Labor Market Search, the Participation Margin and the Business Cycle: A Fresh Look

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first version: May 2006
this version: November 2006

Abstract

This paper considers a real business cycle model with search frictions in the labor market and labor supply which is elastic along the extensive margin. Previous authors have found that such models generate counterfactually procyclical unemployment and a positively-sloped Beveridge curve. This paper presents a sensible calibration which does indeed generate countercyclical unemployment and a negatively correlated unemployment and vacancies despite the presence of a participation margin. In addition, the calibrated model contributes substantially toward resolving the consumption-tightness puzzle described by Ravn (2006).

Many thanks to Michael Burda, Jeff Campbell, Iourii Manovskii, Victor Rios-Rull and Harald Uhlig for helpful comments.Any remaining errors are of course my own. I thank the Department of Economics at the University of Pennsylvania, where much of this paper was written, for its hospitality.

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1 Introduction

Recently, there has been renewed interest in the business cycle properties of models with search frictions and wage bargaining. Beginning with the seminal papers of Shimer (2005) and Hall (2005), a growing body of literature examines the ability of Mortensen-Pissarides search frictions to account for the cyclical variation of labor market variables. One striking feature of this literature is that all models assume that labor supply is inelastic.

Several attempts have been made to calibrate Real Business Cycle models with labor search frictions and labor supply which is elastic along the participation margin. However, previous authors have been unable to match key qualitative facts on the cyclical behavior of unemployment and vacancies. Veracierto (2002), Tripier (2003) and Ravn (2006) all find that their models contradict the data by generating procyclical unemployment and a positively-sloped Beveridge curve (i.e. a positive correlation between unemployment and vacancies). The difficulty is simple but vexing: In response to a positive shock, some workers may wish to enter the labor market by commencing search, swelling the ranks of the unemployed. If the flows of workers between non-participation and search are large enough, then unemployment becomes procyclical and is positively correlated with the procyclical vacancies.

The main contribution of this paper is to show that a carefully calibrated RBC model with search frictions and a participation margin is able to generate both countercyclical unemployment rates and a negative correlation between unemployment and vacancies (a negatively sloped Beveridge curve). The key is a new calibration strategy. First, the abovementioned authors choose the elasticity of labor supply to be either infinite or to match the volatility of employment. In contrast, I calibrate this elasticity to the participation volatility. In the body of the paper, I will show that the two calibration strategies would only be equivalent if they generated equal unemployment volatilities and correlations between unemployment and employment. This is not the case, however, so that there is a meaningful distinction between targetting the employment volatility and targetting the volatility of the participation rate.

This subtle but important difference in calibration strategies turns out to be crucial. The participation rate is only about 1/5 as volatile as GDP. This low volatility of the participation rate requires that the labor supply elasticity be sufficiently low. It turns out that such a low labor supply elasticity implies that the flows of workers into and out of the labor force in reaction to shocks are slow enough to guarantee countercyclical unemployment and a negatively sloped Beveridge curve.

A second key element of the calibration strategy involves time aggregation. The BLS measures unemployment by considering one reference week each month. Quarterly data is obtained by averaging these monthly observations. Hence, it is possible that a technology shock raises unemployment in the impact week or month, but that this is reversed quickly. As a result, the procyclical impact reaction of unemployment would be washed out by subsequent countercyclical movements, so that unemployment is countercyclical on average. I will find this to be the case, as demonstrated by impulse-response functions of unemployment.

A third element of the calibration strategy involves matching the wage volatility. The strategy here is to choose parameters so that the volatility of wages matches its value in the data. Matching the wage volatility is also important in generating countercyclical unemployment rates and a negatively-sloped Beveridge curve, despite the presence of a participation margin. The reason is that if wages react too strongly to productivity shocks, the incentives to enter the labor market are artificially high, increasing the tendency toward procyclical unemployment. I use a calibration strategy which is very similar to that employed by Hagedorn and Manovskii (2006) in order to guarantee that wages are as just as volatile as in the data.

The carefully calibrated RBC model with search frictions can also be used to gain a new perspective on the debate over whether or not Mortensen-Pissarides-style serach frictions can account well for the cyclical variation in labor market variables. Using differing calibration strategies, Shimer (2005) and Hagedorn and Manovskii (2006) find that the stylized version of the Mortensen-Pissarides model can explain practically none or all of the cyclical variation in labor market variables, respectively. Clearly, introducing labor supply elasticity makes it more difficult for a given productivity shock to lead to highly volatile labor market tightness.

The second contribution of this paper is to examine to what extent elastic labor supply diminishes the fraction of cyclical variation in unemployment, vacancies and tightness that can be accounted for by the model with Mortensen-Pissarides search frictions. I find that the results are highly sensitive to the labor supply elasticity assumed. The model with inelastic labor can account for virtually all of the volatility in tightness and about 85% of that in unemployment. When labor supply elasticity is increased (i.e. when the calibration target is employment volatility), the model can account for less than 1/20th of the volatility of tightness in the data. The reason is that a strong negative correlation between unemployment and volatilities is required to ensure that their ratio fluctuates sufficiently. Such a strong negative correlation cannot be generated when labor supply elasticity is too high. However, in my preferred calibration, which involves targetting participation volatility and results in a relatively low labor supply elasticity, the model can still account for about two-thirds of the volatility in tightness.

The third contribution of this paper is to contribute to resolving the "consumption-tightness puzzle" described by Ravn (2006). Ravn (2006) derives that the volatility of labor market tightness in the model should be equal to the volatility of consumption multiplied by the inverse of the intertemporal elasticity of substitution in consumption: $\sigma_\theta = \eta\sigma_c$. This implies that the model cannot reconcile the very low volatility of consumption with the high volatility of labor market tightness, unless η is very high. Very high values for η are problematic, however, for a variety of reasons. Ravn's consumption-tightness relationship was derived under the assumption of infinitely elastic labor supply. I establish that a more general formulation with lower degrees of labor supply elasticity contributes to resolving this puzzle, increasing the amount of volatility in tightness that can be explained under log utility from about one-twentieth of that in the data to nearly one-half.

This paper also relates to an earlier literature which integrated search frictions into business cycle models. Merz (1995) and Andolfatto (1996) showed that business cycle models with search frictions could be quite successful at accounting for the cyclical properties of macro variables, as well as for the subset of the labor variables they considered. However, neither of these models allows for a participation margin. Merz (1995) also encounters the difficulty of a positively-sloped Beveridge curve when allowing for endogenous search intensity.

Also, in independent work, Haefke and Reiter (2006) allow for heterogeneous productivity in home production, combined with idiosyncratic productivity shocks, to restrict the flow of workers into unemployment due to a positive technology shock. Although their approach also is able to replicate the key features of the data, the heterogeneity increases the complexity considerably. The approach presented in the present paper is based on homogeneous agents, making it considerably more straightforward and tractable.

