Technical Appendix to
Optimal Sticky Prices under Rational Inattention

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March 2007

1 Introduction

This Technical Appendix contains proofs of three results that appear in the paper “Optimal Sticky Prices under Rational Inattention.” In Section 2 of the Technical Appendix we derive equation (38). In Section 3 of the Technical Appendix we prove that, after the log-quadratic approximation to the profit function, Gaussian signals are optimal. In Section 4 of the Technical Appendix we derive the relevant equations for the problem studied in Section 8.2.

2 Equilibrium price level in the white noise case

In Section 5, we start from the guess

\[ p_t = \alpha q_t \]  \hspace{1cm} (1)

and we obtain the actual law of motion

\[ p_t^* = \left( 1 - 2^{-2\kappa^*_1} \right) \Delta_t, \]  \hspace{1cm} (2)

where

\[ \kappa^*_1 = \begin{cases} 
\kappa & \text{if } \frac{\sigma^2}{\left(\frac{14}{11}\right)^2} \sigma^2_z \geq 2^{2\kappa} \\
\frac{1}{2}\kappa + \frac{1}{4} \log_2 \left( \frac{\sigma^2}{\left(\frac{14}{11}\right)^2} \sigma^2_z \right) & \text{if } \frac{\sigma^2}{\left(\frac{14}{11}\right)^2} \sigma^2_z \in \left[ 2^{-2\kappa}, 2^{2\kappa} \right] \\
0 & \text{if } \frac{\sigma^2}{\left(\frac{14}{11}\right)^2} \sigma^2_z \leq 2^{-2\kappa}
\end{cases} \]
and
\[ \Delta_t = p_t + \frac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|} q_t \]
\[ = \left[ \alpha + \frac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|} (1 - \alpha) \right] q_t. \]

The equilibrium price level is the fixed point of the mapping between the perceived law of motion (1) and the actual law of motion (2). Since the optimal allocation of attention can be a corner solution we have to distinguish three possible cases.

First, suppose that in equilibrium firms allocate no attention to aggregate conditions, \( \kappa_1^* = 0 \). Then the actual law of motion for the price level is
\[ p_t^* = 0. \]

The fixed point of the mapping between the perceived law of motion and the actual law of motion is
\[ \alpha = 0. \quad (3) \]

At the fixed point
\[ \Delta_t = \frac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|} q_t, \]
which implies that \( \kappa_1^* = 0 \) is an optimal choice at the fixed point if and only if
\[ \frac{\sigma^2_\Delta}{(\hat{\pi}_{14}/\hat{\pi}_{11})^2 \sigma^2_z} = \left( \frac{\hat{\pi}_{14}}{\hat{\pi}_{11}} \right)^2 \frac{\sigma^2_q}{\sigma^2_z} \leq 2^{-2\kappa}. \]

Assuming \( \hat{\pi}_{13} > 0 \), the weak inequality can also be expressed as
\[ \frac{\hat{\pi}_{13} \sigma_q}{|\hat{\pi}_{14}| \sigma_z} \leq 2^{-\kappa}. \quad (4) \]

Hence, there exists an equilibrium with \( \kappa_1^* = 0 \) if and only if the parameters satisfy (4). The equilibrium is given by (3).

Second, suppose that in equilibrium firms allocate all attention to aggregate conditions, \( \kappa_1^* = \kappa \). Then the actual law of motion for the price level is
\[ p_t^* = (1 - 2^{-2\kappa}) \left[ \alpha + \frac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|} (1 - \alpha) \right] q_t. \]
The fixed point of the mapping between the perceived law of motion and the actual law of motion is characterized by the equation

\[
\alpha = (1 - 2^{-2\kappa}) \left( \alpha + \frac{\hat{\pi}_{13}}{|\pi_{11}|} (1 - \alpha) \right),
\]

which has the unique solution

\[
\alpha = \frac{(2^\kappa - 1) \frac{\hat{\pi}_{13}}{|\pi_{11}|}}{1 + (2^\kappa - 1) \frac{\hat{\pi}_{13}}{|\pi_{11}|}}.
\]  \hspace{1cm} (5)

