $X_{(1)}, X_{(2)}$ by $g_{12}(x, y) = 2f(x)f(y)$. And we denote the equilibrium duopoly profits of the winner and the loser by π_W and π_L , respectively; and the equilibrium profit when both firms abstain from participation in the mechanism game by π_{nn} .

3. Model I: Complete information in the duopoly game

Following Jehiel and Moldovanu (2000) we first consider a highly stylized model in which firms learn about each other's cost reductions before they play the duopoly game. In the absence of royalty contracts for losers, the mechanism reduces to a standard second-price auction with reserve price.

³This assures that the duopoly game has a unique equilibrium (see Szidarovszky and Yakowitz, 1977).

⁴A sufficient condition for the log-concavity of both F and $1 - F$ is the log-concavity of f . See Lemma A2 in Goeree and Offerman (2003) together with Theorem 1 in Bagnoli and Bergstrom (2005).

3.1. *Benchmark: The game without royalty contract for the loser*

The auctioning of one license without royalty contract for the loser has already been analyzed in Jehiel and Moldovanu (2000, Sect.4). Their main finding was that in the presence of negative externalities, using a reserve price becomes less attractive for the auctioneer. Among other results we will show that the reserve price plays a crucial role if one replaces the simple license auction considered by Jehiel and Moldovanu (2000) by the generally superior mechanism proposed here.

We briefly review their results for the case of a duopoly, which serve as a benchmark.

Because only the winner of the auction has access to the innovation, the equilibrium profits in the duopoly subgames depend only on the cost reduction of the winner of the auction, denoted by x . And the equilibrium profit of the winner, $\pi_W(x)$, is increasing and that of the loser, $\pi_L(x)$, is decreasing in x .

In a second-price auction with reserve price, R , the winner pays the reserve price if he is the only bidder and otherwise pays the maximum of the second highest bid and the reserve price. The game has an equilibrium in weakly dominant strategies. There, bidders play cutoff strategies, and bid truthfully if and only if their cost reduction is at least as high as the cutoff value r , and do not bid otherwise. The equilibrium cutoff value is such that the marginal bidder, with cost reduction equal to r , is indifferent between bidding and not bidding, i.e., $\pi_W(r) - R = \pi_{nn}$. This yields a unique relationship between r and R , that allows us to eliminate R , and compute the innovator's expected profit as a function of r .

The equilibrium bid function, β_n , and the innovator's expected revenue, $G_n(r)$ are (see Sect. 4, Jehiel and Moldovanu, 2000),

$$\beta_n(x) = \pi_W(x) - \pi_L(x), \quad \text{for } x \geq r \quad (6)$$

$$G_n(r) = 2F(r)(1 - F(r))(\pi_W(r) - \pi_{nn}) + \int_r^c (\pi_W(y) - \pi_L(y)) g_2(y) dy. \quad (7)$$

And the optimal cutoff value r_n solves the following condition:

$$\frac{1 - F(r)}{f(r)} = \frac{\pi_W(r) - \pi_{nn}}{\pi'_W(r)} + \frac{1 - F(r)}{F(r)} \frac{(\pi_{nn} - \pi_L(r))}{\pi'_W(r)}. \quad (8)$$

It is useful to compare this with a hypothetical world in which there is no negative externality in the sense that the loser's equilibrium duopoly profit is not affected by the winner's cost reduction. In that hypothetical world, the equilibrium bidding strategy and the innovator's expected revenue would be equal to

$$\beta_0(x) = \pi_W(x) - \pi_{nn} \quad (9)$$

$$G_0(r) = 2F(r)(1 - F(r))(\pi_W(r) - \pi_{nn}) + \int_r^c (\pi_W(y) - \pi_{nn}) g_2(y) dy, \quad (10)$$

yielding the optimal cutoff value r_0 that solves the equation,

$$\frac{1 - F(r)}{f(r)} = \frac{\pi_W(r) - \pi_{nn}}{\pi'_W(r)}. \quad (11)$$

Comparing (8) and (11) and using the assumed hazard rate monotonicity one obtains $r_n < r_0$, and because R is strictly increasing in r , it follows immediately:

Proposition 1 (Jehiel and Moldovanu (2000)). *The optimal reserve price with negative externality is lower than that in a standard auction without negative externality.*

Essentially, in the presence of negative externalities, the innovator has an incentive to lower the reserve price, because a lower reserve price makes it more likely that both firms bid, and the winner pays $\pi_W(r) - \pi_L(r)$, which is more than the reserve price, $R = \pi_W(r) - \pi_{nn}$ that the winner pays if only one firm bids.

3.2. The game with royalty contract for the loser

An innovation is a public good because it does not involve rivalry in consumption. If the innovator awards one fixed-fee license to the winner he may also let the loser use that innovation, without affecting the winner's cost reduction. This suggests that the innovator leaves money on the table if he excludes the loser from using the innovation.

Under complete information the innovator can easily extract that surplus by letting the loser use the innovation in exchange for a royalty rate per output unit equal to the cost reduction induced by the innovation, without losing auction revenue (Giebe and Wolfstetter, 2008).

For a moment, suppose cost reductions become common knowledge after the auction to firms *as well as* to the innovator. Then neither the duopoly nor the bidding game is affected by adding the royalty contract. And it follows immediately:

Proposition 2. *When costs reductions are common knowledge after the auction, adding the royalty scheme is profitable for the innovator and increases welfare.*

Adding the royalty scheme leaves the loser's cost unchanged, because the royalty rate is equal to his cost reduction. Therefore, neither the duopoly nor the bidding game is affected. The innovator then earns the same income in the auction, yet the royalty contract adds a positive royalty income. The equilibrium payoffs of the duopolists remain unchanged, but the innovator's equilibrium expected revenue increases; hence, welfare increases.

However, if cost reductions become common knowledge only among firms, whereas the innovator remains uninformed, the innovator can only update his prior beliefs about cost reductions from observed messages, and then set the royalty rate equal to the reported cost reduction of the loser. This, in turn, induces firms to use their messages to influence the innovator's beliefs about their cost reduction and thus the royalty rate they have to pay in the event when they lose.

If firms' cost reductions are unknown both to its rival firm and to the innovator, charging royalties from the loser exerts a downward pressure on the winner's equilibrium transfer. The question then is whether the innovator can extract more royalty income from the loser than his loss in revenue due to the reduced equilibrium transfer paid by the winner.

We employ the following procedure to solve the game. As a working hypothesis assume the mechanism has a symmetric and monotone increasing transfer rule (referred to as bid function), $\beta : [r, c] \rightarrow \mathfrak{R}$ (which we will confirm) that induces truth-telling as a Bayesian Nash equilibrium. Consider one firm, say

firm 1, with cost reduction x , which unilaterally deviates from truth-telling, and compute its payoff function for all possible messages $z \in (r, c)$. We then choose β in such a way that no such deviation from truth-telling is profitable. For this purpose, one must first solve all duopoly subgames that may occur.

3.2.1. Downstream duopoly “subgames”

Suppose firm 1 has drawn cost reduction x but reports cost reduction $z \geq x$, while firm 2 tells the truth.⁵ In the continuation duopoly game, the following “subgames” occur, depending upon the true and pretended cost reductions of firm 1, x, z , and the true cost reduction of firm 2, y .

When both firms participated and firm 1 won. Let $z > y$ and $x, y \geq r$. The innovator observes the message of firm 2, y , and charges it a royalty rate equal to y . It is then common knowledge among firms that the profile of unit costs is $(c_1, c_2) = (c - x, c)$. Denote the Cournot equilibrium strategies for this cost profile by $(q_{W_1}(x), q_{L_2}(x))$. Therefore, the reduced form profit function of firm 1, conditional on having won, is $\pi_W(x) := (P(q_{W_1}(x) + q_{L_2}(x)) - c + x)q_{W_1}(x)$.

When both firms participated and firm 1 lost. Let $y > z$ and $x, y \geq r$. The innovator charges firm 1 a royalty rate equal to z . It is then common knowledge among firms that the profile of unit costs is $(c_1, c_2) = (c - x + z, c - y)$. Denote the equilibrium strategies of that duopoly subgame by $(q_{L_1}(x, z, y), q_{W_2}(x, z, y))$. Therefore, the reduced form profit function of firm 1, conditional on having lost, is

$$\pi_L(x, z, y) := (P(q_{W_2}(x, z, y) + q_{L_1}(x, z, y)) - c + x - z)q_{L_1}(x, z, y).$$

On the equilibrium path, i.e., for $z = x$, the equilibrium strategy of firm 1 when it lost and the associated reduced form payoff are only a function of firm 2’s cost reduction, y ; therefore, we write: $q_L^*(y) := q_{L_1}(x, z, y)|_{z=x}$ and $\pi_L^*(y) := \pi_L(x, z, y)|_{z=x}$. Similarly, we write $q_W^*(y) := q_{W_2}(x, z, y)|_{z=x}$.