The paper is organized as follows: Section 2 presents the model, whose equilibrium is found in section 3. The calibration strategy is described in section 4, while quantitative results are presented in section 5 and section 6 concludes.

2 Model

This section presents the basic model. It is a standard real business cycle, augmented by labor market frictions and wage bargaining. Labor supply is elastic along the extensive (participation) margin. The bargaining setup involves firms bargaining individually with each worker. Agents are risk averse. The agents are organized into large households which provide full insurance against idiosyncratic consumption fluctuations. The production technology is Cobb-Douglas with labor and capital as inputs. This model can be seen as the natural extension of the RBC literature to allow for search frictions and decentralized wage bargaining.

2.1 Household's Problem

Each household consists of a number of individuals which is large enough to guarantee perfect insurance over consumption. The household maximizes its discounted expected utility from consumption as:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left[\ln c_t + \phi \frac{l_t^{1-\frac{1}{\nu}}}{1-\frac{1}{\nu}} \right] \tag{1}$$

subject to the large-family budget and time constraints

$$w_t h_t + r_t k_{t-1} \geq c_t + i_t \tag{2}$$

$$k_t = (1-\delta) k_{t-1} + i_t \tag{3}$$

$$1 = h_t + u_t + l_t \tag{4}$$

$$h_{t+1} = (1-\chi) h_t + f_t u_t \tag{5}$$

where the fraction h_t family members earn the wage w_t, while u_t are unemployed and l_t are either consuming leisure or engaged in home production. The household owns the capital stock, which it rents at market rate r_t to firms.

Households obtain period utility from the consumption of goods c_t and the fraction of members l_t who enjoy leisure. Equivalently, this can be interpreted as households' obtaining utility from market goods c_t and non-market goods

s_t, where household goods are produced using labor only according to a decreasing returns to scale technology $s_t = \frac{l_t^{1-\frac{1}{\nu}}}{1-\frac{1}{\nu}}$. Under either interpretation, ν represents the intertemporal elasticity of substitution over time use, and l_t is the fraction of household members who do not participate in the labor market.

Two further aspects of the participation decision are important. First, households are assumed to gain utility only from non-market time, so that the disutilities to search and employment are equal. The idea is that searching for a job is itself a full-time job, which both precludes leisure and household production activities. Second, it is assumed that all transitions into and out of the labor force are channeled through unemployment. Clearly, workers who leave non-participation to enter the labor force must first pass through at least one period of search in order to find a job. In addition, it is assumed that workers who exit the workforce are culled from the ranks of the unemployed, so that workers never quit a job to exit the labor force.

The solution to the family's problem takes the form of two Euler equations. The first is the standard Euler equation for consumption.

$$1 = \beta E_t \left\{ \frac{c_t}{c_{t+1}} \left[r_{t+1} + 1 - \delta \right] \right\} \tag{6}$$

The second Euler equation reflects the household's participation decision.

$$\underbrace{\phi l_t^{-\frac{1}{\nu}}}_{u_l(c_t, l_t)} = f_t \beta E_t \left\{ \frac{w_{t+1}}{c_{t+1}} - \phi l_{t+1}^{-\frac{1}{\nu}} + \phi l_{t+1}^{-\frac{1}{\nu}} \frac{1-\chi}{f_{t+1}} \right\} \tag{7}$$

The left-hand side of (7) reflects the marginal disutility to increasing the family's labor force participation. The right hand side captures the discounted marginal benefit to employment, scaled by the rate at which searching workers find jobs f_t. Those marginal benefits to employment include the utility from the consumption due to the additional wage income, net of the disutility to work, and plus the relative continuation value from current employment, captured by $u_l(c_{t+1}, l_{t+1}) \frac{1-\chi}{f_{t+1}}$. The larger is the rate at which workers find jobs at $t+1$, the lower the option value of employment.

2.2 Search and Matching in the Labor Market

The labor market is characterized by a standard search and matching framework. Aggregate stocks of unemployed workers U_t and vacancies V_t are converted into job matches by a constant returns to scale matching function $m(U_t, V_t) = s \cdot U_t^{\eta} V_t^{1-\eta}$. Defining labor market tightness as $\theta_t \equiv \frac{V_t}{U_t}$, the firm meets unemployed workers at rate $q_t = s\theta_t^{-\eta}$, while the unemployed workers meet vacancies at rate $f_t = s\theta_t^{1-\eta}$. Aggregate unemployment evolves as

$$U_{t+1} = U_t + [1 - f_t - \chi] U_t \tag{8}$$

where χ is the exogenous match destruction rate.

Workers are identical and bargaining is individual. Define $\widetilde{\beta}_{t+1} \equiv \beta \frac{u_c(c_{t+1})}{u_c(c_t)}$ to be the households' stochastic discount factor. A worker's value of employment is:

$$V_t^E = w_t - u_l(c_t.l_t) + E_t \left\{ \widetilde{\beta}_{t+1} \left[(1-\chi) V_{t+1}^E + \chi V_{t+1}^U \right] \right\} \tag{9}$$

The value of unemployment is also standard.

$$V_t^U = b - u_l(c_t.l_t) + E_t \left\{ \widetilde{\beta}_{t+1} \left[f_t V_{t+1}^E + (1-f_t) V_{t+1}^U \right] \right\} \tag{10}$$

where b denotes some non-tradeable flow value to being unemployed, expressed in units of output. Defining $V_t^W \equiv V_t^E - V_t^U$ yields an expression for the worker's surplus to employment[1]:

$$V_t^W = w_t - b + (1 - \chi - f_t) E_t \left\{ \widetilde{\beta}_{t+1} V_{t+1}^W \right\} \tag{11}$$

2.3 Firm's Problem

There is a continuum of identical firms on the unit interval. Firms are perfectly competitive and produce using a constant returns to scale Cobb-Douglas technology. I abandon the standard one-worker-per-firm assumption in favor of a more general framework with multiple-worker firms.

Firms maximize the discounted value of future profits. Firms adjust employment by varying the number of workers [extensive margin] rather than the number of hours per worker. This is the appropriate margin, since about 2/3 of the fluctuations in employment can be attributed to the extensive margin.[2] The firm's date t state variable is the number of workers currently employed, h_t. The firm's key employment decision is the number of vacancies v_t. Firms open as many vacancies as necessary to hire the desired number of workers next period, while taking into account that the real cost to opening a vacancy is κ. Firms are assumed to own their capital stock and firms take aggregate variables summarized by Γ as given. The firm's problem becomes:

$$V^J(z_t, h_t|\Gamma) = \max_{v_t, i_t} \left[\begin{array}{c} y_t - w_t h_t - r_t k_{t-1} - \kappa v_t \\ +E_t \left\{ \widetilde{\beta}_{t+1} V^J(z_{t+1}, h_{t+1}|\Gamma) \right\} \end{array} \right] \tag{12}$$

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[1] Recall that $\widetilde{\beta}_{t+1}$ represents the stochastic discount factor $\beta u_c(c_{t+1}, l_{t+1}) / u_c(c_t, l_t)$, so that the worker's value functions can be rewritten as

$$u_c(c_t, l_t) V_t^W = u_c(c_t, l_t)(w_t - b) + \beta(1 - \chi - f_t) E_t \left\{ u_c(c_{t+1}, l_{t+1}) V_{t+1}^W \right\}$$

Hence, V_t^W should be interpreted as the value of employment in real units rather than in terms of utility. The firm's value is also stated in real terms, as the two values must be expressed consistently for the Nash bargaining below.