At the fixed point

\[
\Delta_t = \frac{2^{2\kappa} \frac{\hat{\pi}_{13}}{|\pi_{11}|} q_t}{1 + (2^{2\kappa} - 1) \frac{\hat{\pi}_{13}}{|\pi_{11}|}},
\]

which implies that \( \kappa_1^* = \kappa \) is an optimal choice at the fixed point if and only if

\[
\frac{\sigma^2_{\Delta}}{(\frac{\hat{\pi}_{14}}{|\pi_{11}|})^2 \sigma^2_z} \geq 2^\kappa.
\]

Assuming \( \hat{\pi}_{13} > 0 \), the weak inequality can also be expressed as

\[
\frac{\hat{\pi}_{13} \sigma_q}{|\pi_{14}| \sigma_z} \geq 2^{-\kappa} + (2^\kappa - 2^{-\kappa}) \frac{\hat{\pi}_{13}}{|\pi_{11}|}.
\]  \hspace{1cm} (6)

Hence, there exists an equilibrium with \( \kappa_1^* = \kappa \) if and only if the parameters satisfy (6). The equilibrium is given by (5).

Third, suppose that in equilibrium firms allocate attention to aggregate and idiosyncratic conditions, \( \kappa_1^* \in (0,1) \). Then the actual law of motion for the price level is

\[
p_t^* = \left( 1 - 2^{-2\kappa_1^*} \right) \left[ \alpha + \frac{\hat{\pi}_{13}}{|\pi_{11}|} (1 - \alpha) \right] q_t
\]

\[
= \left( 1 - 2^{-\kappa} \sqrt{\frac{(\frac{\hat{\pi}_{14}}{|\pi_{11}|})^2 \sigma^2_z}{\sigma^2_{\Delta}}} \right) \left[ \alpha + \frac{\hat{\pi}_{13}}{|\pi_{11}|} (1 - \alpha) \right] q_t
\]

\[
= \left( 1 - 2^{-\kappa} \sqrt{\frac{(\frac{\hat{\pi}_{14}}{|\pi_{11}|})^2 \sigma^2_z}{\alpha + \frac{\hat{\pi}_{13}}{|\pi_{11}|} (1 - \alpha) \sigma^2_q}} \right) \left[ \alpha + \frac{\hat{\pi}_{13}}{|\pi_{11}|} (1 - \alpha) \right] q_t.
\]
The fixed point of the mapping between the perceived law of motion and the actual law of
motion is characterized by the equation

\[
\alpha = \left(1 - 2^{-\kappa} \sqrt{\frac{(\hat{\pi}_{14}/\pi_{11})^2 \sigma_z^2}{\alpha + \frac{\hat{\pi}_{13}}{\pi_{11}} (1 - \alpha)^2 \sigma_q^2}}\right) \left[ \alpha + \frac{\hat{\pi}_{13}}{\pi_{11}} (1 - \alpha) \right],
\]

which can also be written as

\[
\alpha = \left[ \alpha + \frac{\hat{\pi}_{13}}{\pi_{11}} (1 - \alpha) \right] - 2^{-\kappa} \sqrt{\frac{\left(\frac{\hat{\pi}_{14}}{\pi_{11}}\right)^2 \sigma_z^2}{\sigma_q^2}} \left[ \alpha + \frac{\hat{\pi}_{13}}{\pi_{11}} (1 - \alpha) \right].
\] (7)

Now there are two possibilities. The first possibility is \(\alpha + \frac{\hat{\pi}_{13}}{\pi_{11}} (1 - \alpha) > 0\). In this case, equation (7) becomes

\[
\alpha = \left[ \alpha + \frac{\hat{\pi}_{13}}{\pi_{11}} (1 - \alpha) \right] - 2^{-\kappa} \sqrt{\frac{\left(\frac{\hat{\pi}_{14}}{\pi_{11}}\right)^2 \sigma_z^2}{\sigma_q^2}},
\]

which has the unique solution

\[
\alpha = 1 - 2^{-\kappa} \sqrt{\frac{\left(\frac{\hat{\pi}_{14}}{\pi_{13}}\right)^2 \sigma_z^2}{\sigma_q^2}}.
\] (8)