When at least one firm did not participate. If no one participated, the game is just the default game without innovation, and the equilibrium profit of firm 1 is equal to π_{nn} . If firm 1 was the only one that participated, its equilibrium profit is the same as in the event when both firms participated and firm 1 won, and if firm 2 was the only one that participated, the equilibrium profit of firm 1 is the same as in the game without royalty scheme, and is exclusively a function of the winner’s cost reduction, $\pi_L(y)$, as explained in Section 3.1.

Lemma 1. *In the relevant duopoly subgames, $\partial_z \pi_L(x, z, y)|_{z=x} = -q_L^*(y)\gamma(y)$, where*

$$\gamma(y) := 1 - (P'(q_{W_2}(\cdot) + q_{L_1}(\cdot))\partial_z q_{W_2}(x, z, y))|_{z=x} > 1. \quad (12)$$

and $\pi_W'(x) > 0$, $\pi_L^{*'}(y) < 0$. If demand is linear, $\gamma(y) = 4/3$ (for a summary account of the linear model see Appendix E and Appendix F).

The proof is in Appendix A.

⁵The case of $z \leq x$ is slightly different, yet yields the same payoff function, $\Pi(x, z)$ and differential equation, and hence is omitted.

3.2.2. Licensing “subgame”

We now characterize the transfer rule β (bid function) that induces truth-telling as a Bayesian Nash equilibrium and firms’ participation strategy. As a working hypothesis assume that participation strategy is a cutoff strategy according to which firms participate if and only if their cost reduction is greater than or equal to the cutoff value r . We then find β and the relationship between r and R , and finally confirm that there is a unique cutoff strategy.

Lemma 2. *Suppose firms play a cutoff participation strategy with cutoff value r . Then there is a unique relationship between r and the reserve price R which is implicitly defined as the solution of:*

$$R = \pi_W(r) - \pi_{nn}. \quad (13)$$

Proof. Consider the marginal firm, with cost reduction $x = r$. If that firm participates, and the other firm tells the truth (if it participates), that firm’s payoff is $\Pi_p(r)$, whereas if it does not participate, its payoff is $\Pi_{np}(r)$:

$$\begin{aligned} \Pi_p(r) &= F(r) (\pi_W(r) - R) + \int_r^c \pi_L^*(y) dF(y) \\ &= -F(r)R + F(r)\pi_W(r) + \int_r^c \pi_L^*(y) dF(y) \\ \Pi_{np}(r) &= F(r)\pi_{nn} + \int_r^c \pi_L^*(y) dF(y). \end{aligned}$$

And the assertion follows immediately. \square

As a working hypothesis suppose β is strictly monotone increasing (which we will confirm below). Consider a firm, say firm 1, that has drawn cost reduction x , but deviates from equilibrium by reporting cost reduction $z \geq x$,⁶ while firm 2 reports its true cost reduction. Then, using the equilibria of the continuation duopoly subgames, the payoff function of firm 1 in the mechanism with royalty scheme is

$$\begin{aligned} \Pi(x, z) &= F(r) (\pi_W(x) - R) + \int_r^z (\pi_W(x) - \beta(y)) dF(y) \\ &\quad + \int_z^c \pi_L(x, z, y) dF(y), \quad \text{where } R = \pi_W(r) - \pi_{nn}. \end{aligned} \quad (14)$$

The equilibrium β must be such that truth-telling is a Bayesian Nash equilibrium. Therefore, β must be such that $x = \arg \max_z \Pi(x, z)$, for all $x \in [r, c]$.

Proposition 3. *The equilibrium transfer rule β (bid function) that induces truth-telling as a Bayesian Nash equilibrium for all $x \geq r$ is strictly increasing and equal to*

$$\beta(x) = \pi_W(x) - \pi_L^*(x) + \frac{1}{f(x)} \int_x^c \partial_z \pi_L(x, z, y)|_{z=x} dF(y). \quad (15)$$

Moreover, $\beta(x) < \beta_n(x), \forall x \in [r, c)$, and $\beta_n(x) - \beta(x)$ is decreasing in x with $\beta(c) = \beta_n(c)$. And firms play a unique cutoff participation strategy with a cutoff value r .

⁶The “downward” deviations, $z \leq x$, yield the same differential equation.

Proof. 1) Assuming, as a working hypothesis that firms play a cutoff participation strategy, induced by R (as defined in Lemma 2). Then, a firm with $x \geq r$ that reports message $z \geq x$ participates and obtains the payoff function $\Pi(x, z)$ stated in (14). Differentiating it with respect to z , and then setting $z = x$, one obtains,

$$(\pi_W(x) - \beta(x)) f(x) + \int_x^c \partial_z \pi_L(x, z, y)|_{z=x} dF(y) - \pi_L^*(x) f(x) = 0. \quad (16)$$

This implies (15) and, by Lemma 1, $\beta(x) < \beta_n(x)$ for all $x \geq r$.

2) The proof of the asserted monotonicity of β is in Appendix B. The proof of the monotonicity of $\beta_n(x) - \beta(x)$ is similar to the proof of Proposition 8 and hence omitted.

3) Having established sufficient conditions for the monotonicity of β one must also confirm that the underlying first-order conditions (16) yield a global maximum for each x . This is assured if the function $\Pi(x, z)$ is pseudoconcave in z , which requires $\partial_{zx}\Pi(x, z) \geq 0$.⁷ As we explain below, this “second-order condition” is satisfied if the innovator adopts a sufficiently high reserve price.

4) Finally, we need to confirm that the game has a unique cutoff participation strategy, which we prove in Appendix C. \square

From the point of view of the innovator, adding the royalty scheme has an adverse effect on the winner’s equilibrium transfer. This signalling effect has the following interpretation. If the introduction of the royalty scheme did not affect that equilibrium transfer, each firm would benefit from reporting a lower than the true cost reduction. This incentive to signal can only be eliminated by pointwise lowering the equilibrium β , and by introducing a sufficiently high reserve price. This indicates that the innovator faces a trade-off between income earned from transfers paid by the winner and royalty income.

To see why a sufficiently high reserve price is needed, we show that:

Proposition 4. $\Pi(x, z)$ is pseudoconcave if r is sufficiently large.

Proof. The cross derivative of $\Pi(x, z)$ is equal to

$$\partial_{zx}\Pi(x, z) = (\partial_x \pi_W(x) - \partial_x \pi_L(x, z, z)) f(z) + \int_z^c \partial_{xz} \pi_L(x, z, y) dF(y). \quad (17)$$

We show that it is positive when x, z are sufficiently large; therefore, since $z \geq x \geq r$, Π is pseudoconcave if r is sufficiently large.

Let $r \rightarrow c$. Then the integral on the RHS of (17) vanishes; however, the first term does not vanish and is positive. Indeed, as we show in Appendix D,

$$\lim_{x, z \rightarrow c} (\partial_x \pi_W(x) - \partial_x \pi_L(x, z, z)) = \frac{(4P'(Q) + P''(Q)Q)(q_{W_1}(c) - q_{L_1}(c, c, c))}{3P'(Q) + P''(Q)Q} > 0. \quad (18)$$

⁷A function of one variable is pseudoconcave if it is increasing to the left of the stationary point and decreasing to the right. Firms’ payoff function $\Pi(x, z)$ is pseudoconcave in z if $\partial_{zx}\Pi \geq 0$, for all x, z , because the sign of that cross derivative implies $z < x \Rightarrow \partial_z \Pi(x, z) > \partial_z \Pi(z, z) = 0$, and $z > x \Rightarrow \partial_z \Pi(x, z) < \partial_z \Pi(x, x) = 0$. Pseudoconcavity obviously implies that every stationary point is a global maximum.

Hence, $\partial_{zx}\Pi(x, z) > 0$ for $x = z = c$. By continuity, pseudoconcavity holds true for $x, z \geq r$ for sufficiently large r . \square

To obtain some insight into how large is a “sufficiently large” r , consider the model with linear demand. In that case we can strengthen Proposition 4 to:

Proposition 5. *Suppose demand is linear. Then, $\Pi(x, z)$ is pseudoconcave if and only if $r \geq r_{\min}$, where r_{\min} is implicitly defined as the unique solution of*

$$r - \frac{1 - F(r)}{f(r)} \frac{2}{3} = 0. \quad (19)$$

Specifically, if $F(x) = x/c$ (uniform distribution), $r_{\min} = 2c/5$.