[2] cf. Hansen (1985), whose results can also be replicated with more recent data.

subject to

$$\begin{aligned}
\text{production function} &: & y_t = Ae^{z_t} h_t^{1-\alpha} k_{t-1}^{\alpha} && (13)\\
\text{transition function } h &: & h_t = (1-\chi)\, h_{t-1} + q_{t-1} v_{t-1} && (14)\\
\text{wage curve} &: & w_t = w\left(h_t, k_{t-1}, z_t | \Gamma\right) && (15)\\
\text{technology shock} &: & z_t = \rho z_{t-1} + \varepsilon_t && (16)
\end{aligned}$$

where the wage curve is the result of individual Nash bargaining as described in the following sub-section.

The first order condition for capital choice is standard:

$$r_t = \frac{\partial y_t}{\partial k_{t-1}} = \alpha \frac{y_t}{k_{t-1}} \tag{17}$$

The first order condition for vacancies states that the marginal value of an additional worker must equal the cost of searching for that worker:

$$\frac{\kappa}{q\left(\theta_t\right)} = E_t \left\{ \widetilde{\beta}_{t+1} \frac{\partial V^J\left(z_{t+1}, h_{t+1}, k_t | \Gamma\right)}{\partial h_{t+1}} \right\}. \tag{18}$$

Combining (18) with the envelope condition for employment h_t leads to an optimality condition for the firm's choice of labor input:

$$\frac{\kappa}{q_t} = E_t \left\{ \widetilde{\beta}_{t+1} \left[(1-\alpha) \frac{y_{t+1}}{h_{t+1}} - w_{t+1} + (1-\chi) \frac{\kappa}{q_{t+1}} \right] \right\} \tag{19}$$

Equation (19) equates the cost of hiring a worker (left hand side) to the discounted expected marginal benefits of hiring that worker. These marginal benefits are the worker's marginal product net of wages, minus the impact of hiring an additional worker on the bargained wage, and plus the avoided next-period search cost if a new hire stays with the firm, which happens with probability $(1-\chi)$.

Finally, the envelope condition for employment h_t and (18) also lead to an expression for the marginal value of a worker:

$$\frac{\partial V^J(z_t, h_t)}{\partial h_t} = (1-\alpha)\frac{y_t}{h_t} - w_t + (1-\chi)\frac{\Phi_V}{q_t} \tag{20}$$

Equation (20) will be the firm's surplus when bargaining with each worker.

2.4 Individual Wage Bargaining

The key assumption of the individual bargaining framework is that firms cannot commit to long-term employment contracts, and may renegotiate wages with each worker at any time. This makes each worker effectively the marginal worker.[3] Hence, the firm's outside option is not remaining idle, but rather producing with one worker less, so that firm's surplus is the marginal value of a worker. Also, individual bargaining involves bargaining over wages only, since an individual worker can only deprive the firm of her own marginal product, which does not give the worker sufficient leverage to negotiate hiring.

Individual bargaining is the appropriate bargaining setup when studying the business cycle properties of the US economy for two reasons. First, "employment at will" is dominant in US labor markets. Under employment at will, both firms and workers can terminate the employment relationship at any time, without justification.[4] Less than 10% of private sector workers are currently covered by a collective bargaining agreement, according to CPS data reported in Hirsch and Macpherson (2003). Hence, US labor markets

[3]The individual bargaining framework was introduced by Stole and Zwiebel (1996, 1996a). It has previously been applied to settings with decreasing returns to scale by Smith (1999), to multiple worker types by Cahuc et. al. (2004) and to settings with monopolistic competition in goods markets by Ebell and Haefke (2004, 2005).

[4]Some states do restrict the ability of the firm to terminate the worker without due cause, but these restrictions are weak and generally only require a notice period for unjustified dismissals of 2 to 8 weeks. No states impose any significant restrictrictions on the ability of the worker to terminate at any time without cause. The only leverage a firm may exercise in some states is to withhold compensation for accrued unused vacation time if the worker does not give at least 2 weeks notice.

are better characterized by individual than by collective bargaining. Second, individual bargaining is the natural extension of the Mortensen-Pissarides framework to multi-worker firms

The individual Nash bargaining problem maximizes the weighted sum of log surpluses

$$\max_{w_t} \mu \ln V_t^W + (1-\mu) \ln \frac{\partial V^J}{\partial h_t}$$

subject to firm surplus (20) and worker's surplus (11). Worker's bargaining power is given by μ. The first order condition of the bargaining problem is:

$$\begin{aligned} w_t &= \mu \left[(1-\alpha) \frac{y_t}{h_t} + (1-\chi) \frac{\kappa}{q_t} \right] \\ &\quad + (1-\mu) \left[b - (1-\chi-f_t) E_t \left\{ \widetilde{\beta}_{t+1} V_{t+1}^W \right\} \right] \end{aligned} \tag{21}$$

The important assumption made in deriving (21) is that $E_t \left\{ \widetilde{\beta}_{t+1} V_{t+1}^W \right\}$ is not a function of the firm-level bargained wage w_t. We now proceed to confirm this assumption, and then derive the wage curve. First, solve (21) for $(1-\chi-f_t) E_t \left\{ \widetilde{\beta}_{t+1} V_{t+1}^W \right\}$ and substitute into the difference equation for worker's surplus (11):

$$V_t^W = \frac{\mu}{1-\mu} \left[(1-\alpha) \frac{y_t}{h_t} - w_t + (1-\chi) \frac{\kappa}{q_t} \right] \tag{22}$$

Next, take (22) ahead one step, multiply both sides by $\widetilde{\beta}_{t+1}$ and take expectations to obtain:

$$E_t \left\{ \widetilde{\beta}_{t+1} V_{t+1}^W \right\} = \frac{\mu}{1-\mu} E_t \left\{ \widetilde{\beta}_{t+1} \left[(1-\alpha) \frac{y_t}{h_t} - w_t + (1-\chi) \frac{\kappa}{q_t} \right] \right\}$$

Since both parties engaged in wage bargaining are aware of the firm's optimization problem, we can use the firm's Euler equation for optimal labor choice to simplify this last equation, obtaining a closed form expression for expected discounted future worker's surplus:

$$E_t\left\{\widetilde{\beta}_{t+1}V_{t+1}^W\right\}=\frac{\mu}{1-\mu}\frac{\kappa}{q_t} \tag{23}$$

Hence, we have confirmed that $E_t\left\{\widetilde{\beta}_{t+1}V_{t+1}^W\right\}$ is independent of the current bargained wage w_t. In fact, future surplus depends only upon aggregate variables. The reason is that the expected worker's surplus is a search rent, whose value depends only upon the cost of searching for a new worker $\frac{\kappa}{q_t}$.

