On Coexistence of Vertical Separation and Vertical Integration[*]

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Abstract

We analyze Vertical Separation game under very mild assumptions on demand and cost functions. We proof that whenever diseconomy of scale is high for at least one firm, there are two equilibria, where one firm separates and another integrates. Also we show that asymmetric equilibria exist even in a completely symmetric game.

JEL classification: L22, L42.

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1 Introduction

Commonly, "strategic separation" refers to a manufacturer's decision to sell its good through an exclusive retailer, which takes a final decision on a quantity or a price, and "strategic delegation" means that an owner delegates a decision on a final output or a price to a manager within a firm. If the agent (the retailer or the manager) has no bargaining power then the principal (the manufacturer or the owner) completely controls his objective through contract terms. A strategic role of both separation and delegation is the same: by manipulating the agent's objective the principal alters its agent's as well as its rivals' behaviors in order to achieve more preferable

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outcome. In contrast to separation, "integration" implies that the profit maximizing principal takes a decision on a final output or price directly.

Classical papers of Vickers (1985), Sklivas (1987), Fershtman and Judd (1987), Bonanno and Vickers (1988) demonstrate that it is in a private interest of each principal to delegate a decision to an agent.[1] Hence in a unique equilibrium each principal takes an advantage of delegation and coexistence of vertical separation and vertical integration is never an equilibrium outcome. This result is robust in respect to a nature of competition (price or quantity competition), number of firms, and a demand function specification. Moreover, the result holds under different assumptions on a type of agents' objectives: linear combinations of profit and quantity (Vickers (1985)) or profit and revenue (Sklivas (1987), Fershtman and Judd (1987)), or retailer's profit (Bonanno and Vickers (1988)). In recent papers Jansen et all (2007) and Ritz (2008) assume that agents maximize a linear combination of the principal's profit and its market share and also obtain that in equilibrium each owner delegates a decision to a manager.

A possibility of coexistence of vertical separation and vertical integration is shown by Basu (1995) and Jansen (2003). To obtain the result authors assume that separated firms bear an additional fixed cost. That effectively implies that separation alters a firm's technology. Thus, in their models, separation relates to both a choice of technology and a choice of the agent's incentive scheme.

A common feature of all models mentioned above is assumption on linearity of cost functions. We extend existing analysis considering a demand and cost functions in general form. We analyze a case of duopolists competing in quantities and, following a standard approach, assume that each separated firm uses two-part tariff in a trade with its retailer. We demonstrate that vertical separation and vertical integration coexist whenever a cost function of at least one firm exhibits a sufficient degree of a scale diseconomy (or, in other words, a marginal cost curve is steep enough). Also we show the existence of asymmetric equilibria in a perfectly symmetric game.

The intuition for our results is the following. If only one firm separates then, effectively, it has an advantage of a first mover and gets a Stackelberg leader's profit. Thus, each firms prefers to separate given its rival integrates. If both firms separate it may make a competition between

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[1] If firms compete in prices delegation leads to a Pareto efficient outcome while if firms compete in quantities delegation lowers a profit of at least one firm.

their retailers very strong and under a decreasing economy of scale it leads to profits lower than a Stackelberg follower's profit. In this case each firm prefers to integrate given its rival separates and therefore there exist two equilibria, where one firm separates and another integrates.

The rest of the paper is organized as follows. Section 2 describes the model and characterizes subgame outcomes. Section 3 provides equilibrium analysis and main results and Section 4 concludes. All proofs are relegated to Appendix.

2 Model

We consider two firms, $i = 1, 2$, producing homogeneous good and competing in quantities. A demand function $P(Q)$, with $Q = q_1 + q_2$,where q_i is an output of the firm i, and cost functions $C_i(q_i)$ $i = 1, 2$ are such that:

A1. $\exists \overline{Q} > 0$: $P(Q) > 0$ for $Q \in [0, \overline{Q})$ and $P(Q) = 0$ for $Q \geq \overline{Q}$; $P''(Q)$ is continuous; $P(0) = \overline{P} > 0$, $-P'(Q) > \delta > 0$, $P'(Q) + P''(Q)q_i < 0$ for $Q \in [0, \overline{Q})$.

A2. $C_i(q)$ is a twice continuously differentiable increasing convex function, $C_i(0) = 0$, $C_i'(0) = 0$, $0 < C_i''(Q) < b$ for all $q_i \in (0, \overline{Q}]$ and some $b > 0$.