Proof. Substituting $\pi_W(x)$, and $\pi_L(x, z, y)$ from Appendix E into (17), one can easily confirm that $\forall x, z \geq r$:

$$\partial_{zx}\Pi(x, z) = \frac{4}{3}zf(z) - \frac{8}{9}(1 - F(z)) \geq 0 \iff x, z \geq r \geq r_{\min}. \quad (20)$$

Existence and uniqueness of r_{\min} follow from the fact that the LHS of (19) is negative at $r = 0$, positive at $r = c$, and strictly increasing in r by the assumed hazard rate monotonicity. \square

Using the example of linear demand, a uniform distribution, $c = 0.49$, and $x = 0.296$, the role of the reserve price is illustrated in Figure 1. There we plot $\Pi(x, z)$ for $r = 0$ (figure on the left) and for $r = r_{\min}$ (figure on the right). If $r = 0$, the stationary point $\partial_z\Pi(x, z)|_{z=x} = 0$ is a local but not a global maximum for the assumed $x = 0.296$. Therefore, the best reply of a firm with $x = 0.296$ is to report a cost reduction equal to zero, thus become loser, yet obtain the innovation for free. Whereas if $r = r_{\min}$, that stationary point is a global maximum (see the right side of Figure 1).

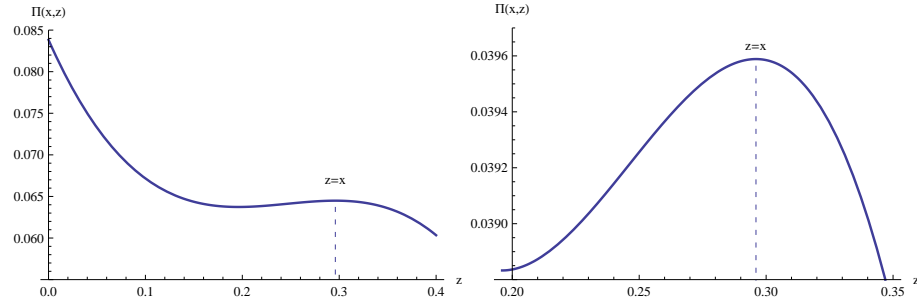


Figure 1: Left: Stationary point at $z = x$ is not a global maximum for $r = 0$; Right: Stationary point at $z = x$ is a global maximum for $r = r_{\min}$

The purpose of a sufficiently high reserve price is to deter firms from reporting a zero cost reduction. Of course, the second order conditions can also be assured by assuming that the support of the distribution is sufficiently bounded away from zero, employing the family of truncated distributions from below, denoted by H , with support $[d, c]$, $0 \leq d < c$, defined in (5).⁸

⁸To prove this, replace F by H in (20). Then, that condition becomes a condition concerning d , which is exactly the same as the condition on r_{\min} .

3.2.3. Is it profitable to add royalty contracts for the loser?

For the innovator, adding the royalty scheme has a benefit and a cost. The benefit is that he earns royalty income from the loser whenever both firms participate, without directly affecting the payoff of the winner in the downstream duopoly game, since, in equilibrium, the loser pays a royalty rate equal to his cost reduction. The cost is that the innovator needs to set a relatively high reserve price, which involves the risk that only one or even no firm participates, and that the equilibrium β is pointwise lower than β_n . Nevertheless, adding the royalty scheme is profitable for all standard probability distributions, even though one can construct examples in which it is not profitable. The latter occurs if the probability distribution exhibits a high concentration on low cost reductions.

The innovator's expected revenue in the mechanism with royalty scheme, $G(r)$, has three components: the expected transfer when only one firm participates, the expected transfer of the winner when both participate, and the expected transfer of the loser when both participate,

$$G(r) = 2F(r)(1 - F(r))R + \int_r^c \beta(y)g_2(y)dy + \int_r^c \int_r^x yq_L^*(x)g_{12}(x, y)dydx.$$

Define $\Delta(r) := G(r) - G_n(r)$. Substituting $G_n(r)$ (see (7)), β , and $R = \pi_W(r) - \pi_{nn}$, one finds after changing the order of integration and a bit of rearranging:

$$\begin{aligned} \Delta(r) &= 2 \int_r^c \int_y^c \left(yq_L^*(x)f(y) + (1 - F(y)) \partial_z \pi_L(y, z, x)|_{z=y} \right) dF(x)dy \\ &= 2 \int_r^c \int_y^c q_L^*(x) \left(y - \frac{1 - F(y)}{f(y)} \gamma(x) \right) dF(x)dF(y). \end{aligned} \quad (21)$$

To obtain further results, we now assume that the demand function is linear. In that case, γ is constant and equal to $4/3$ (see Lemma 1).

Proposition 6 (Sufficient Condition). *Consider truncations of F from below, $H : [d, c] \rightarrow [0, 1]$, $d \geq 0$. Then, there exists $d^* \in (0, c)$ such that $\Delta(r) > 0$, $\forall r$ and $\forall d \geq d^*$. Hence, $G(r^*) > G_n(r_n^*)$, where r^* is the maximizer of $G(r)$ and r_n^* that of $G_n(r)$.*

Proof. Define

$$\phi(x) := x - \frac{(1 - H(x))}{H'(x)} \gamma = x - \frac{4(1 - F(x))}{3f(x)}.$$

By the assumed hazard rate monotonicity, ϕ is strictly increasing. Therefore, $\phi(x) \geq 0, \forall x \in [d, c]$ if and only if $\phi(d) \geq 0$. Because $\phi(0) < 0$, $\phi(c) > 0$, and ϕ is strictly increasing, it follows that there exists a unique d^* such that $\phi(d^*) = 0$, and we conclude that $\Delta(r) > 0, \forall r \in [d, c]$ if $d > d^*$. For example, if F is the uniform distribution, then $d^* = 4c/7$. \square

In order to understand intuitively why truncations from below guarantee the profitability of the royalty scheme, note that (again assuming linear demand):

Proposition 7. *Truncations from below do not affect the gap between $\beta_n(x)$ and $\beta(x)$.*

Proof.

$$\begin{aligned}
\beta_n(x) - \beta(x) &= \frac{1}{H'(x)} \frac{4}{3} \int_x^c q_L^*(y) H'(y) dy \\
&= \frac{1 - F(d)}{f(x)} \frac{4}{3} \int_x^c q_L^*(y) \frac{f(y)}{1 - F(d)} dy \\
&= \frac{1}{f(x)} \frac{4}{3} \int_x^c q_L^*(y) f(y) dy.
\end{aligned}$$

□

Now recall that truncations from below imply a first-order stochastic dominance shift of the distribution function. Therefore, in view of Proposition 7, truncations from below shift probability mass from cost reductions at which the royalty scheme involves a large loss in revenue earned from the winner, because of a large gap between β_n and β , to those where that loss is small. At the same time, equilibrium royalty rates are smaller for low than for high cost reductions. Therefore, truncations from below also increase royalty income. Both effects unambiguously contribute to increase the profitability of the royalty scheme.

Altogether, adding the royalty scheme is profitable for many standard probability distributions, including the uniform distribution with support $[0, c]$. Starting from the uniform distribution, one finds that if probability mass is shifted to high cost reductions, the royalty scheme becomes even more profitable. Whereas, if the probability distribution exhibits a sufficiently high concentration on low cost reductions, the royalty scheme becomes less profitable, and the revenue ranking can be reversed.

We close with two examples, one in which the royalty scheme is profitable, and one where it is not. Both examples assume a truncated exponential distribution and $c = 0.3$. These distributions are consistent with the assumed log-concavity of reliability functions. Obviously, the probability distribution in Figure 2 exhibits a concentration on high cost reductions; whereas that in Figure 3 exhibits a concentration on low cost reductions.⁹

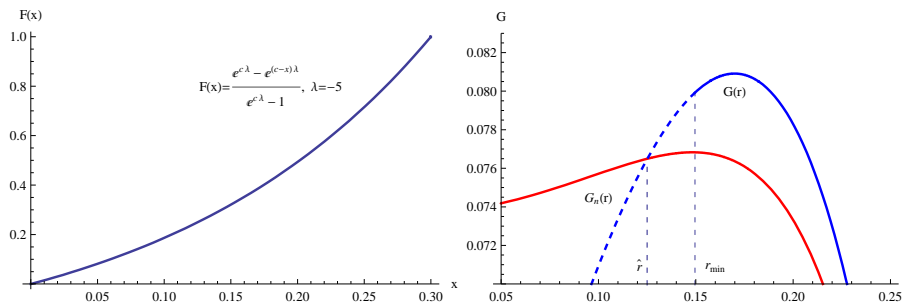


Figure 2: Truncated exponential distribution with a concentration on high cost reductions: c.d.f. (left) and the associated expected revenue of the innovator (right)

Figures 2 and 3 additionally plot the innovator's expected revenue for the corresponding probability distributions. Evidently, as one skews the distribution

⁹In these graphs we denote the root of $\Delta(r)$ by \hat{r} .

towards high cost reductions, the royalty scheme is highly profitable; whereas, if one concentrates probability mass on low cost reductions, the revenue ranking is reversed and it no longer pays to adopt the royalty scheme.