Finally, use (23) in conjunction with (21) to obtain the wage curve:

$$w_t=(1-\mu)\,b+\mu\left[(1-\alpha)\frac{y_t}{k_{t-1}}+\kappa\theta_t\right] \tag{24}$$

3 Equilibrium

An equilibrium is defined as sequences of prices and labor market tightnesses which solve the firm's, the household's and the bargaining problem and which let markets clear. The solution satisifes the household's Euler equations (6) and (7), the household constraints (2)-(5), the firm's optimality conditions (19) and (17), the wage curve (24), the transition equation for aggregate unemployment (8) and appropriate market-clearing conditions.

This definition of equilibrium yields a system of thirteen equations in the thirteen unknowns $(h_t,l_t,k_t,f_t,q_t,\theta_t,u_t,v_t,w_t,y_t,c_t,i_t,z_t)$. All equilibrium equations are listed in the appendix in their log-linearized form. The log-linearized system is solved by the method of undetermined coefficients, implemented using Uhlig's toolkit.

4 Calibration

The key element of the calibration strategy is the use of the cyclical variation in the participation rate to pin down the elasticity of labor supply ν. This is a novel calibration strategy, and plays an important role in establishing the model's ability to generate countercyclical unemployment rates and a negatively sloped Beveridge curve, despite the presence of elastic labor supply along the participation margin.[5] Otherwise, the calibration strategy is similar to that of Hagedorn and Manovskii (2006). It turns out that there exists a continuum of pairs of worker's bargaining power and vacancy cost shares in national income (μ, Π_V) that match the target wage volatility. Data on hiring costs per worker is used to discriminate among these pairs, and pin down the value of worker's bargaining power μ.[6]

The baseline calibration is summarized in Table 1. Period length is one week. There are fourteen parameters to pin down: the technology parameter A, the intertemporal elasticity of substitution over time use ν, the weight on non-market time in the utility function ϕ, the matching elasticity η, vacancy costs κ, worker's bargaining power μ, the output elasticity of capital α, the flow value of unemployment b, the depreciation rate δ, the match destruction rate χ, and the matching scale parameter s, the discount factor β and the two parameters of the productivity shock ρ and σ_ε.

Without loss of generality, the technology parameter A is normalized to one. The parameters of the weekly productivity process are chosen to match the autocorrelation and volatility of TFP in post-war quarterly US data. Choosing weekly autocorrelation $\rho_w = 0.9895$ and weekly standard deviation of the innovation $\sigma_{\varepsilon,w} = 0.0034$ leads to quarterly values $\rho_q = 0.765$ and $\sigma_{z,q} = 0.013$.[7] The discount factor β is chosen to match an annual risk-free rate of 4%. The depreciation rate for capital is chosen so that the investment

[5] The calibration is numeric, and relies upon a monotone relationship between ν and σ_p, as illustrated in Figure 1.

[6] Hagedorn and Manovskii (2006) fix vacancy costs by specifying a target for the share of vacancy costs in national income Π_V, and then pin down μ by the wage elasticity of productivity. In the present paper the calibration is numeric, and the continuum of pairs of (Π_V, μ) which lead to the target wage volatility is depicted in Figure 2.

[7] These values are identical to those chosen in Hagedorn and Manovskii (2006), who base them on the autocorrelation and standard deviation of HP-filtered quarterly data.

share of income $\frac{i}{y} = 0.25$, matching its value in the post-war data reported by Francis and Ramey (2001).[8] I follow Hagedorn and Manovskii (2006) in setting the weekly separation rate χ to 0.0081, which corresponds to the quarterly rate of $\chi = 0.10$ estimated by Shimer (2005). Similarly, the target for the monthly job-finding rate f is 0.139, which corresponds to Shimer (2005)'s monthly value of 0.45. I target a steady-state tightness of $\theta = 0.63$. Together, f and θ pin down the job-filling rate q at $q = \frac{f}{\theta} = 0.22$. The choices for θ and q are innocuous: they are simply a normalization, as Shimer (2005) points out.

The target for the labor share is $\Pi_{\mathrm{l}} = 0.64$, as implied by the data. Factor shares add up to one, so that $\Pi_{\mathrm{l}} + \Pi_{\mathrm{k}} + \Pi_{\mathrm{v}} = 1$, where Π_{v} is the share of vacancy costs in national income. The steady-state capital share is determined as $\Pi_{\mathrm{k}} = \alpha$. Numerically, there is a continuum of pairs (μ, Π_V) which match any given volatility of wages, as illustrated by Figure 2. Fortunately, there is data available on Π_V, which makes it possible to pin down the pair (μ, Π_V) which is consistent with both the data on wage volatility and the data on vacancy costs. We follow Hagedorn and Manovskii (2006) in estimating that hiring a worker costs 7.6 % of the worker's annual wage[9]. The resulting vacancy costs are $\kappa = 0.29$. Given these vacancy costs, worker's bargaining power $\mu = 0.122$ guarantees that the volatility of wages relative to output is $\sigma_{w/y} = 0.42$, its value in the data. Finally, together, the choices for θ, and f pin down the parameters m and b. The scaling parameter of the matching function becomes $m = 0.175$ and the value of non-participation is

[8] The Francis and Ramey (2001) data is appropriate for calibration here because it focuses exclusively on the private sector, as does the present model.

[9] The cost of filling a vacancy has two components in Hagedorn and Manovskii (2006)'s formulation, labor and capital costs. In Hagedorn and Manovskii (2006), labor costs are 4.5% of quarterly wages, corresponding to 1.1% of annual wages. Capital costs are given as 47.5% of weekly average labor productivity to post each vacancy. Since a vacancy must be posted for $\frac{1}{q} = 4.6$ weeks to hire each worker, capital costs are 2.2 times weekly average labor productivity, which corresponds to 3.4 times weekly wages at a wage share of 0.64. Finally, dividing by 52 yields capital costs per hire of 6.5% of annual wages. Adding the two components yields total vacancy costs per hire of 7.6% of annual wages.