A3. $P^{(3)}(Q) \geq 0$ for all $Q \in [0, \overline{Q})$.

Assumptions A1-A2 are sufficient conditions for existence of a unique equilibrium in a Cournot game[2] and together with Assumption 3 ensure existence of a solution in a whole game.

We consider the following two stage game. At the first stage, each firm i chooses $m_i \in \{Separate, Integrate\} \equiv \{S, I\}$. If $m_i = I$ then the firm i becomes the retailer of its own good. If $m_i = S$ then the firm i sets the terms of a two-part tariff contact $\{w_i, A_i\}$ where w_i is a per unit price and A_i is a franchise fee.

At the second stage, all decisions are observed[3] and retailers choose quantities simultaneously and independently to maximize their own profits.

The profit of the integrated firm i is $\pi_i = P(Q)q_i - C_i(q_i)$. If the firm i separates, its own and its retailer's profits are $\pi_i^F = w_i q_i(w_i) + A_i - C_i(q_i(w_i))$ and $\pi_i^R = P(Q)q_i - w_i q_i - A_i$,

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[2] See Van Long and Soubeyranb (2000) for details.

[3] We assume that decisions are observable and irreversible and thus there is no commitment problem. For a discussion of observability and commitment in a delegation game see Katz (1991), Bagwell (1995), Corts and Neher (2003).

respectively. Clearly, that in equilibrium the separated firm extracts whole its retailer's profit using franchise fee. Thus $\pi_i^{*R} = 0$ and $\pi_i^{*F} = P(Q^*)q_i^* - C_i(q_i^*)$, where $Q^*, q_i^*, \pi_i^{*F}, \pi_i^{*R}$ denote equilibrium values.

Let $\Gamma(P, C_1, C_2)$ denote the described above game for a given demand function $P(Q)$ and cost functions $C_1(q_1)$, $C_2(q_2)$.

Subgame outcomes

There are four subgames depending on the choice $m_i \in \{S, I\}, i = 1, 2$ of each firm at the first stage. If both firms integrate they play the Cournot game. A unique equilibrium is determined by first order conditions

$$\left\{ \begin{array}{c} P'q_i + P - C_i' = 0 \\ i = 1, 2. \end{array} \right. . \tag{1}$$

Let $\{q_1^{*C}, q_2^{*C}\}$ and $\{\pi_1^{*C}, \pi_1^{*C}\}$ denote equilibrium values and let q_i^{*M} be an output of the firm i if it were a monopolist.

If the firm 1 separates and the firm 2 integrates then the retailers' game

$$\left\{ \begin{array}{l} \max\limits_{q_1} \pi_1^R = Pq_1 - w_1 q_1 \\ \max\limits_{q_2} \pi_2^R = Pq_2 - C_2 \end{array} \right.$$

has first order conditions

$$\left\{ \begin{array}{l} P'q_1 + P - w_1 = 0 \\ P'q_2 + P - C_2' = 0 \end{array} \right. .$$

The retailer 2 has the Cournot reaction curve while a position of the retailer 1's reaction curve depends on the firm 1's choice of w_1. Thus, by choosing w_1, the firm 1 determines a point of intersection of reaction curves. Clearly, the optimal w_1 is such that an equilibrium outcome replicates the Stackelberg outcome of the $[I, I]$-subgame. Therefore, the solution of the $[S, I]$-subgame may be characterized as the following: $q_2^F(q_1)$ solves

$$P'q_2 + P - C_2' = 0$$

and q_1^L is such that

$$P'q_1 + P - C_1' + P'q_1 \frac{dq_2^F(q_1)}{dq_1} = 0. \tag{2}$$

By the implicit function theorem we have that

$$\frac{dq_2^F}{dq_1} = -\frac{P''q_2^F + P'}{P''q_2^F + 2P' - C_2''}.$$

Let $w_1^{*L}, \{q_1^{*L}, q_2^{*F},\}$ and $\{\pi_1^{*L}, \pi_2^{*F}\}$ be equilibrium values of the $[S, I]$-subgame. The same arguments and notation apply for the $[I, S]$-subgame.