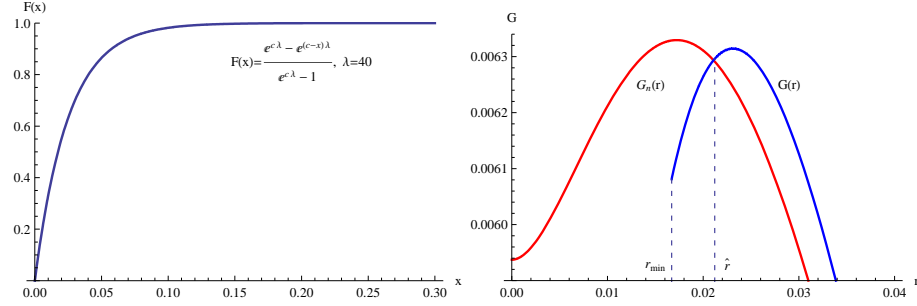


Figure 3: Truncated exponential distribution with a concentration on low cost reductions: c.d.f. (left) and the associated expected revenue of the innovator (right)

4. Model II: Incomplete information in the duopoly game

We now turn to the more plausible model in which firms do not learn each other's cost reduction after licenses have been awarded and before they play the duopoly game. In this case, the duopoly game is one of incomplete information, and firms' messages in the licensing game affect both the beliefs of the innovator and of the rival firm. Like in model I, the innovator sets the royalty rate equal to the cost reduction reported by the loser; yet, unlike in model I, firms also use their beliefs to predict their rival's cost reduction and the royalty rate paid by the loser.

Therefore, like in model I, firms would like to signal weakness to the innovator, and make him believe that their cost reduction is "low", because a low cost reduction translates into a low royalty rate in the event when they lose. However, unlike in model I, firms would also like to signal strength to their rival and make him believe that its cost reduction is "high", since this induces the rival to play less aggressively in the duopoly game. Both signaling considerations affect the equilibrium transfer rule β .

In the following we solve model II, employing the same methodology as in model I. We show that model II yields more general results, since none of our results assume linearity of inverse demand, and adding the royalty scheme is more profitable in model II than in model I.

Since the game without royalty scheme corresponds to the game analyzed by Goeree (2003) and Das Varma (2003), we focus on the game with royalty scheme and only mention casually what changes if no royalty scheme is adopted.

4.1. Downstream duopoly "subgames"

Suppose firm 1 has drawn cost reduction x but reports cost reduction $z \geq x$, while firm 2 tells the truth.¹⁰ In the continuation duopoly game, the following "subgames" occur, depending upon x, z and the true cost reduction of firm 2, y .

¹⁰Like in model I, the case of $z \leq x$ is slightly different, yet yields the same payoff function, $\Pi(x, z)$, and differential equation, and hence is omitted.

4.1.1. *When both firms participated and firm 1 won*

Let $z > y$ and $x, y \geq r$. Firm 1 privately knows its cost reduction is x ; whereas firm 2 (the loser) believes that the winner's cost reduction is z . Therefore, firm 2 believes to play a duopoly subgame with the profile of unit costs $(c_1, c_2) = (c - z, c)$. Denote the associated equilibrium strategies of that game the loser believes to play by $(q_W(z), q_{L_2}(z))$.

Firm 1 anticipates that the loser plays $q_{L_2}(z)$. But since firm 1 privately knows that its cost reduction is x rather than z it plays the best reply:

$$q_{W_1}(x, z) = \arg \max_q (P(q + q_{L_2}(z)) - c + x) q. \quad (22)$$

The reduced form profit function of firm 1, conditional on having won, is

$$\pi_W(x, z) := (P(q_{W_1}(x, z) + q_{L_2}(z)) - c + x) q_{W_1}(x, z).$$

4.1.2. *When both firms participated and firm 1 lost*

Let $y > z$ and $x, y \geq r$. Firm 2 believes to play a Cournot duopoly subgame with unit costs $(c_1, c_2) = (c, c - y)$. Denote the associated equilibrium strategies of the game that firm 2 (the winner) believes to play by $(q_L(y), q_{W_2}(y))$.

If the royalty scheme is adopted, firm 1 privately knows that its cost reduction is x yet pays a royalty rate z that exceeds its cost reduction x . Therefore, firm 1 plays the following best reply to $q_{W_2}(y)$:

$$q_{L_1}(x, z, y) = \arg \max_q (P(q + q_{W_2}(y)) - c + x - z) q. \quad (23)$$

The associated reduced form profit function of firm 1 conditional on having lost is then $\pi_L(x, z, y) := (P(q_{W_2}(y) + q_{L_1}(x, z, y)) - c + x - z) q_{L_1}(x, z, y)$. Also note that on the equilibrium path, for $z = x$, that payoff is only a function of the cost reduction of firm 2, y ; therefore, we write $q_L^*(y) := q_{L_1}(x, z, y)|_{z=x}$ and $\pi_L^*(y) := \pi_L(x, z, y)|_{z=x}$.

Whereas if no royalty scheme is used, the equilibrium play of firm 1 depends only on its rival's cost reduction y . Hence, the reduced form profit function of firm 1 conditional on having lost is $\pi_L(y) := (P(q_{W_2}(y) + q_L(y)) - c) q_L(y)$.

We stress that in model II the equilibrium output of firm 2 in the event when firm 1 has lost, q_{W_2} , is only a function of its own cost reduction, y , whereas in model I it is a function of x , z , and y . This is due to the fact that in model II both firm 2 and the innovator believe that the cost reduction of firm 1 is equal to z , and therefore firm 2 believes that the profile of effective unit costs is $(c_1, c_2) = (c - z + z = c, c - y)$. Whereas in model I, the innovator believes that the cost reduction of firm 1 is equal to z but firm 2 knows that it is equal to x ; therefore, firm 2 believes that the profile of effective unit costs is $(c_1, c_2) = (c - x + z, c - y)$.

4.1.3. *When at least one firm did not participate*

If no firm participated, the game is just the default game without innovation, and the equilibrium profit of firm 1 is π_{nn} . If firm 1 was the only one that participated, its equilibrium profit is the same as in subgame 4.1.1, and if firm 2 was the only one that participated, the equilibrium profit of firm 1 is the same as in the game without royalty scheme, and is exclusively a function of the winner's cost reduction, $\pi_L(y)$, as explained in Section 4.1.2.

Lemma 3. *In the relevant duopoly subgames, 1) $\partial_z \pi_L(x, z, y)|_{z=x} = -q_L^*(y)$ and 2) $\frac{d}{dx} \pi_W(x, x) > 0$, $\pi_L^*(y) < 0$.*

Proof. The proof of part 1) is the same as the proof of Lemma 1 (spelled out in Appendix A) except that in model II one has $\partial_z q_{W_2}(\cdot) = 0$; therefore, $\gamma(y) = 1$, see (12). Part 2) follows from the envelope theorem:

$$\begin{aligned} \frac{d}{dx} \pi_W(x, x) &= \left(P'(\cdot) \partial_x q_{L_2}(z) q_{W_1}(x, z) + q_{W_1}(x, z) + P'(\cdot) \partial_z q_{L_2}(z) q_{W_1}(x, z) \right) \Big|_{z=x} \\ &= \left(q_{W_1}(x, z) + P'(\cdot) \partial_z q_{L_2}(z) q_{W_1}(x, z) \right) \Big|_{z=x} \quad (\text{since } \partial_x q_{L_2}(z) = 0) \\ &> 0 \quad (\text{since } P'(\cdot) < 0 \text{ and } \partial_z q_{L_2}(z) < 0) \\ \pi_L^*(y) &= P'(\cdot) \partial_y q_{W_2}(y) q_{L_1}(x, z, y) < 0 \quad (\text{since } \partial_y q_{W_2}(y) > 0). \end{aligned}$$

□

4.2. Licensing “subgame”

To solve the winner’s equilibrium transfer rule, β , we use the same solution procedure as in model I. Using the equilibria of the continuation duopoly subgames, the payoff function of firm 1 in the mechanism with royalty scheme is

$$\Pi(x, z) = F(r)(\pi_W(x, z) - R) + \int_r^z (\pi_W(x, z) - \beta(y)) dF(y) + \int_z^c \pi_L(x, z, y) dF(y),$$

where similar to model I, the relationship between the reserve price R and the cutoff value r is: $R = \pi_W(r, r) - \pi_{nn}$.