95.6% of wages. This high value is similar to that resulting from Hagedorn and Manovskii (2006)'s calibration. Although such high values for the replacement rate have proven to be controversial, it is useful to note that they are an improvement on the values implicitly assumed in the classic contributions of Rogerson (1985) and Hansen (1985). In these latter setups, agents participate in lotteries over employment status, requiring them to be indifferent between employment and unemployment. Here, in contrast, households strictly prefer employment, albeit by a relatively small margin.

Finally, the utility parameters ϕ and ν remain to be set. The weight on non-market generated utility ϕ is chosen so that the steady-state fraction $1 - l$ of family members who participate in the labor market matches the average rate of labor market participation in the US from 1964 to 2006 at 64%. The intertemporal elasticity of substitution of participation is set so that the volatility of the participation rate matches the data. The calibration is numeric: Figure 1 shows the relationship between labor supply elasticity ν and participation volatility. The resulting elasticity of time use is $\nu = 0.0058$. That this value is very low is also a consequence of calibrating to a weekly frequency. In the equivalent quarterly calibration, presented in Table 5, the labor supply elasticity is $\nu = 0.064$, corresponding to a coefficient of relative risk aversion over time use of 15.6.

In addition, Figure 1 illustrates that the relationship between participation elasticity ν and participation volatility $\sigma_{p/y}$ is very steep around the point estimate from the data of $\sigma_{p/y} = 0.20$. This implies that even significant mismeasurement of participation volatility would lead to only small changes in the calibrated value for ν, suggesting that the use of such low values for ν is quite robust. Conversely, the use of only moderately higher values for ν would lead to strongly counterfactual participation elasticities.

5 Results

Results of the baseline calibration are presented in Table 2. In what follows, I will first discuss the model's success at generating countercyclical unemployment and a negatively sloped Beveridge curve despite labor supply which is elastic along the extensive margin. Next, the impact of elastic labor supply on the ability of the model to account for the the volatility of labor market variables over the cycle is discussed. Finally, I relate my results to the 'consumption-tightness puzzle' described by Ravn (2006), and explain why the baseline calibration is able to help to resolve this puzzle.

5.1 Countercyclical Unemployment

The baseline calibration generates unemployment which is nearly as countercyclical as in the data, $\rho_{\text{model}}(u, y) = -0.85$ versus $\rho_{\text{data}}(u, y) = -0.88$. It also generates a negatively sloped Beveridge curve, although the contemporaneous correlation between unemployment and vacancies $\rho_{\text{model}}(u, v) = -0.49$ falls somewhat short of its value in the data. The mere fact that model unemployment is strongly countercyclical and the model Beveridge curve negatively sloped is surprising. Previous authors studying RBC models with search frictions and elastic labor supply along the participation margin (Veracierto (2002), Tripier (2003) and Ravn (2005)) have consistently found their models to generate procyclical unemployment and a positively sloped Beveridge curve, contradicting the stylized facts.

The model presented here succeeds where others have failed for two reasons: the calibration strategy and time aggregation. In what follows, I discuss each of these factors in detail.

5.1.1 Targetting Participation Volatility

The first reason that the model presented here succeeds at generating countercyclical unemployment and a negatively-sloped Beveridge curve is the calibration strategy for the labor supply elasticity. Here, I choose ν so as to match the relative volatility of the participation rate $\sigma_{p/y} = 0.20$, leading to a low degree of participation elasticity ν. In contrast, Veracierto (2002), Tripier (2003) and Ravn (2005) have all chosen higher values for ν. Ravn focuses on utility functions that are linear in leisure, and hence are characterized by infinitely elastic labor supplies. Veracierto (2002) calibrates ν to match the volatility of *employment* rather than participation, resulting in a more elastic labor supply in his model.[10]

The difference between targetting participation and employment volatility is subtle but important. In what follows, I will argue that targetting participation volatility is preferable to targetting employment volatility. First, note that the direct impact of an increase in ν is to make agents more willing to shift in and out of the labor force, increasing the volatility of the participation rate. This is not equivalent to an increase in volatility of the employment rate. To see this note that participation p_t is equal to the sum of employment h_t and unemployment u_t.[11] As a result, the volatility of the participation rate is given as

$$p^2\sigma_p^2 = u^2\sigma_u^2 + h^2\sigma_h^2 + 2hu \cdot cov\left(\widehat{u}_t, \widehat{h}_t\right)$$

Matching the volatility of the participation rate σ_p is only equivalent to matching the volatility of employment σ_h if the model generates both an unemployment volatility σ_u and a covariance of unemployment and employment $cov\left(\widehat{u}_t, \widehat{h}_t\right)$ that match those in the data. Otherwise, the two calibration strategies yield different results.

The volatilities of unemployment generated by the alternative targets

[10] Tripier (2003) reports results to one calibration in which labor supply is infinitely elastic, and one in which he chooses labor supply elasticity to match employment volatility, as in Veracierto (2002).

[11] In the log-linearized model, this corresponds to $p\widehat{p}_t = u\widehat{u}_t + h\widehat{h}_t$, where p is the steady-state participation rate and $\widehat{p}_t$ is the log-deviation.

$\sigma_h = 0.56$ and $\sigma_p = 0.20$ are similar and roughly in line with the data.[12] However, the correlation between unemployment and employment generated by the two targets varies considerably. In the data, this correlation is strongly negative at $\rho_{u,h} = -0.95$. Figure 3 shows that targetting the participation rate $\sigma_{p/y} = 0.20$ leads to a strong negative correlation between unemployment and employment, as in the data ($\rho_{u,h}$(data) $= -0.96$ vs. $\rho_{u,h}$ (model) $= -0.95$). In contrast, targetting an employment volatility of $\sigma_h = 0.56$, as in Veracierto (2002), causes unemployment and employment to be nearly uncorrelated in the model. Hence, targetting the employment volatility σ_h is equivalent to targetting too high a volatility in the participation rate σ_p. Indeed, Veracierto finds that his model generates volatility of the participation rate that is nearly three times as large as that observed in the data.[13]

In addition, participation volatility σ_p is much more sensitive to labor supply elasticity ν than employment volatility σ_h, as can be seen from Figure 4. This implies that calibrating to σ_p leads to a smaller deviation from σ_h than vice-versa.

Why does the low labor supply elasticity implied by targetting σ_p help to generate countercyclical unemployment and a negatively-sloped Beveridge curve? To see this, compare impulse-response functions for the low-elasticity scenario (targetting σ_p) and and the high-elasticity scenario (targetting σ_h), shown in Figures 5 and 6 respectively. When labor supply is quite elastic, the response of u to a technology shock is large and positive, as agents respond

[12] The intuition for the u-shaped unemployment volatility in Figure 3 is straightforward. At low levels of labor supply elasticity ν, the positive contemporaneous impact of a technology shock on unemployment is small, while the negative lagged effect is relatively large. As ν increases, the magnitude of the contemporaneous impact increases, while that of the lagged impact declines. Due to the fact that the standard deviation weights large deviations overproportionately, the lowest unemployment volatility is at intermediate values of ν, where both the contemporaneous and the lagged impacts of a technology shock on u are moderate.