At the fixed point

\[
\Delta_l = \left[ 1 - 2^{-\kappa} \sqrt{\frac{\left(\frac{\hat{\pi}_{14}}{\pi_{13}}\right)^2 \sigma_z^2}{\sigma_q^2}} + \frac{\hat{\pi}_{13}}{\pi_{11}} \right] - 2^{-\kappa} \sqrt{\left(\frac{\hat{\pi}_{14}}{\pi_{13}}\right)^2 \sigma_z^2} \sigma_q^2 q_t
\]

and \(\kappa_1^* \in (0, 1)\) is an optimal choice at the fixed point if and only if

\[
2^{-2\kappa} < \left(\frac{\hat{\pi}_{14}}{\pi_{11}}\right)^2 \sigma_z^2 + \frac{\hat{\pi}_{13}}{\pi_{11}} \left(2^{-\kappa} \sqrt{\left(\frac{\hat{\pi}_{14}}{\pi_{13}}\right)^2 \sigma_z^2} \sigma_q^2 \right)^2 < 2^{2\kappa}.
\]

Assuming \(\hat{\pi}_{13} > 0\), these inequalities can also be expressed as

\[
2^{-\kappa} < \frac{\hat{\pi}_{13} \sigma_q}{\pi_{14} \sigma_z} < 2^{-\kappa} + (2^\kappa - 2^{-\kappa}) \frac{\hat{\pi}_{13}}{\pi_{11}}.
\] (9)

The second possibility is \(\alpha + \frac{\hat{\pi}_{14}}{\pi_{11}} (1 - \alpha) < 0\). In this case, equation (7) becomes

\[
\alpha = \left[ \alpha + \frac{\hat{\pi}_{13}}{\pi_{11}} (1 - \alpha) \right] + 2^{-\kappa} \sqrt{\frac{\left(\frac{\hat{\pi}_{14}}{\pi_{13}}\right)^2 \sigma_z^2}{\sigma_q^2}}.
\]
which has the unique solution

\[ \alpha = 1 + 2^{-\kappa} \sqrt{\left( \frac{\hat{\pi}_{14}}{\hat{\pi}_{13}} \right)^2 \frac{\sigma_z^2}{\sigma_q^2}}. \]

At the fixed point

\[ \Delta_t = \left[ 1 + 2^{-\kappa} \sqrt{\left( \frac{\hat{\pi}_{14}}{\hat{\pi}_{13}} \right)^2 \frac{\sigma_z^2}{\sigma_q^2}} - \frac{\hat{\pi}_{13}}{\hat{\pi}_{11}} \left( 2^{-\kappa} \sqrt{\left( \frac{\hat{\pi}_{14}}{\hat{\pi}_{13}} \right)^2 \frac{\sigma_z^2}{\sigma_q^2}} \right) \right] q_t \]

and \( \kappa^*_1 \in (0, 1) \) is an optimal choice at the fixed point if and only if

\[ 2^{-2\kappa} < \left( \frac{\hat{\pi}_{14}}{\hat{\pi}_{11}} \right)^2 \frac{\sigma_z^2}{\sigma_q^2} < 2^{2\kappa}. \]

Assuming \( \hat{\pi}_{13} > 0 \), these inequalities can also be expressed as

\[ 2^{-\kappa} < -\frac{\hat{\pi}_{13} \sigma_q}{\hat{\pi}_{14} |\sigma_z|} < 2^{-\kappa} + \left( 2^\kappa - 2^{-\kappa} \right) \frac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|}. \]

The first inequality in (10) can never be satisfied. Hence, there exists an equilibrium with \( \kappa^*_1 \in (0, 1) \) if and only if the parameters satisfy (9). The equilibrium is given by (8).

Collecting results yields equation (38) in the paper. Note that there is always a unique linear rational expectations equilibrium.

### 3 Optimality of Gaussian signals

#### 3.1 The white noise case

So far we have only allowed Gaussian signals. Now we relax this assumption. We assume that the conditional distribution of the variables of interest up to time \( t \) given the signals up to time \( t \) has a density function. We continue to assume that the joint distribution of the variables of interest up to time \( t \) and the signals up to time \( t \) is independent of time. In this subsection, we assume that the variables of interest follow a white noise process. After the log-quadratic approximation to the profit function, Gaussian signals are optimal.