In the mechanism without royalty scheme, the payoff function is the same, except that in the last term $\pi_L(x, z, y)$ is replaced by $\pi_L(y)$, as explained in the above analysis of the relevant duopoly subgames.

Proposition 8. *The equilibrium transfer rules with and without royalty scheme, β, β_n (bid functions), are, for all $x \geq r$ (provided r is sufficiently large):*

$$\beta(x) = \beta_n(x) + \frac{1}{f(x)} \int_x^c \partial_z \pi_L(x, z, y) \Big|_{z=x} dF(y) \quad (24)$$

$$\beta_n(x) = \pi_W(x, x) - \pi_L^*(x) + \frac{F(x)}{f(x)} \partial_z \pi_W(x, z) \Big|_{z=x}. \quad (25)$$

Whereas β_n is strictly monotone increasing for all r , β is strictly monotone increasing only if r is sufficiently large. Moreover, $\beta_n(x) - \beta(x)$ is decreasing in x , and $\beta(x) < \beta_n(x), \forall x \in [r, c]$ with $\beta(c) = \beta_n(c)$.

Proof. For the solution and proof of monotonicity of β_n see Goeree (2003, Proposition 2).

The derivation of β is similar to that in model I.

To prove monotonicity of β , it is sufficient to show that $\beta(x) - \beta_n(x)$ is strictly monotone increasing, i.e. $(\beta(x) - \beta_n(x))'$, is positive, as we confirm below:

$$\frac{d}{dx} \left(\frac{1}{f(x)} \int_x^c \partial_z \pi_L(x, z, y) \Big|_{z=x} dF(y) \right)$$

$$\begin{aligned}
&= \frac{1}{f(x)} \int_x^c (\partial_{zx}\pi_L(x, z, y)|_{z=x} + \partial_{zz}\pi_L(x, z, y)|_{z=x}) dF(y) \\
&\quad - \partial_z\pi_L(x, z, x)|_{z=x} - \frac{f'(x)}{f(x)^2} \int_x^c \partial_z\pi_L(x, z, y)|_{z=x} dF(y) \\
&= -\partial_z\pi_L(x, z, x)|_{z=x} - \frac{f'(x)}{f(x)^2} \int_x^c \partial_z\pi_L(x, z, y)|_{z=x} dF(y) \quad (\text{step a}) \\
&> -\partial_z\pi_L(x, z, x)|_{z=x} + \frac{1}{1-F(x)} \int_x^c \partial_z\pi_L(x, z, y)|_{z=x} dF(y) \quad (\text{step b}) \\
&> -\partial_z\pi_L(x, z, x)|_{z=x} + \frac{1}{1-F(x)} \int_x^c \partial_z\pi_L(x, z, x)|_{z=x} dF(y) \quad (\text{step c}) \\
&= -\partial_z\pi_L(x, z, x)|_{z=x} + \partial_z\pi_L(x, z, x)|_{z=x} = 0.
\end{aligned}$$

The different steps in this assessment are explained as follows: step a) follows from the facts that $\partial_{zx}\pi_L(x, z, y)|_{z=x} = -\partial_x q_L^*(y) = 0$ and $\partial_{zz}\pi_L(x, z, y)|_{z=x} = -\partial_z q_L^*(y) = 0$ (see Lemma 3); step b) follows from the assumed log-concavity of the reliability function, which implies that $f'(x) > -f(x)^2/(1-F(x))$, together with the fact that $\partial_z\pi_L(x, z, y)|_{z=x} < 0$; step c) follows from the fact that $\partial_z\pi_L(x, z, y)|_{z=x} = -q_L^*(y)$ (by Lemma 3), which is monotone increasing in y . Therefore, the proof of the monotonicity of β applies to all concave inverse demand functions. Also note that $\beta(x) < \beta_n(x)$ by Lemma 3.

The role of the reserve price, or the cutoff value r , to assure that the second-order conditions are satisfied, is explained below. \square

Similar to model I, we find that a sufficiently high reserver price is required for pseudoconcavity of the payoff function $\Pi(x, z)$. Unlike in model I, this condition is not only sufficient but also necessary.¹¹

Proposition 9. $\Pi(x, z)$ is pseudoconcave if and only if r is sufficiently large.

Proof. The cross derivative of the payoff function is

$$\begin{aligned}
\partial_{zx}\Pi(x, z) &= (\partial_x\pi_W(x, z) - \partial_x\pi_L(x, z, z))f(z) + \partial_{xz}\pi_W(x, z)F(z) \\
&\quad + \int_z^c \partial_{xz}\pi_L(x, z, y)dF(y). \tag{26}
\end{aligned}$$

1) Sufficiency: We show that this cross-derivative is positive if x, z are sufficiently large. Therefore, Π is pseudoconcave if r is sufficiently large.

Let $r \rightarrow c$. Then the integral on the RHS of (26) vanishes; however, the sum of the first and second terms on the RHS of (26) is positive. By the envelope theorem,

$$\partial_x\pi_W(x, z) = q_{W_1}(x, z), \quad \partial_x\pi_L(x, z, z) = q_{L_1}(x, z, z). \tag{27}$$

Therefore, $\lim_{x, z \rightarrow c} (\partial_x\pi_W(x, z) - \partial_x\pi_L(x, z, z)) = q_{W_1}(c, c) - q_{L_1}(c, c, c) > 0$. Since $\partial_{xz}\pi_W(x, z)|_{x=z=c} > 0$, it follows that for $x = z = c$, $\partial_{zx}\Pi(x, z) > 0$. By continuity, pseudoconcavity holds true for all $x, z \geq r$ when r is sufficiently large.

¹¹In model I this condition was only sufficient, whereas necessity applies only in the case of linear demand.

2) Necessity: We show that pseudoconcavity is violated if r is not sufficiently large. For this purpose, suppose $r = 0$. Consider a firm with cost reduction $x > 0$ that reports $z = 0$. We show that $\partial_z \Pi(x, z)|_{z=0} < 0$, which contradicts pseudoconcavity, since pseudoconcavity requires that $\Pi(x, z)$ is monotone increasing for all $z < x$.

Differentiating $\Pi(x, z)$ with respect to z , and using the candidate equilibrium β stated in Proposition 8, one has

$$\begin{aligned} \partial_z \Pi(x, z)|_{z=0} &= (\pi_W(x, 0) - \pi_L(x, 0, 0)) f(0) + (\pi_L^*(0) - \pi_W(0, 0)) f(0) \\ &\quad + \int_0^c \left(\partial_z \pi_L(x, z, y)|_{z=0} - \partial_z \pi_L(x, z, y)|_{x=0, z=0} \right) dF(y) \\ &= \int_0^c \int_0^x \partial_{\tau z} \pi_L(\tau, z, y)|_{z=0} d\tau dF(y) \\ &= \int_0^c \int_0^x \partial_z q_{L_1}(\tau, z, y)|_{z=0} d\tau dF(y) < 0. \end{aligned}$$

There, the second equality follows from $\pi_W(x, 0) \equiv \pi_L(x, 0, 0)$, $\pi_L^*(0) \equiv \pi_W(0, 0)$, the definition of the definite integral and a change in the order of differentiation of π_L ; the third equality follows from (27), and the final inequality follows from the fact that $\partial_z q_{L_1}(\tau, z, y)|_{z=0} < 0$. \square

Having established existence and monotonicity of β and β_n , we now show that:

Proposition 10. *The introduction of the royalty scheme reduces the equilibrium transfers $\beta(x)$ pointwise by a smaller amount than in model I.*

Proof. Distinguish the equilibrium transfer rules β in models I and II by writing β_n^I, β^I and β_n^{II}, β^{II} , and define $\Delta\beta^I := \beta_n^I(x) - \beta^I(x)$, and $\Delta\beta^{II} := \beta_n^{II}(x) - \beta^{II}(x)$. Recall that due to $\gamma(y) > 1$:

$$\partial_z \pi_L^I(x, z, y)|_{z=x} = -q_L^*(y)\gamma(y) < -q_L^*(y) \quad (\text{model I}) \quad (28)$$

$$\partial_z \pi_L^{II}(x, z, y)|_{z=x} = -q_L^*(y) \quad (\text{model II}) \quad (29)$$

Therefore, for all x from the intersection of the domains of these functions,

$$\begin{aligned} \Delta\beta^I &= -\frac{1}{f(x)} \int_x^c \partial_z \pi_L^I(x, z, y)|_{z=x} dF(y) \\ &= \frac{1}{f(x)} \int_x^c \gamma(y) q_L^*(y) dF(y) \quad (\text{by Lemma 1}) \\ &> \frac{1}{f(x)} \int_x^c q_L^*(y) dF(y) \quad (\text{since } \gamma(y) > 1) \\ &= \Delta\beta^{II} > 0. \end{aligned}$$

\square

The intuition for this result is as follows. Consider the equilibrium β in model II without royalty scheme, which by definition of an equilibrium exhibits equality of the marginal benefit and the marginal cost of an incremental change

in the transfer from $\beta_n^{II}(x)$ to $\beta_n^{II}(x + \varepsilon)$.¹² Now introduce the royalty scheme, while maintaining the candidate equilibrium β function. Then, the marginal benefit of reporting a higher cost reduction does not change whereas it gives rise to a positive marginal cost, due to the fact that when the firm loses, it pays a royalty rate that exceeds its cost reduction.