If both firms separate then retailers maximization problems are

$$\max_{qi} \pi_i^R = Pq_i - w_i q_i, i = 1, 2$$

where w_1, w_2 are set by firms at the previous stage. As the feasibility constraint, $q_i \geq 0$, implies that $q_i = 0$ for all $w_i \geq \overline{P}$ we assume without loss of generality that $w_i \leq \overline{P}$. Then a solution of the retailers' problem is determined by the system of first order conditions:

$$\begin{cases} P'q_1 + P - w_1 = 0 \\ P'q_2 + P - w_2 = 0 \end{cases}. \tag{3}$$

The Jacobian matrix of (3) is

$$\mathbf{J} = \begin{pmatrix} P''q_1 + 2P' & P''q_1 + P' \\ P''q_2 + P' & P''q_2 + 2P' \end{pmatrix}$$

with $\det(\mathbf{J}) > 0$ for any (q_1, q_2) and by the implicit function theorem we have that

$$\frac{d\mathbf{q}}{d\mathbf{w}} = \frac{1}{\det(\mathbf{J})} \begin{pmatrix} P''q_2 + 2P' & -(P''q_1 + P') \\ -(P''q_2 + P') & P''q_1 + 2P' \end{pmatrix}, \tag{4}$$

where $\mathbf{q} = (q_1, q_2)$ and $\mathbf{w} = (w_1, w_2)$.

Let $\{q_1^s(w_1,w_2), q_2^s(w_1,w_2)\}$ denote the solution of (3). Then optimal values of w_1, w_2 satisfy:

$$\frac{\partial \pi_i^S}{\partial w_i} = P'\left(\frac{\partial q_i^s}{\partial w_i} + \frac{\partial q_j^s}{\partial w_i}\right) q_i^s + P\frac{\partial q_i^s}{\partial w_i} - C_i'\frac{\partial q_i^s}{\partial w_i} = 0, i \neq j. \tag{5}$$

Directly differentiating (5) in respect to w_i and using (4) one can obtain that $\partial^2\pi_i/\partial w_i^2 < 0$ under Assumptions 1 and 2 and provided $P^{(3)}(Q) \geq 0$. Therefore Assumptions 1-3 ensure existence of a pure strategy equilibrium in the $[S,S]$-subgame. It is convenient to rewrite (5) in the form

$$P'q_i^s + P - C_i' + P'q_i^s\left(\frac{\partial q_j^s}{\partial w_i}/\frac{\partial q_i^s}{\partial w_i}\right) = 0$$

where

$$\frac{\partial q_j^s}{\partial w_i}/\frac{\partial q_i^s}{\partial w_i} = -\frac{P''q_j^s + P'}{P''q_j^s + 2P'}. \tag{6}$$

Let $\{q_1^{*S}, q_2^{*S}\}$ and $\{\pi_1^{*S}, \pi_1^{*S}\}$ be the equilibrium values in the $[S,S]$-subgame. Now we may represent the game as the following table.

		Firm 2	
		Separate	*Integrate*
Firm 1	*Separate*	π_1^{*S}, π_2^{*S}	π_1^{*L}, π_2^{*F}
	Integrate	π_1^{*F}, π_2^{*L}	π_1^{*C}, π_2^{*C}

3 Equilibrium

First of all, let's note that the Stackelberg leader's profit is always greater than the Cournot profit, $\pi_i^{*L} > \pi_i^{*C}$, and therefore we have the following result.

Proposition 1 *The $[I,I]$-subgame is never played in equilibrium.*

The equilibrium is determined by a relation of manufacturers' profits π_i^{*L}, π_i^{*F} and π_i^{*S}. More specifically, in equilibrium $[S,I]$ is played if $\pi_2^{*F} \geq \pi_2^{*S}$; $[I,S]$ if $\pi_1^{*F} \geq \pi_1^{*S}$; $[S,S]$ if both $\pi_1^{*F} \leq \pi_1^{*S}$ and $\pi_2^{*F} \leq \pi_2^{*S}$ hold.

Now, let's consider a family of games $\{\Gamma(P, C_1, \alpha C_2)\}|_{\alpha>0}$.

Lemma 1 *As* $\alpha \to \infty$*, both* $q_2^{*F}(q_1, \alpha)$ *and* $dq_2^F/dq_1(q_1, \alpha)$ *uniformly converges to zero.*

The statement of Lemma is quite intuitive: the steeper is the firm 2's marginal curve (the greater is α) the more negligibly are both its output, $q_2^{*F}(q_1, \alpha)$, and its response to changes in q_1, $dq_2^F/dq_1(q_1, \alpha)$, for any $q_1 \in [0, \overline{Q})$.