[13] Here, I refer to Table 6 in Veracierto (2002), which gives the results of the Mortensen-Pissarides search model.

to the increased wages and increased probability of job-finding by streaming into search (unemployment). When labor supply is less elastic, the initial impact of a technology shock on u is still positive, but smaller, because of agents' lower willingness to substitute leisure over time, putting a brake on the flows into search. As a result of the small increase in u, combined with a strong increase in vacancies, tightness and hence job-finding rates increase strongly. The increased job-finding rates ensure that the inflows of searching workers are 'mopped up' quickly and transit into employment, so that net inflows of workers to unemployment become negative within two weeks. In addition, the quick reversal of unemployment's behavior, coupled with an increase in tightness, help keep the Beveridge curve negatively sloped.

In contrast, in the high labor supply elasticity scenario, flows into unemployment upon impact are nearly has high as the increases in vacancies. As a result, tightness and job-finding rates do not increase much, so that job-seekers transit to employment at a lower rate. It then takes nearly half a year for the net inflows into unemployment to become negative. Not only does this lead to procyclical unemployment, but the strong correlation between unemployment and vacancies leads to a strongly positively-sloped Beveridge curve.

5.1.2 Time Aggregation

The second reason that our model succeeds at generating realistic behavior of unemployment has to do with time aggregation and data collection. The BLS samples unemployment and vacancies for one reference week each month.[14] That is, subjects are asked whether they were searching for work not during the entire month, but only during the reference week. As a result, it is possible that a worker enters the labor force between reference weeks, searches for up to 3 weeks, finds a job, and is never recorded as unemployed. This is

[14] I refer here to collection procedures for the Current Population Survey, described on the BLS website under www.bls.gov/cps/cps_htgm.htm.

especially relevant in good times, when job-finding rates are high.

In addition, since productivity data is available quarterly, we can only assess the cyclical behavior of unemployment at a quarterly frequency. The quarterly data is obtained as an average of monthly values. Hence, a small upward tick in unemployment on impact of a positive technology shock would be averaged with the lagged downward movements in unemployment. As a result, the average unemployment rate over the quarter might respond negatively to a positive productivity shock.

To address these issues, I calibrate the model to weekly data, aggregate the results to a quarterly frequency by taking averages, HP-filter the quarterly series, and then calculate the correlations and the standard deviations.

5.2 Volatilities of Unemployment and Vacancies

Next, I discuss the ability of the model with elastic labor supply to account for the volatilities of labor market variables over the cycle. In recent work, Hagedorn and Manovskii (2006) show that a model with Mortensen-Pissarides labor search frictions and inelastic labor supply is able to match the data on the cyclical variation in unemployment, vacancies and tightness very well. Here, I employ a calibration strategy that is very similar to that of Hagedorn and Manovskii (2006), while allowing for elastic labor supply. Clearly, elastic labor supply makes it more difficult for the model to generate highly volatile unemployment and vacancies. The quantitative question is: How much of the volatility of labor market variables in the data can be accounted for by a full RBC model with elastic labor supply?

Table 2 compares the results of the baseline calibration with elastic labor supply to those with inelastic labor supply. Even the modest degree of labor supply elasticity in the present calibration does decrease the ability of Mortensen-Pissarides search frictions to account for the volatilities of unemployment and vacancies somewhat. While the model with inelastic labor supply can account for nearly all of the volatility of tightness, the model with labor supply elasticity is now only able to account for about two-thirds of it. Similarly, the model with inelastic labor supply can account for about 85% of the volatility of unemployment, as compared to 62% for the model with elastic labor supply. However, the model with labor supply elasticity continues to account very well for the volatility of vacancies.

In contrast, Table 3 shows that when labor supply elasticity ν is increased sufficiently to match employment volatility $\sigma_{h/y}$, the performance of the model at matching the volatility of tightness θ deteriorates dramatically, despite the fact that the model is still able to account for nearly 70% of the volatilities of unemployment and vacancies in the data. The reason is that the model which targets $\sigma_{h/y}$ generates unemployment which is nearly perfectly *positively* correlated with vacancies, so that the ratio of vacancies to unemployment (tightness) is nearly constant.

To summarize: The model which targets partcipation volatility is able to account for the bulk of the volatility in labor market tightness, while the model which targets employment volatility is not.

5.3 The Consumption-Tightness Puzzle

In recent and provocative work, Ravn (2006) derives a relationship between labor market tightness θ and the intertemporal elasticity of substitution over consumption.[15]

$$\theta = \frac{1-\mu}{\mu}\frac{1}{\kappa}\frac{\varphi}{u_c} \tag{25}$$

where φ is a constant which gives the marginal disutility to the family of one worker's search activity. Ravn (2006)'s consumption-tightness equation is valid for the special case of utility which is linear in leisure. It is straightforward to show that for a general utility function $u(c,l)$, his consumption-

[15] I use the notation of this paper. Ravn's original equation is $\theta = \frac{\upsilon}{1-\upsilon}\frac{\omega}{c^{-\eta}}$ where υ is firm's bargaining power, $c^{-\eta}$ is the marginal utilty to consumption and $\omega = \frac{H(1)-H(1-s)}{\kappa}$ is a constant given the ratio between marginal search costs and vacancy posting costs κ.

tightness relationship becomes:

$$\theta = \frac{1-\mu}{\mu}\frac{1}{\kappa}\frac{u_l}{u_c} \tag{26}$$

The only difference between (25) and (26) is the marginal disutility to search activity term. In Ravn's formulation, the infinite elasticity of labor supply leads to a constant marginal disutility to search. In my formulation labor supply elasticity is ν, so that the marginal disutility of labor term is $u_l = \phi l^{-\frac{1}{\nu}}$.