This is also true in model I. However, in model II the rival firm believes that the effective cost is equal to c , whereas in model I the rival knows that the royalty rate exceeds the cost reduction so that the effective cost is higher than c . To reestablish an equilibrium, the equilibrium transfer rule β has to be lowered pointwise, to bring the unchanged marginal benefit in balance with the higher marginal cost. But since that marginal cost is higher in model I than in model II, the equilibrium transfer rule β has to be lowered more in model I than in model II.

4.3. The innovator's expected revenue

The above result suggests that adding the royalty scheme is more profitable in model II than in model I, since it has a smaller adverse effect on the equilibrium β and thus on the innovator's revenue earned from the winner.

The expected revenues of the innovator in the mechanism with and without royalty scheme, $G(r)$ and $G_n(r)$, are

$$\begin{aligned} G_n(r) &= 2(1 - F(r))F(r)R + \int_r^c \beta_n(y)g_2(y)dy \\ G(r) &= 2(1 - F(r))F(r)R + \int_r^c \beta(y)g_2(y)dy + \int_r^c \left(\int_r^x yq_L^*(x)g_{12}(x, y)dy \right) dx \\ &= G_n(r) + \int_r^c (\beta(y) - \beta_n(y))g_2(y)dy + \int_r^c \left(\int_r^x yq_L^*(x)g_{12}(x, y)dy \right) dx. \end{aligned}$$

Define $\Delta(r) := G(r) - G_n(r)$. After substituting β and R and using the fact that $\partial_z \pi_L(y, z, x)|_{z=y} = -q_L^*(x)$ by Lemma 3, one obtains,

$$\Delta(r) = 2 \int_r^c \int_y^c q_L^*(x) \left(y - \frac{1 - F(y)}{f(y)} \right) dF(x)dF(y),$$

Proposition 11. *In model II the introduction of the royalty scheme increases the innovator's expected revenue more than in model I, for all r from the intersection of the domains of these functions.*

Proof. Distinguish Δ in models I and II by writing $\Delta^I(r)$ and $\Delta^{II}(r)$. Using (28), (29), and the fact that $\gamma(y) > 1$ in model I, one finds for all r from the intersection of the domains of these functions,

$$\begin{aligned} \Delta^I(r) &= 2 \int_r^c \int_y^c q_L^*(x) \left(y - \frac{1 - F(y)}{f(y)} \gamma(x) \right) dF(x)dF(y) \\ &< 2 \int_r^c \int_y^c q_L^*(x) \left(y - \frac{1 - F(y)}{f(y)} \right) dF(x)dF(y) = \Delta^{II}(r). \end{aligned}$$

□

¹²The benefit is the expected revenue from winning, net after deducting the expected price, and the cost is the loss in the event of losing.

The following results hold for all concave inverse demand functions (unlike in model I where similar results hold only for a class of linear demand functions).

Proposition 12 (Sufficient Condition). *Consider truncations of F from below, $H : [d, c] \rightarrow [0, 1], d \geq 0$. Then, there exists $d^* \in (0, c)$ such that $\Delta(r) > 0, \forall r$ and $\forall d \geq d^*$. Hence, $G(r^*) > G_n(r_n^*)$, where r^* is the maximizer of $G(r)$ and r_n^* that of $G_n(r)$.*

Proof. The proof is the same as that of Proposition 6, except that now $\phi(x) := x - (1-H(x))/H'(x)$, independent of the form of demand function. \square

Similar to model I:

Proposition 13. *Truncations from below do not affect the gap between $\beta_n(x)$ and $\beta(x)$. Therefore, truncations from below unambiguously increase the innovator's revenues earned from both the winner and the loser.*

The proof is similar to that of Proposition 7 and thus omitted.

Altogether we see that truncations from below shift probability mass to high values in the sense of first-order stochastic dominance, without changing the gap between the functions β and β_n . Therefore, truncations from below unambiguously raise the revenues earned from both firms. This is similar to what we observed in model I. However, in model I we had to assume linear demand, whereas in model II these results hold for all concave inverse demand functions.

Finally, we illustrate the performance of the royalty scheme with two examples, which employ the same demand function and probability distributions as in model I. In Figures 4, the figure on the left corresponds to the probability distribution plotted in Figure 2, and the figure on the right corresponds to the distribution plotted in Figure 3. Unlike in model I, adding the royalty scheme is profitable even for the probability distribution that exhibits a concentration on low cost reductions.

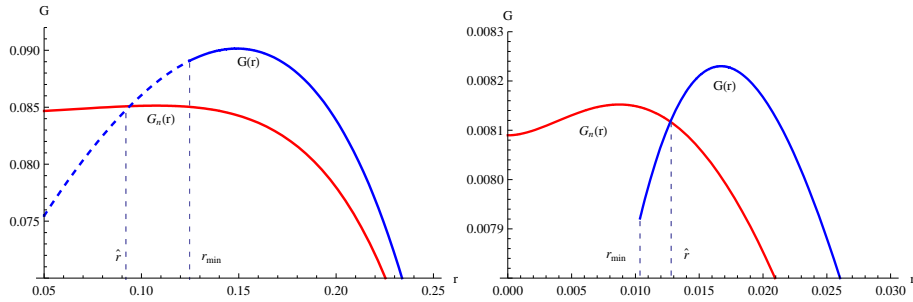


Figure 4: Expected revenue for truncated exponential distributions with a concentration on high cost reductions (left) and on low cost reductions (right)

Altogether, these examples indicate that adding the royalty scheme is more profitable in model II than in model I, and is particularly appealing if the probability distribution exhibits a concentration on high cost reductions, as already explained intuitively at the end of Section 3. The enhanced profitability in model II is due to the fact that the potential to signal strength to one's rival exerts an upward pressure on the equilibrium β function, which counters the downward pressure due to the potential to signal to the innovator.

5. Robustness

The current analysis focuses on two firms, and one may wonder whether it can be extended to more than two firms.

If one increases the number of firms, two issues come up: the innovator must optimize the number of fixed-fee licenses, and he must choose among various possible transfer rules that apply if more than one firm is awarded a fixed-fee license. The first of these issues has been at center stage in the classical literature on patent licensing under complete information (see Kamien, 1992; Giebe and Wolfstetter, 2008), but is more challenging in the present framework of incomplete information.

In order to confirm robustness, we carried out a full scale analysis of the case of three firms, assuming that the innovator employs a non-discriminating transfer rule β if two licenses are awarded. The analysis is a bit more involved because it covers a larger number of “subgames”. Due to space limitations we only review the results of a simple example, and relegate the detailed analysis to a technical supplement (see Fan, Jun, and Wolfstetter, 2011b).

Figure 5 illustrates our findings with an example that assumes linear demand, unit cost $c = 0.49$, and uniformly distributed cost reductions, $F(x) = x/c$. In the left panel of Figure 5 we plot the innovator’s expected revenues with and without royalty scheme, $G(r), G_n(r)$ for the case of *one* fixed-fee license.¹³ Evidently, adding the royalty scheme is profitable in this case. In the right panel we plot the innovator’s expected revenues for the case of *two* fixed-fee licenses, with and without royalty scheme, and the case of one fixed-fee license with royalty scheme.