In general, separation has two effects. First, it allows a firm to manipulate its retailer's reaction curve and thus gives a strategic advantage to a separated firm. Second, it makes competition between retailers stronger. This is due to the fact that retailers' reaction curves (determined by (3)) are steeper then firm's reaction curves (determined by (1)). While the first always increases the firm's profit, the second is harmful. Thus, a decision on separation depends on which effect dominates. The following proposition states that if a firm cost function exhibits high enough diseconomy of scale then the firm preferes to integrate given its rival separates.

Proposition 2 $\exists \overline{\alpha} > 0$ *such that for any* $\alpha \geq \overline{\alpha}$ *the game* $\Gamma(P, C_1, \alpha C_2)$ *has equilibrium where* $[S, I]$ *played at the first stage.*

The intuition for the result is the following. As the firm 2's cost function exhibits high diseconomy of scale, an advantage of manipulating the retailer's objective is small while more aggressive behavior of the retailer 1 lowers the firm 2's profit significantly, bellow the Stackelberg follower's profit. Thus, the firm 2 prefers to integrate given the firm 1 separates.

In contrast to Proposition 2, Proposition 3 states that if the firm 2 being inefficient separates then the firm 1 prefers to integrate.

Proposition 3 $\exists \widehat{\alpha} > 0$ *such that for any* $\alpha \geq \widehat{\alpha}$ *the game* $\Gamma(P, C_1, \alpha C_2)$ *has equilibrium where* $[I, S]$ *played at the first stage.*

The reason is the following. As the firm 2 is inefficient, the firm 1 being a Stackelberg follower gets almost monopolistic profit. If it separates then its own and the firm 2's retailers compete more aggressively, which leads to an excessive output and significantly lowers the firm 1's profit. Thus the firm 1 even being more efficient prefers to integrate given the firm 2 separates.

Combining Propositions 2 and 3 we have

Corollary 1 *For any $\alpha \geq \max\{\widehat{\alpha}, \overline{\alpha}\}$ the game $\Gamma\left(P, C_1, \alpha C_2\right)$ has two equilibria where one firm separates and another integrates.*

Thus, there exist *two equilibria* whenever at least *one firm's* marginal curve is sufficiently steep.

Let's note that Corollary 1 does not imply existence of asymmetric equilibria in a symmetric game. To cover this case we formulate the following proposition.

Proposition 4 $\exists \widehat{\alpha} > 0$ *such that for any $\alpha > \widehat{\alpha}$ the symmetric game $\Gamma\left(P, \alpha C, \alpha C\right)$ has two asymmetric equilibria with $[S, I]$ and $[I, S]$ played at the first stage.*

Proposition 4 says that in the symmetric game, whenever the slop of marginal cost curve for each firm is steep enough, the losses from more aggressive retailers' behavior dominates a gain from separation. Thus, each firm prefers to integrate given its rival separates and hence there exist two asymmetric equilibria in a symmetric game.

4 Conclusion

We consider a separation game under very mild assumptions on a demand and cost functions. We demonstrate that vertical integration and vertical separation coexist whenever diseconomy of scale is high enough at least for one firm. Also we demonstrate that there may exist two asymmetric equilibria in a completely symmetric game.

The intuition for results is the following. If only one firm separates then it gets a Stackelberg leader's profit while its rival gets a Stackelberg follower's profit. As the Stackelberg leader's profit is always greater than a Cournot profit, each firm prefers to separate given its rival integrates.

If both firms separate then they compete in per unit prices. Retailers' reaction curves in this case are steeper than integrated firms' reaction curves and therefore an increase in an output of own retailer leads to greater reduction in an output of the rival's retailer. This results in a tough competition in per unit prices between separated firms and may lead to an excessive production and low final profits.

If separated firm, say firm 2, is sufficiently inefficient (that is it has a steep enough marginal cost curve) then its rival, the firm 1, being a Stackelberg follower gets almost monopolistic profit. Separation is not profitable for the firm 1 as it would raise its retailer's output beyond a monopolistic output and thus would lower the firm 1's profit. If, in contrast, the firm 1 being more efficient separates then the firm 2 prefers to integrate as a tough competition in per unit prices would lower its profit below the Stackelberg follower's profit.

In a symmetric case, the higher is a diseconomy of scale the lower is a gain each firm get by manipulating own retailers objectives. Moreover the higher are losses from though competition. Thus if the diseconomy of scale is sufficiently high each firm prefers to integrate given its rival separates.

To obtain results we apply limit analysis. While this approach provides existence results, it does not allow derivation of sufficient conditions. We leave it for a future research.