Let \( \kappa_2 \) denote the information flow allocated to idiosyncratic conditions

\[ \kappa_2 = \mathcal{I} \left( \{z_{it}\}; \{s_{2it}\} \right) \]

\[ = \lim_{T \to \infty} \frac{1}{T} I \left( z_i^T; s_{2i}^T \right). \]
The mutual information can be expressed as

\[
I (z^T_i; s^T_{2i}) = H (z_{i1}, \ldots, z_{iT}) - H (z_{i1}, \ldots, z_{iT} | s^T_{2i})
\]

\[
= H (z_{i1}) + \ldots + H (z_{iT}) - H (z_{i1}, \ldots, z_{iT} | s^T_{2i})
\]

\[
= H (z_{i1}) + \ldots + H (z_{iT}) - [H (z_{i1} | s^T_{2i}) + \ldots + H (z_{iT} | z_{i1}, \ldots, z_{iT-1}, s^T_{2i})]
\]

\[
\geq H (z_{i1}) + \ldots + H (z_{iT}) - [H (z_{i1} | s^T_{2i}) + \ldots + H (z_{iT} | s^T_{2i})]
\]

\[
= TI (z_{it} ; s^T_{2i}) .
\]

The first equality follows from the fact that mutual information equals the difference between entropy and conditional entropy. The second equality follows from the fact that the entropy of independent random variables equals the sum of their entropies. The third equality follows from the chain rule for entropy. The weak inequality follows from the fact that conditioning reduces entropy. See Cover and Thomas (1991), p. 232, for these results. The last equality follows from the stationarity assumption. Furthermore,

\[
I (z_{it} ; s^T_{2i}) = H (z_{it}) - H (z_{it} | s^T_{2i})
\]

\[
= H (z_{it}) - E \left[ H (z_{it} | s^T_{2i} = \tilde{s}^T_{2i}) \right]
\]

\[
\geq H (z_{it}) - \frac{1}{2} \log_2 \left( 2\pi e \sigma^2_{z | s^T_{2i} = \tilde{s}^T_{2i}} \right)
\]

\[
\geq H (z_{it}) - \frac{1}{2} \log_2 \left( 2\pi e E \left[ \sigma^2_{z | s^T_{2i} = \tilde{s}^T_{2i}} \right] \right)
\]

\[
= \frac{1}{2} \log_2 \left( 2\pi e \sigma^2_{z} \right) - \frac{1}{2} \log_2 \left( 2\pi e E \left[ \sigma^2_{z | s^T_{2i} = \tilde{s}^T_{2i}} \right] \right).
\]

The first equality follows from the fact that mutual information equals the difference between entropy and conditional entropy. The second equality follows from the definition of conditional entropy, where \( \tilde{s}^T_{2i} \) denotes a realization of \( s^T_{2i} \). The first weak inequality follows from the fact that the normal density maximizes entropy over all densities with the same variance. See Cover and Thomas (1991), chapter 11. The second weak inequality follows from Jensen’s inequality. The last equality follows from the equation for the entropy of a normal distribution. Together these results imply

\[
\kappa_2 \geq \frac{1}{2} \log_2 \left( \frac{\sigma^2_{z}}{E \left[ \sigma^2_{z | s^T_{2i} = \tilde{s}^T_{2i}} \right]} \right).
\]
After the log-quadratic approximation to the profit function, the expected period \( t \) loss in profits due to imperfect tracking of idiosyncratic conditions equals
\[
\frac{|\hat{\pi}_{11}|}{2} \left( \frac{\hat{\pi}_{14}}{\hat{\pi}_{11}} \right)^2 E \left[ (z_{it} - E[z_{it} | s_{21}])^2 \right] = \frac{|\hat{\pi}_{11}|}{2} \left( \frac{\hat{\pi}_{14}}{\hat{\pi}_{11}} \right)^2 E \left[ \sigma_z^2 | s_2^T = s_2^T \right].
\]

The weak inequality given above implies that
\[
E \left[ \sigma_z^2 | s_2^T = s_2^T \right] \geq 2^{-2\kappa_2} \sigma_z^2.
\]

It is easy to verify that a Gaussian white noise signal of the form \( s_{2it} = z_{it} + \psi_{it} \) attains this bound. See Section 5 of the paper or simply note that in this case all the weak inequalities given above hold with equality. Hence, a Gaussian white noise signal is optimal.

The same arguments yield that, after the log-quadratic approximation to the profit function, a Gaussian white noise signal of the form \( s_{1it} = \Delta_t + \varepsilon_{it} \) is optimal.