Altogether, these figures indicate that the innovator’s expected profit is maximized if he supplies *one* fixed-fee license, combined with the proposed royalty scheme.¹⁴

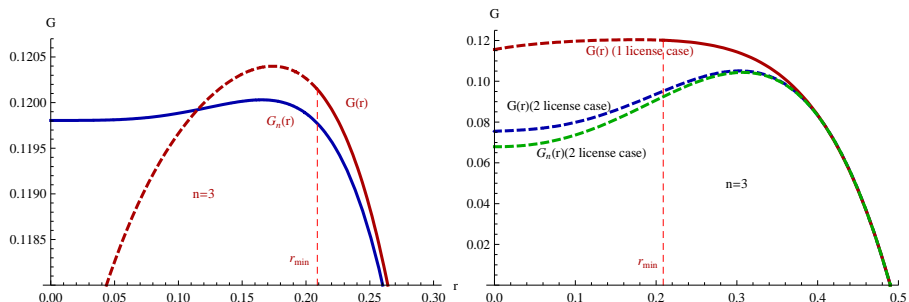


Figure 5: Left: one fixed-fee license with/without royalty; right: one vs. two fixed-fee licenses

Another concern is whether the private value paradigm is appropriate to analyze the cost reductions for firms that serve the same market and employ

¹³As one can easily confirm, second-order conditions (pseudoconcavity of $\pi(x, z)$ in z) are satisfied only if $r \geq r_{\min} = 0.2089$.

¹⁴Note, the different shapes of $G(r)$ (one license case with royalty scheme) in the left and the right panel are due to the different scales we use in these two figures.

similar technologies. If instead one assumes a common value framework, each firm's expected cost reduction is a function of the signals observed by all firms, and so is the royalty rate set by the innovator, unless the largest signal is a sufficient statistic of the unknown cost reduction, in which case only the largest signal matters. In a companion paper we show that the main results of the present paper also extend to this common value framework (see Fan, Jun, and Wolfstetter, 2011a).

6. Discussion

In the present paper we reconsider the licensing of a process innovation to a Cournot duopoly. Unlike the previous literature, we assume incomplete information of the private values type, and analyze a licensing mechanism that awards a limited number of fixed-fee licenses together with royalty contracts for those who failed to win a fixed-fee license.

We consider two specifications of the model: one in which cost reductions become common knowledge among firms after licenses have been awarded and before the oligopoly game is played, and one in which firms' cost reductions remain private information and firms can only update their beliefs about each other's cost reductions from observed messages. In both models, the innovator charges losers a royalty rate equal to their reported cost reduction.

Our main finding is that adding the royalty scheme exerts a downward pressure on equilibrium transfers paid by winners, which contributes to lower the innovator's expected revenue, yet is generally profitable unless cost reductions are highly concentrated on low values.

One crucial feature of our paper is that we restrict transfers from the loser. Rather than allowing arbitrary transfers, we assume that the royalty rate is equal to the cost reduction reported by the loser and that the mechanism does not prescribe outputs. As we show in our *Technical Supplement* (using the linear model with uniformly distributed cost reductions), these restrictions are justified on the following grounds:

If one would permit royalty rates that exceed the loser's cost reduction, the innovator would charge the loser an excessively high royalty rate in order to extract more surplus from the winner. In the extreme, the innovator would set the royalty rate so high that it induces losers' exit from the market.¹⁵ Of course, antitrust authorities do not permit such anti-competitive conduct.

Moreover, we show that a royalty rate that is smaller than the loser's cost reduction is not optimal for the innovator.

These observations also suggest that an economically meaningful theory of optimal licensing must incorporate restrictions that reflect antitrust concerns. In the present framework, solving the optimal mechanism design problem is further complicated by the fact that, unlike in the Myerson (1981), firms are subject to externalities because the payoffs of losers are not independent of the cost reduction of the winner. Solving the optimal mechanism design problem raises fundamental issues that are beyond the scope of the present paper.

¹⁵The same outcome could be achieved more directly if the mechanism could prescribe outputs.

Appendix A. Proof of Lemma 1

Proof. We show that $\partial_z \pi_L(x, z, y)|_{z=x} = -q_L^*(y)\gamma(y)$, where $\gamma(y) > 1$ for all y .

By the envelope theorem we have

$$\begin{aligned} \partial_z \pi_L(x, z, y)|_{z=x} &= -q_{L_1}(x, z, y)|_{z=x} \left(1 - P'(q_{W_2}(\cdot) + q_{L_1}(\cdot))\partial_z q_{W_2}(x, z, y)\right)|_{z=x} \\ &= -q_L^*(y)\gamma(y) \end{aligned}$$

$$\begin{aligned} \text{where } \gamma(y) &:= 1 - \left(P'(q_{W_2}(\cdot) + q_{L_1}(\cdot))\partial_z q_{W_2}(x, z, y)\right)|_{z=x} \\ &> 1 \quad (\text{because } P' < 0 \text{ and } \partial_z q_{W_2}(x, z, y)|_{z=x} > 0). \end{aligned}$$

Note, both $P'(\cdot)$ and $\partial_z q_{W_2}(x, z, y)|_{z=x}$ are only functions of y .

If demand is linear, $P'(\cdot) = -1$, $\partial_z q_{W_2}(x, z, y)|_{z=x} = 1/3$; hence, $\gamma(y) = 4/3$ (see Appendix E).

Next we show that $\pi'_W(x) > 0$ and $\pi'_L(y) < 0$. Again, using the envelope theorem and the fact that $q'_{L_2}(x) < 0$ and $q'^*_{W_2}(y) > 0$ one has:

$$\begin{aligned} \pi'_W(x) &= P'(\cdot)q'_{L_2}(x)q_{W_1}(x) + q_{W_1}(x) > 0 \\ \pi'_L(y) &= P'(\cdot)q'^*_{W_2}(y)q_L^*(y) < 0. \end{aligned}$$

□

Appendix B. Part 2 of the proof of Proposition 3

Compute $\beta'(x)$ from (15). By Lemma 1 and the assumed log-concavity of the reliability function, which implies that $f'(x) > -f(x)^2/(1-F(x))$, this derivative can be written as

$$\begin{aligned} \beta'(x) &= (\pi'_W(x) - \pi'_L(x)) + \frac{d}{dx} \left(\frac{1}{f(x)} \int_x^c \partial_z \pi_L(x, z, y)|_{z=x} dF(y) \right) \\ &> (\pi'_W(x) - \pi'_L(x)) + q_L^*(x)\gamma(x) - \frac{1}{1-F(x)} \int_x^c q_L^*(y)\gamma(y)dF(y). \end{aligned}$$

There, $\pi_W(x)$ and $\pi_L^*(x)$ are the equilibrium profits in the Cournot subgames when the winner's cost reduction is x :

$$\begin{aligned} \pi_W(x) &= \max_q (P(q + q_L^*(x)) - c + x)q, \\ \pi_L^*(x) &= \max_q (P(q_W^*(x) + q) - c)q, \end{aligned}$$

and $q_W^*(x)$ and $q_L^*(x)$ are the corresponding equilibrium outputs.¹⁶

The first-order conditions of the above maximization problem are:

$$\begin{aligned} P'(q_W^*(x) + q_L^*(x))q_W^*(x) + P(q_W^*(x) + q_L^*(x)) - c + x &= 0 \\ P'(q_W^*(x) + q_L^*(x))q_L^*(x) + P(q_W^*(x) + q_L^*(x)) - c &= 0. \end{aligned} \tag{B.1}$$

¹⁶Note that $q_W^*(x) = q_{W_1}(x) = q_{W_2}(x, z, y)|_{z=x=y}$, $q_L^*(x) = q_{L_2}(x) = q_{L_1}(x, z, y)|_{z=x=y}$.

Differentiating (B.1) w.r.t. x , one obtains

$$\begin{aligned} (P''(\cdot)q_W^*(x) + P'(\cdot))(q_W^{*\prime}(x) + q_L^*(x)) + P'(\cdot)q_W^{*\prime}(x) &= -1 \\ (P''(\cdot)q_L^*(x) + P'(\cdot))(q_W^*(x) + q_L^{*\prime}(x)) + P'(\cdot)q_L^{*\prime}(x) &= 0, \end{aligned}$$

from which one can derive

$$\begin{aligned} q_W^{*\prime}(x) &= -\frac{2P'(\cdot) + P''(\cdot)q_L^*(x)}{P'(\cdot)(3P'(\cdot) + P''(\cdot)(q_W^*(x) + q_L^*(x)))} \\ q_L^{*\prime}(x) &= \frac{P'(\cdot) + P''(\cdot)q_L^*(x)}{P'(\cdot)(3P'(\cdot) + P''(\cdot)(q_W^*(x) + q_L^*(x)))}. \end{aligned} \quad (\text{B.2})$$

By the envelope theorem, one obtains

$$\begin{aligned} \pi_W'(x) &= (P'(\cdot)q_L^{*\prime}(x) + 1)q_W^*(x) \\ \pi_L^{*\prime}(x) &= P'(\cdot)q_W^{*\prime}(x)q_L^*(x). \end{aligned} \quad (\text{B.3})$$