A Appendix

Proof of Lemma 1. $q_2^F(q_1,\alpha)$ solves $P'q_2 + P - \alpha C_2'(q_2) = 0$ for all $\alpha > 0$ and all $q_1 \in [0,\overline{Q})$. Both $P'(\cdot)$ and $P(\cdot)$ are bounded, therefore $\alpha C_2'(q_2^F(q_1,\alpha))$ is bounded also for all $q_1 \in [0,\overline{Q})$ and thus $q_2^F(q_1,\alpha) \underset{\alpha\to\infty}{\to} 0$ uniformly. Moreover,

$$\lim_{\alpha\to\infty} \alpha C_2'(q_2^F(q_1,\alpha)) = \lim_{\alpha\to\infty} P(q_1) > 0$$

for all $q_1 \in [0,\overline{Q})$.

Now, let's show that

$$\lim_{\alpha\to\infty} \alpha C_2'(q_2^F(q_1,\alpha)) \in (0,\infty)$$

implies

$$\lim_{\alpha\to\infty} \alpha C_2^{''}(q_2^F(q_1,\alpha)) = \infty$$

for all $q_1 \in [0,\overline{Q})$. If $C''(0) = 0$ then by the Taylor theorem $\exists \eta \in [0,q]$: $C_i'(q) = C_i^{''}(\eta)q$. As $C_i'''(q) > 0$ for any $q > 0$, we have $C_i'''(q)$ is increasing at $q = 0$ and hence $C_i^{''}(\eta) < C_i^{''}(q)$ for small enough q. Thus,

$$\lim_{q\to 0} \frac{C_i'(q)}{C_i^{''}(q)} \leq \lim_{q\to 0} \frac{C_i^{''}(\eta)}{C_i^{''}(q)} q \leq \lim_{q\to 0} q = 0.$$

If $C''(0) > 0$ then $\lim\limits_{q\to 0} C_i'(q)/C_i^{''}(0) = 0$.

Therefore, for any $q_1 \in [0,\overline{Q})$ we have that

$$\lim_{\alpha\to\infty} \alpha C_2^{''}(q_2^F(q_1,\alpha)) = \lim_{\alpha\to\infty} \left(\alpha C_2^{'}(q_2) \frac{C_2^{''}(q_2)}{C_2^{'}(q_2)} \right)\Bigg|_{q_2 = q_2^F(q_1,\alpha)} =$$

$$= P(q_1) \left(\lim_{\alpha\to\infty} \frac{C_2^{'}(q_2)}{C_2^{''}(q_2)} \right)^{-1} \Bigg|_{q_2 = q_2^F(q_1,\alpha)} = \infty.$$

Finally, this provides that $\frac{dq_2^F}{dq_1} = -\frac{P''q_2^F + P'}{P''q_2^F + 2P' - \alpha C_2''}$ uniformly converges to zero as $\alpha \to \infty$. ■

Proof of Propositions 2 and 3. Let's consider $[S,I]$-subgame. By Lemma 1, $dq_2^F/dq_1 \to 0$ and $q_2^F(q_1,\alpha) \to 0$ uniformly as $\alpha \to \infty$ and therefore (2) uniformly converges to the monopolist's first order condition:

$$P'(Q)q_1 + P(Q) - C_1'(q_1) + P'(Q)q_1\frac{dq_2^F}{dq_1} \underset{\alpha\to\infty}{\to} P'(q_1)q_1 + P(q_1) - C_1'(q_1). \tag{7}$$

Thus, $\{q_1^{*L}(\alpha), q_2^{*F}(\alpha)\} \to \{q^{*M}, 0\}$ as $\alpha \to \infty$.

Now, let's consider $[S,S]$-subgame. As there exist an internal solution of $\partial\pi_2^S(\alpha)/\partial w_2 = 0$ for all $\alpha > 0$, we have that $\alpha C'(q_2^{*S})$ is bounded and thus $q_2^S(\alpha) \to 0$ as $\alpha \to \infty$. Therefore we have that $-\frac{P''(q_1^S)q_1^S + P'(q_1^S)}{P''(q_1^S)q_1^S + 2P'(q_1^S)} \to -\frac{1}{2}$ and, in the limit, the firm 1's first order condition takes a form:

$$P'(Q^S)q_1^S + P(Q^S) - C_1'(q_1^S) + P'(Q^S)q_1^S\left(\frac{\partial q_2^S}{\partial w_1^S}\Big/\frac{\partial q_1^S}{\partial w_1^S}\right) \underset{\alpha\to\infty}{\to}$$
$$\underset{\alpha\to\infty}{\to} P'(q_1^S)q_1^S/2 + P(q_1^S) - C_1'(q_1^S) = 0. \tag{8}$$