### 3.2 The general case

We now turn to the general case where the variables being tracked follow arbitrary stationary Gaussian processes. We again assume that the conditional distribution of the variables of interest up to time \( t \) given the signals up to time \( t \) has a density function. Furthermore, we continue to assume that the joint distribution of the variables of interest up to time \( t \) and the signals up to time \( t \) is independent of time. We also continue to assume that firms receive a long sequence of signals in period one. We prove the following result. After the log-quadratic approximation to the profit function, Gaussian signals are optimal.

Let \( \kappa_2 \) denote the information flow allocated to idiosyncratic conditions
\[
\kappa_2 = I(\{z_{it}\};\{s_{2it}\}) = \lim_{T \to \infty} \frac{1}{T} I(z_i^T; s_{2i}^T),
\]
where \( z_i^T \equiv (z_{i1}, \ldots, z_{iT}) \) and \( s_{2i}^T \equiv (s_{21}, s_{22}, \ldots, s_{2iT}) \). The mutual information can be
expressed as

\[
I(z_i^T; s_{2i}) = H(z_i^T) - H(z_i^T | s_{2i})
\]

\[
= H(z_i^T) - E[H(z_i^T | s_{2i} = \tilde{s}_{2i}^T)]
\]

\[
\geq H(z_i^T) - E\left[\frac{1}{2} \log_2 \left( (2\pi e^T \det \Omega_{zz|s_{2i}^T = \tilde{s}_{2i}^T} \right) \right]
\]

\[
= H(z_i^T) - \frac{1}{2} \log_2 \left( (2\pi e^T) \right) - \frac{1}{2} E \left[ \log_2 \left( \det \Omega_{zz|s_{2i}^T = \tilde{s}_{2i}^T} \right) \right]
\]

\[
\geq H(z_i^T) - \frac{1}{2} \log_2 \left( (2\pi e^T) \right) - \frac{1}{2} \log_2 \left( \det E \left[ \Omega_{zz|s_{2i}^T = \tilde{s}_{2i}^T} \right] \right). \tag{11}
\]

The first equality follows from the fact that mutual information equals the difference between entropy and conditional entropy. The second equality follows from the definition of conditional entropy, where \( \tilde{s}_{2i}^T \) denotes a realization of \( s_{2i}^T \). The weak inequality (11) follows from the fact that the multivariate normal density maximizes entropy over all densities with the same covariance matrix. See Cover and Thomas (1991), chapter 11. The weak inequality (12) follows from Ky Fan’s inequality which states that the log of the determinant of a symmetric nonnegative definite matrix is a concave function. See Cover and Thomas (1991), page 501. If \( z_i^T \) and \( s_{2i}^T \) have a multivariate normal distribution then the conditional distribution of \( z_i^T \) given \( s_{2i}^T \) is a normal distribution and the conditional covariance matrix of \( z_i^T \) given \( s_{2i}^T \) is independent of the realization of \( s_{2i}^T \). In this case, the weak inequalities (11) and (12) hold with equality. Hence, for a given expected conditional covariance matrix \( E \left[ \Omega_{zz|s_{2i}^T = \tilde{s}_{2i}^T} \right] \), the mutual information \( I(z_i^T; s_{2i}^T) \) is minimized by a multivariate normal distribution for \( z_i^T \) and \( s_{2i}^T \).

After the log-quadratic approximation to the profit function, the expected discounted sum of losses in profits due to imperfect tracking of idiosyncratic conditions equals

\[
E\left[ \sum_{t=1}^{\infty} \beta^t \frac{\hat{\beta}_{11}^t}{\hat{\pi}_{11}^t} \left( \frac{\hat{\pi}_{14}^t}{\hat{\pi}_{11}^t} \right)^2 (z_{it} - E[z_{it}|s_{2i}^t])^2 \right]
\]

\[
= \sum_{t=1}^{\infty} \beta^t \left( \frac{\hat{\beta}_{11}^t}{\hat{\pi}_{11}^t} \right)^2 \left( \frac{\hat{\pi}_{14}^t}{\hat{\pi}_{11}^t} \right)^2 E \left[ (z_{it} - E[z_{it}|s_{2i}^t])^2 \right] \tag{13}
\]