To compute $\gamma(x)$, consider the Cournot subgame in the event when both firms participate and firm 1 has lost. The first-order conditions of that subgame are:

$$\begin{aligned} P'(\cdot)q_{W_2}(\cdot) + P(\cdot) - c + y &= 0 \\ P'(\cdot)q_{L_1}(\cdot) + P(\cdot) - c + x - z &= 0. \end{aligned} \quad (\text{B.4})$$

Differentiating (B.4) w.r.t. z , one obtains

$$\partial_z q_{W_2}(x, z, y) = -\frac{P'(\cdot) + P''(\cdot)q_{W_2}(x, z, y)}{P'(\cdot)(3P'(\cdot) + P''(\cdot)(q_{W_2}(x, z, y) + q_{L_1}(x, z, y)))}. \quad (\text{B.5})$$

Setting $z = y = x$ and using definition (12), we get

$$\gamma(x) = 1 + \frac{P'(\cdot) + P''(\cdot)q_W^*(x)}{3P'(\cdot) + P''(\cdot)(q_W^*(x) + q_L^*(x))} \quad (\text{B.6})$$

Combining (B.2), (B.3) and (B.6), we have

$$\begin{aligned} (\pi_W'(x) - \pi_L^{*\prime}(x)) + q_L^*(x)\gamma(x) &= 2q_L^*(x) + q_W^*(x) \\ &+ \frac{(P'(\cdot) + P''(\cdot)q_L^*(x))q_W^*(x)}{3P'(\cdot) + P''(\cdot)(q_W^*(x) + q_L^*(x))} \\ &> 2q_L^*(x) + q_W^*(x) \quad (\text{since } P' \leq 0, P'' \leq 0). \end{aligned} \quad (\text{B.7})$$

Evaluation (B.5) at $z = x$ and using (12), one obtains $\gamma(y)$, which has the same form as (B.6) with x replaced by y . Hence, we have

$$-q_L^*(y)\gamma(y) = -2q_L^*(y) + \frac{(2P'(\cdot) + P''(\cdot)q_L^*(y))q_L^*(y)}{3P'(\cdot) + P''(\cdot)(q_W^*(y) + q_L^*(y))} > -2q_L^*(y). \quad (\text{B.8})$$

From (B.7) and (B.8), it follows that

$$\begin{aligned} \beta'(x) &> 2q_L^*(x) + q_W^*(x) - \frac{1}{1 - F(x)} \int_x^c 2q_L^*(y)dF(y) \\ &> 2q_L^*(x) + q_W^*(x) - \frac{1}{1 - F(x)} \int_x^c 2q_L^*(x)dF(y) \\ &= q_W^*(x) > 0, \end{aligned}$$

the second inequality holds because $q_L^*(\cdot)$ is a decreasing function.

Appendix C. Part 4 of the proof of Proposition 3

Proof. Consider a firm with cost reduction x . If that firm participates, and the other firm tells the truth (if it participates), that firm's payoff is $\Pi_p(x)$, whereas if it does not participate, its payoff is $\Pi_{np}(x)$:

$$\begin{aligned}\Pi_p(x) &= F(r) (\pi_W(x) - R) + \int_r^x (\pi_W(x) - \beta(y)) dF(y) + \int_x^c \pi_L^*(y) dF(y) \\ &= -F(r)R + F(x)\pi_W(x) - \int_r^x \beta(y) dF(y) + \int_x^c \pi_L^*(y) dF(y) \\ \Pi_{np}(x) &= F(r)\pi_{nn} + \int_r^c \pi_L^*(y) dF(y).\end{aligned}$$

Let

$$\begin{aligned}\psi(x) &:= \Pi_p(x) - \Pi_{np}(x) \\ &= -F(r)(\pi_{nn} + R) + F(x)\pi_W(x) - \int_r^x (\pi_L^*(y) + \beta(y)) dF(y).\end{aligned}$$

Differentiate ψ with respect to x , and one obtains, using (15):

$$\begin{aligned}\psi'(x) &= f(x)\pi_W(x) + F(x)\pi'_W(x) - (\pi_L^*(x) + \beta(x)) f(x) \\ &= F(x)\pi'_W(x) - \int_x^c \partial_z \pi_L(x, x, y) dF(y) > 0.\end{aligned}$$

By definition of r , one has $\psi(r) = 0$; hence, r is implicitly defined as the solution of (13), and we conclude that firms participate if and only if $x \geq r$. \square

Appendix D. Supplement to the proof of Proposition 4

Here we prove inequality (18): $\lim_{x, z \rightarrow c} (\partial_x \pi_W(x) - \partial_x \pi_L(x, z, z)) > 0$.
By the envelope theorem one has

$$\partial_x \pi_W(x) = q_{W_1}(x) (1 + P'(Q) \partial_x q_{L_2}(x)) \quad (\text{D.1})$$

$$\partial_x \pi_L(x, z, z) = q_{L_1}(x, z, z) (1 + P'(Q) \partial_x q_{W_2}(x, z, z)). \quad (\text{D.2})$$

Note, in the subgame that underlies the first equation, the profile of unit costs is $(c_1, c_2) = (c - x, c)$, whereas in that of the second equation it is $(c_1, c_2) = (c - x + z, c - z)$. In both cases, the sum of unit costs is equal to $2c - x$. By a well-known fact, the aggregate equilibrium output, Q , is only a function of the sum of unit costs (see Bergstrom and Varian, 1985). Therefore, Q is the same in both equations.

The first order conditions concerning the subgame that underlies the first equation are:

$$P(Q) + P'(Q)q_{W_1}(x) - (c - x) = 0 \quad (\text{D.3})$$

$$P(Q) + P'(Q)q_{L_2}(x) - c = 0. \quad (\text{D.4})$$

Differentiating (D.3) and (D.4) with respect to x and solving for $\partial_x q_{L_2}(x)$ one has

$$\partial_x q_{L_2}(x) = \frac{P'(Q) + P''(Q)q_{L_2}(x)}{P'(Q) (3P'(Q) + P''(Q)Q)}. \quad (\text{D.5})$$

Similarly, one finds

$$\partial_x q_{W_2}(x, z, z) = \frac{P'(Q) + P''(Q)q_{W_2}(x, z, z)}{P'(Q)(3P'(Q) + P''(Q)Q)}. \quad (\text{D.6})$$

Combining (D.1)-(D.6) and using the facts that $q_{L_1}(c, c, c) = q_{L_2}(c)$, $q_{W_1}(c) = q_{W_2}(c, c, c)$ proves inequality (18).

Appendix E. Model I with linear demand

In the duopoly games with royalty scheme, the equilibrium output strategies of firm 1 are: $q_{W_1}(x) = (1-c+2x)/3$, $q_{L_1}(x, z, y) = (1-c+2x-2z-y)/3$. The associated equilibrium profits are $\pi_W(x) = q_{W_1}(x)^2$ and $\pi_L(x, z, y) = q_{L_1}(x, z, y)^2$. The equilibrium profit when both firms do not participate is $\pi_{nn} = (1-c)^2/9$.

In the game without royalty scheme, $q_{L_1}(x, z, y)$ should be replaced by $q_L(y) = (1-c-y)/3$ and $\pi_L(x, z, y)$ by $\pi_L(y) = q_L(y)^2$.

The equilibrium β , β_n functions are,

$$\beta_n(x) = \frac{x(2-2c+x)}{3}, \quad \beta(x) = \beta_n(x) - \frac{4}{9f(x)} \int_x^c (1-c-y)dF(y).$$

The relationship between the reserve price R and the critical valuation r induced by R is: $R = \pi_W(r) - \pi_{nn} = 4(1-c+r)r/9$.

Appendix F. Model II with linear demand

In the game with royalty scheme, the equilibrium output strategies of firm 1, are $q_{W_1}(x, z) = (2-2c+3x+z)/6$, $q_{L_1}(x, z, y) = (2-2c+3x-3z-2y)/6$. The associated equilibrium profits are $\pi_W(x, z) = q_{W_1}(x, z)^2$ and $\pi_L(x, z, y) = q_{L_1}(x, z, y)^2$. The equilibrium profit when both firms do not participate, π_{nn} , is the same as in model I.

In the game without royalties, $q_{L_1}(x, z, y)$ should be replaced by $q_L(y) = (1-c-y)/3$ and $\pi_L(x, z, y)$ by $\pi_L(y) = (q_L(y))^2$.

The equilibrium transfer rules β , β_n are,

$$\beta_n(x) = \frac{x(2-2c+x)}{3} + \frac{(1-c+2x)F(x)}{9f(x)}$$

$$\beta(x) = \beta_n(x) - \frac{1}{3f(x)} \int_x^c (1-c-y)dF(y).$$

The relationship between R and r is the same as in model I.

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