From (25), Ravn goes on to derive an expression relating the volatility of tightness to the intertemporal elasticity of substitution.

$$\sigma_\theta = \eta\sigma_c \tag{27}$$

where η is the intertemporal elasticity of substitution in consumption (i.e. $u_c = c^{-\eta}$). Ravn concludes from (27) that the high degree of tightness volatility $\sigma_\theta = 23.66\%$ observed in the data can only be reconciled with the low volatility of consumption $\sigma_c = 1.23\%$ if η is very high, so that the intertemporal elasticity of substitution of consumption $\frac{1}{\eta}$ is very low. In particular, when consumption is log (as it must be to satisfy the King-Plosser-Rebelo conditions for balanced growth), $\eta = 1$, so that the implied tightness volatility is 1.23%, only about $\frac{1}{20}$th its value in the data. When labor supply is inelastic and $u(c,l) = \frac{c^{1-\eta}}{1-\eta} + \phi\frac{l^{1-\frac{1}{\nu}}}{1-\frac{1}{\nu}}$, however, the relationship between tightness and consumption volatility becomes:

$$\sigma_\theta = \sqrt{\eta^2\sigma_c^2 + \frac{1}{\nu^2}\sigma_l^2 - 2\frac{1}{\nu}cov\left(\widehat{l}_t, \widehat{c}_t\right)} \tag{28}$$

Now, taking $\nu = 0.064$ to match the volatility of the participation rate[16], and using that the volatility of non-participation $\sigma_l = 0.58$ % and the co-

[16] I use the larger value of labor supply elasticity $\nu = 0.064$ implied by the quarterly calibration described in Table 5.

variance $cov\left(\widehat{l}_t, \widehat{c}_t\right) = \rho_{l,c}\sigma_l\sigma_c = -0.19\%$ implies that even under log utility in consumption, the implied volatility of tightness σ_θ is 9.5%.[17] This is a substantial improvement, as now the model can account for nearly half of the volatility of tightness observed in the data. Allowing for higher values of η, as in Ravn (2006)[18], leads to only modest further increases in the implied volatility of tightness that can be generated, as can be seen in Table 4.

6 Conclusions

This paper has integrated Mortensen-Pissarides style search frictions with decentralized wage bargaining into a standard business cycle model with risk averse households, capital and labor supply which is elastic along a participation margin. In contrast to the extant literature, I find that the model with search frictions and a participation margin can indeed replicate key qualitative properties of labor market variables. In particular, when calibrated carefully the model generates countercyclical unemployment and a negative correlation between vacancies and unemployment, in accordance with the stylized facts.

In addition, the model does quite well at accounting for the cyclical variation in macro variables. The finding here is that Mortensen-Pissarides can account for about two-thirds of the volatility in unemployment and tightness, while accounting for nearly all of the volatility of vacancies over the cycle. This suggests that Mortensen-Pissarides search frictions are an important source of labor market fluctuations, while leaving room for additional factors. Finally, the model contributes substantially to resolving the 'consumption-tightness puzzle' described by Ravn (2006).

[17] To obtain the volatility of deviations in non-participation from the volatility of participation, recall that log-linearizing $p = 1 - l$ yields $p\widehat{p}_t = -l\widehat{l}_t$, so that $\sigma_l = \frac{p}{l}\sigma_p$. Use that steady-state participation is $p = 0.65$ and that $\sigma_p = 0.31$ to obtain $\sigma_l = 0.58$. The covariance is obtained using that $\rho_{l,c} = -\rho_{p,c} = -0.27$ and $cov\left(\widehat{c}_t, \widehat{l}_t\right) = \rho_{l,c}\sigma_l\sigma_c$.

[18] Only if $\eta = 1$ are the King-Plosser-Rebelo conditions for balanced growth satisfied.

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Table 1: Baseline Calibration (Weekly)

Parameter	Value	Inelastic	Target	Equation
A	1.0	1.0	normalization	—
ν	0.0058	—	$\sigma_{p/y} = 0.20$	see appendix
ρ	0.9895	0.9895	data	—
σ_{ε}	0.34	0.34	data	—
χ	0.0081	0.0081	$u = 5.5\ \%,\ f = 0.139$	$u = \frac{\chi}{\chi+f}$
$\widetilde{\beta}$	$0.988^{1/12}$	$0.988^{1/12}$	$\widetilde{r} = 4.0\ \%$ annually	$\widetilde{r} = \frac{1}{\widetilde{\beta}} - 1$
δ	0.0026	0.0026	$\frac{i}{y} = 0.25$	$\frac{i}{y} = \frac{\delta}{\widetilde{r}+\delta}\Pi_{\text{capital}}$
η	0.50	0.50	data range	—
μ	0.122	0.087	$\sigma_{w/y} = 0.42$	see appendix
α	0.347	0.347	$\Pi_k = 34.74$	$\Pi_k = \alpha$
κ	0.29	0.29	data	$\Pi_v = 0.0126$
m	0.175	0.175	$\theta = 1.8, f = 0.139$	$f = s\theta^{1-\eta}$
$\frac{b}{w}$	0.956	0.970	$\Pi_{\text{labor}} = 0.64$	$b = \Pi_{\text{labor}} A \left(\frac{k}{h}\right)^{\alpha} - \frac{\mu}{1-\mu}\theta \left[1 + \frac{\widetilde{r}+\chi}{f}\right] \kappa$

Table 2: **Baseline Results**

Variable	**Data**	**Model**	**Model Inelastic**	HM	Veracierto
$\rho(u,y)$	-0.88	-0.85	-0.96	–	0.22
$\rho(u,v)$	-0.97	-0.49	-0.81	-0.97	–
$\rho(u,h)$	-0.95	-0.96	-1.00	–	–
$\sigma_{\theta/z}$	19.1	13.2	18.1	22.5	–
$\sigma_{u/z}$	9.5	5.9	8.1	11.2	4.9[1)]
$\sigma_{v/z}$	10.1	9.2	10.9	13.0	–
$\sigma_{c/y}$	0.61	0.29	0.27	–	0.31
$\sigma_{i/y}$	3.79	3.08	3.06	–	4.50
$\sigma_{h/y}$	0.60	0.44	0.36	–	0.56
$\sigma_{w/y}$	0.42	0.42	0.42	–	–
$\sigma_{p/y}$	0.20	0.20	—	–	0.58
μ	—	0.122	0.087	0.052	–
ν	—	0.0058	—	–	—
$\frac{b}{w}$	—	0.956	0.970	0.955	—

Values in the table are the ratio of the volatility of variable x to the volatility of the technology shock z or of output y. Data values for the relative volatilities of θ, v, u and f are taken from Shimer (2005), while data values for the relative volatilities of w, c and i come from Francis and Ramey (2001). Neither of these two sources reports the volatility of employment or participation. These numbers are based upon quarterly BLS data from 1964 Q1-2005 Q4 which has been HP-filtered using Ravn and Uhlig (2004)'s optimal parameter value for quarterly data of 1600. 1) Veracierto (2002) reports the realtive standard deviation of unemployment relative to output, not to productivity. I obtain $\sigma_{u/z} = \frac{\sigma_{u/y}}{\sigma_{z/y}} = \frac{2.26}{0.46}$.