\[
= \sum_{t=1}^{\infty} \beta^t \left( \frac{\hat{\beta}_{11}^t}{\hat{\pi}_{11}^t} \right)^2 \left( \frac{\hat{\pi}_{14}^t}{\hat{\pi}_{11}^t} \right)^2 E \left[ E \left[ (z_{it} - E[z_{it}|s_{2i}^t])^2 | s_{2i}^t \right] \right]
\]

\[
= \sum_{t=1}^{\infty} \beta^t \left( \frac{\hat{\beta}_{11}^t}{\hat{\pi}_{11}^t} \right)^2 \left( \frac{\hat{\pi}_{14}^t}{\hat{\pi}_{11}^t} \right)^2 \left( \sigma_{z_{it}|s_{2i}^t}^2 \right). \tag{14}
\]
The assumption that the joint distribution of $z_t^i$ and $s^T_{2t}$ is independent of $t$ in combination with the assumption that firms receive a long sequence of signals in period one implies that $E\left[\sigma^2_{z_t^i|s^T_{2t}=\tilde{s}^T_{2t}}\right]$ is independent of $t$ for all $t \geq 1$. Equation (13) becomes

$$E\left[\sum_{t=1}^{\infty} \beta^t \frac{\hat{\pi}_{11}}{2} \left(\hat{\pi}_{14} \hat{\pi}_{11}\right)^2 (z_{it} - E[z_{it}|s^T_{2t}])^2\right] = \frac{\beta}{1-\beta} \left(\frac{\hat{\pi}_{14}}{\hat{\pi}_{11}}\right)^2 E\left[\sigma^2_{z_T|s^T_{2T}=\tilde{s}^T_{2T}}\right],$$

for any $T \geq 1$.

Equation (15) implies that the expected discounted sum of losses in profits due to imperfect tracking of idiosyncratic conditions only depends on the expected conditional variance $E\left[\sigma^2_{z_T|s^T_{2T}=\tilde{s}^T_{2T}}\right]$. Furthermore, the expected conditional variance $E\left[\sigma^2_{z_T|s^T_{2T}=\tilde{s}^T_{2T}}\right]$ is the $(T,T)$ element of the expected conditional covariance matrix $E\left[\Omega_{zz}|s^T_{2T}=\tilde{s}^T_{2T}\right]$. Finally, we proved above that, for any expected conditional covariance matrix $E\left[\Omega_{zz}|s^T_{2T}=\tilde{s}^T_{2T}\right]$, the mutual information $I(z^T_i; s^T_{2t})$ is minimized by a multivariate normal distribution for $z^T_i$ and $s^T_{2t}$. Hence, Gaussian signals are optimal.

The same arguments yield that Gaussian signals about aggregate conditions are optimal.

4 Attending to variables that reveal information about both aggregate and idiosyncratic conditions

Let the profit function be given by equation (19) in the paper. Then the price set by firm $i$ in period $t$ is given by equation (20) in the paper and the profit-maximizing price is given by equation (21) in the paper. For simplicity, consider the case where $q_t$ and $z_{it}$ follow Gaussian white noise processes and $p_t = \alpha q_t$. For ease of exposition, assume that $\left(\hat{\pi}_{14}/|\hat{\pi}_{11}|\right) = 1$. Suppose that firm $i$ can choose signals of the form

$$s_{1it} = \Delta_t + \omega z_{it} + \varepsilon_{it},$$
$$s_{2it} = \omega \Delta_t + z_{it} + \psi_{it},$$

where the parameter $\omega \geq 0$ and $\{\varepsilon_{it}\}$ and $\{\psi_{it}\}$ are idiosyncratic Gaussian white noise processes that are mutually independent and independent of $\{\Delta_t\}$ and $\{z_{it}\}$. The price set
by firm \( i \) in period \( t \) equals

\[
p_{it}^* = \mathbb{E}\left[ p_{it}^0 | s_{it}^t \right]
\]

\[
= \frac{(\omega^2 - 1) \left( \omega - 1 \right) \frac{\sigma^2}{\sigma_z^2} \sigma^2_z + \left( 1 + \omega \frac{\sigma^3}{\sigma_z^3} \right) \frac{\sigma^3}{\sigma_z^3}}{(\omega^2 - 1)^2 \frac{\sigma^2}{\sigma_z^2} \sigma^2_z + \left( 1 + \omega \frac{\sigma^2}{\sigma_z^2} \right) \frac{\sigma^2}{\sigma_z^2} + \left( \omega^2 \frac{\sigma^2}{\sigma_z^2} + 1 \right) \frac{\sigma^2}{\sigma_z^2} + 1} s_{1it}
\]

\[
+ \frac{(\omega^2 - 1)^2 \frac{\sigma^2}{\sigma_z^2} \sigma^2_z + \left( 1 + \omega \frac{\sigma^2}{\sigma_z^2} \right) \frac{\sigma^2}{\sigma_z^2} + \left( \omega^2 \frac{\sigma^2}{\sigma_z^2} + 1 \right) \frac{\sigma^2}{\sigma_z^2} + 1} s_{2it}.
\]