Comparing (7) and (8), we get that $\lim_{\alpha\to\infty} q_1^{*S}(\alpha) > q^{*M} = \lim_{\alpha\to\infty} q_1^{*L}(\alpha)$. Thus, if the firm 2 separates then the output of the firm 1 is higher than the Stackelberg leader's output. As the firm 2's profit, $\pi_2(q_1, q_2)$, decreases in q_1 we have that $\pi_2^{*F} = \max_{q_2} \pi_2(q_1^{*L}, q_2) > \max_{q_2} \pi_2(q_1^{*S}, q_2) = \pi_2^{*S}$ whenever $q_1^{*S} > q_1^{*L}$. Thus for α big enough $\pi_2^{*F} \geq \pi_2^{*S}$, which proofs Proposition 2.

Finally, applying similar argument to the $[I,S]$-subgame, we get that $\{q_1^{*F}(\alpha), q_2^{*L}(\alpha)\} \to \{q^{*M}, 0\}$ as $\alpha \to \infty$. Therefore, $\lim_{\alpha\to\infty} q_1^{*S}(\alpha) > q^M = \lim_{\alpha\to\infty} q_1^{*F}(\alpha)$ implies that for α big enough $\pi_1^{*F} \geq \pi_1^{*S}$, which proofs Proposition 3. ■

Proof of Proposition 4. Suppose the firm i separates and the firm j integrates. As $\alpha \to \infty$, both $q_i^{*L}(\alpha), q_i^{*F}(\alpha)$ converges to zero as well as

$$\frac{dq_j^F}{dq_i} = -\frac{P''q_j^F + P'}{P''q_j^F + 2P' - \alpha C''}$$

does. Thus, the term $P'(Q)q_i\left(dq_j^F/dq_i\right)$ has a higher order of smallness than q_i. Therefore, the first order conditions of the Stackelberg game

$$\begin{cases} P'(Q)q_i + P(Q) - \alpha C'(q_i) + P'(Q)q_i\frac{dq_j^F}{dq_i} = 0 \\ P'(Q)q_j + P(Q) - \alpha C'(q_j) = 0 \end{cases}$$

uniformly converges to the first order conditions of the Cournot game:

$$\left\{\begin{array}{l} P'q_i + P - \alpha C' = 0 \\ i = 1, 2 \end{array}\right. . \tag{9}$$

This means that

$$\lim_{\alpha\to\infty} \frac{q_i^L(\alpha)}{q^C(\alpha)} = \lim_{\alpha\to\infty} \frac{q_j^F(\alpha)}{q^C(\alpha)} = 1.$$

The equilibrium values of $\{q_1^S(\alpha), q_2^S(\alpha)\}$ goes to $\{0, 0\}$ as $\alpha \to \infty$ and thus

$$\frac{\partial q_j^s}{\partial w_i} / \frac{\partial q_i^s}{\partial w_i} = -\frac{P'' q_j^s + P'}{P'' q_j^s + 2P'} \to -1/2.$$

Therefore, the first order conditions of the $[S, S]$-subgame

$$\left\{\begin{array}{c} P'q_i^s + P - \alpha C_i' + P'q_i^s \left(\frac{\partial q_j^s}{\partial w_i^s} / \frac{\partial q_i^s}{\partial w_i^s}\right) = 0, \\ i, j = 1, 2, i \neq j \end{array}\right.$$

converges to the system

$$\left\{\begin{array}{c} P'q_i^s + P - \alpha C' - \frac{1}{2}P'q_i^s = 0, \\ i = 1, 2. \end{array}\right. \tag{10}$$

as $\alpha \to \infty$. Comparing (9) and (10) we obtain

$$\lim_{\alpha\to\infty} \frac{q_i^C(\alpha)}{q_i^L(\alpha)} = 1 > \lim_{\alpha\to\infty} \frac{q_i^C(\alpha)}{q_i^S(\alpha)},$$

which implies $q_i^L(\alpha) < q_i^S(\alpha)$ for α big enough and therefore $\pi_j^F > \pi_j^S, \pi_i^L > \pi_i^S$, which proves Proposition. ■

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