The expected period \( t \) loss in profits equals

\[
\frac{\mid \pi_{11} \mid}{2} \mathbb{E}\left[ (p_{it}^0 - p_{it}^*)^2 \right] = \frac{\mid \pi_{11} \mid}{2} \frac{(\omega - 1)^2 \frac{\sigma^2}{\sigma_z^2} \sigma^2_z + (\omega - 1)^2 \frac{\sigma^2}{\sigma_z^2} \sigma^2_z + \frac{\sigma^2}{\sigma_z^2}}{(\omega^2 - 1)^2 \frac{\sigma^2}{\sigma_z^2} \sigma^2_z + \left( 1 + \omega \frac{\sigma^2}{\sigma_z^2} \right) \frac{\sigma^2}{\sigma_z^2} + \left( \omega^2 \frac{\sigma^2}{\sigma_z^2} + 1 \right) \frac{\sigma^2}{\sigma_z^2} + 1}.
\]

The information flow equals

\[
\mathcal{I}(\{P_t, Z_{it}\}; \{s_{it}\})
\]

\[
= \mathcal{I}(\{p_t, z_{it}\}; \{s_{it}\})
\]

\[
= I(p_t, z_{it}; s_{it})
\]

\[
= H(s_{it}) - H(s_{it} | p_t, z_{it})
\]

\[
= H(s_{1it}, s_{2it}) - H(s_{1it}, s_{2it} | \Delta_t, z_{it})
\]

\[
= \frac{1}{2} \log_2 \left( (\omega^2 - 1)^2 \frac{\sigma^2}{\sigma_z^2} \sigma^2_z + \left( 1 + \omega \frac{\sigma^2}{\sigma_z^2} \right) \frac{\sigma^2}{\sigma_z^2} + \left( \omega^2 \frac{\sigma^2}{\sigma_z^2} + 1 \right) \frac{\sigma^2}{\sigma_z^2} + 1 \right). \quad (16)
\]

The second equality follows from the assumption that \( p_t, z_{it} \) and \( s_{it} = (s_{1it}, s_{2it}) \) follow white noise processes. The third equality follows from the fact that mutual information equals the difference between entropy and conditional entropy. The fourth equality follows from the fact that \( p_t \) and \( \Delta_t \) contain the same information in the white noise case. The fifth equality follows from the expressions for the entropy and the conditional entropy of a multivariate normal distribution.

Choosing the signal-to-noise ratios \( (\sigma^2_{\Delta}/\sigma^2_z) \) and \( (\sigma^2_z/\sigma^2_\psi) \) so as to minimize the expected period \( t \) loss in profits (16) subject to a constraint on the information flow (17) is a standard constrained minimization problem. The solution depends on \( \omega \) and \( (\sigma^2_{\Delta}/\sigma^2_z) \). Suppose that \( (\sigma^2_{\Delta}/\sigma^2_z) < 1 \). Then there is a critical value \( \bar{\omega} \in [0, 1) \). For \( \omega \in [0, \bar{\omega}) \) the firm decides to
receive both signals. For \( \omega \in [\bar{\omega}, 1) \) the firm decides to receive only signal two. At \( \omega = 1 \) the firm is indifferent between receiving only signal two and receiving only signal one. For \( \omega \in (1, \frac{1}{\bar{\omega}}] \) the firm decides to receive only signal one. For \( \omega > \frac{1}{\bar{\omega}} \) the firm decides to receive both signals. So long as \( \omega \neq 1 \), the price set by firm \( i \) responds more to idiosyncratic conditions than to aggregate conditions. As \( \omega \to 0 \) or \( \omega \to \infty \) the solution converges to the solution presented in Section 5 of the paper.