Table 3: Targetting Employment Volatility

Variable	Data	Model	Model Inelastic
$\rho(u, y)$	-0.88	0.26	-0.96
$\rho(u, v)$	-0.97	1.00	-0.81
$\rho(u, h)$	-0.95	-0.07	-1.00
$\sigma_{\theta/z}$	19.1	0.7	18.1
$\sigma_{u/z}$	9.5	6.6	8.1
$\sigma_{v/z}$	10.1	6.9	10.9
$\sigma_{c/y}$	0.61	0.33	0.27
$\sigma_{i/y}$	3.79	3.13	3.06
$\sigma_{h/y}$	0.60	0.60	0.36
$\sigma_{w/y}$	0.42	0.42	0.42
$\sigma_{p/y}$	0.20	0.62	—
μ	—	0.737	0.087
ν	—	0.38	—

Values in the table are the ratio of the volatility of variable x to the volatility of the technology shock z or of output y. Data values for the relative volatilities of θ, v, u and f are taken from Shimer (2005), while data values for the relative volatilities of w, c and i come from Francis and Ramey (2001). Neither of these two sources reports the volatility of employment or participation. These numbers are based upon quarterly BLS data from 1964 Q1-2005 Q4 which has been HP-filtered using Ravn and Uhlig (2004)'s optimal parameter value for quarterly data of 1600. 1) Veracierto (2002) reports the realtive standard deviation of unemployment relative to output, not to productivity. I obtain $\sigma_{u/z} = \frac{\sigma_{u/y}}{\sigma_{z/y}} = \frac{2.26}{0.46}$.

Table 4: Intertemporal Elasticity of Consumption and Tightness Volatility

	σ_θ
$\eta = 1$	9.5 %
$\eta = 2$	9.7 %
$\eta = 3$	10.1 %
$\eta = 4$	10.6 %
$\eta = 5$	11.2 %

All tightness volatilities have been calculated assuming an elasticity of labor supply $\nu = 0.064$ and using the data values given in Section 5.3, according to equation (28).

Table 5: Baseline Calibration (Quarterly)

Parameter	**Value**	**Target**	**Equation**
A	1.0	normalization	—
ν	0.064	$\sigma_{p/y} = 0.20$	see appendix
ρ	0.765	data	—
σ_ε	0.83	data	—
χ	0.10	$f = 0.83, u = 5.5\%$	$u = \frac{\chi}{\chi+f}$
$\widetilde{\beta}$	0.988	$\widetilde{r} = 4.0$ % annually	$\widetilde{r} = \frac{1}{\widetilde{\beta}} - 1$
δ	0.031	$\frac{i}{y} = 0.25$	$\frac{i}{y} = \frac{\delta}{\widetilde{r}+\delta}\Pi_{\text{capital}}$
η	0.50	data range	—
μ	0.271	$\sigma_{w/y} = 0.42$	see appendix
α	0.347	$\Pi_k = 34.74$	$\Pi_k = \alpha$
κ	0.16	data	$\Pi_v = 0.0126$
m	0.62	$\theta = 1.8, f = 0.83$	$f = s\theta^{1-\eta}$
$\frac{b}{w}$	0.939	$\Pi_{\text{labor}} = 0.64$	$b = \Pi_{\text{labor}} A \left(\frac{k}{h}\right)^\alpha - \frac{\mu}{1-\mu}\theta\left[1 + \frac{\widetilde{r}+\chi}{f}\right]\kappa$

7 Appendix

7.1 Log-linearized System of Difference Equations

1. Tightness
$$\widehat{\theta}_t - \widehat{v}_t + \widehat{u}_t = 0$$

2. Unmployment and employment
$$h\widehat{h}_t + u\widehat{u}_t + l\widehat{l}_t = 0$$

3. Job-filling rate
$$\widehat{q}_t + \eta\widehat{\theta}_t = 0$$

4. Job finding rate
$$\widehat{f}_t - (1-\eta)\,\widehat{\theta}_t = 0$$

5. Production function
$$\widehat{y}_t - z_t - \alpha\widehat{k}_{t-1} - (1-\alpha)\,\widehat{h}_t = 0$$

6. Wage curve
$$-w\widehat{w}_t + \beta\,(1-\alpha)\,\frac{y}{h}\left(\widehat{y}_t - \widehat{h}_t\right) + \mu\kappa\theta\widehat{\theta}_t = 0$$

7. Household budget constraint
$$-c\widehat{c}_t + y\widehat{y}_t - v\kappa\widehat{v}_t - i\widehat{i}_t = 0$$

8. Evolution of employment
$$-h\widehat{h}_t + (1-\chi-f)\,h\widehat{h}_{t-1} + (1-h)\,f\widehat{f}_{t-1} = 0$$

9. Evolution of capital

$$-\widehat{k}_t + (1-\delta)\,\widehat{k}_{t-1} + \delta\widehat{i}_t = 0$$

10. Firm's optimality for labor

$$0 = \frac{\kappa}{q}\widehat{f}_t - \frac{\kappa}{q}\widehat{\theta}_t + E_t\left\{\left[\begin{array}{c} \widetilde{\beta}\,(1-\alpha)\,\frac{y}{h}\widehat{y}_{t+1} - \widetilde{\beta}\,(1-\alpha)\,\frac{y}{h}\widehat{h}_{t+1} - \widetilde{\beta}w\widehat{w}_{t+1} \\ -\,(1-\chi)\,\widetilde{\beta}\frac{\kappa}{q}\widehat{f}_{t+1} + (1-\chi)\,\widetilde{\beta}\frac{\kappa}{q}\widehat{\theta}_{t+1} \\ +\widetilde{\beta}\gamma\,(\widehat{c}_{t+1} - \widehat{c}_t)\left[w - (1-\chi)\,\frac{\kappa}{q} - (1-\alpha)\,\frac{y}{h}\right] \end{array}\right]\right\}$$

11. Firm's optimality for capital

$$E_t\left\{\alpha\frac{y}{k}\widetilde{\beta}\widehat{y}_{t+1} - \alpha\frac{y}{k}\widetilde{\beta}\widehat{k}_t - \widetilde{\beta}\gamma\left[\alpha\frac{y}{k} + 1 - \delta\right](\widehat{c}_{t+1} - \widehat{c}_t)\right\} = 0$$

12. Household's Euler equation for labor market participation

$$\begin{aligned} 0 \;\;=\;\; & \phi l^{-\frac{1}{\nu}}\frac{1}{\nu}\widehat{l}_t + f\mu\frac{w}{c}\left(\widehat{w}_{t+1} - \widehat{c}_{t+1}\right) + \mu\left[f\frac{w}{c} + \phi l^{-\frac{1}{\nu}}\left(1-\chi-f\right)\right]\widehat{f}_t \\ & -(1-\chi)\,\beta\phi l^{-\frac{1}{\nu}}\widehat{f}_{t+1} - \beta\phi l^{-\frac{1}{\nu}}\frac{1}{\nu}\left[1-\chi-f\right]\widehat{l}_{t+1} \end{aligned}$$

13. Evolution of shocks

$$z_t = \rho z_{t-1} + \varepsilon_t$$

This is a system of 13 linear equations in the 13 unknowns $(h_t, l_t, k_t, f_t, q_t, \theta_t, u_t, v_t, w_t, y_t, c_t, i_t, z_t)$.

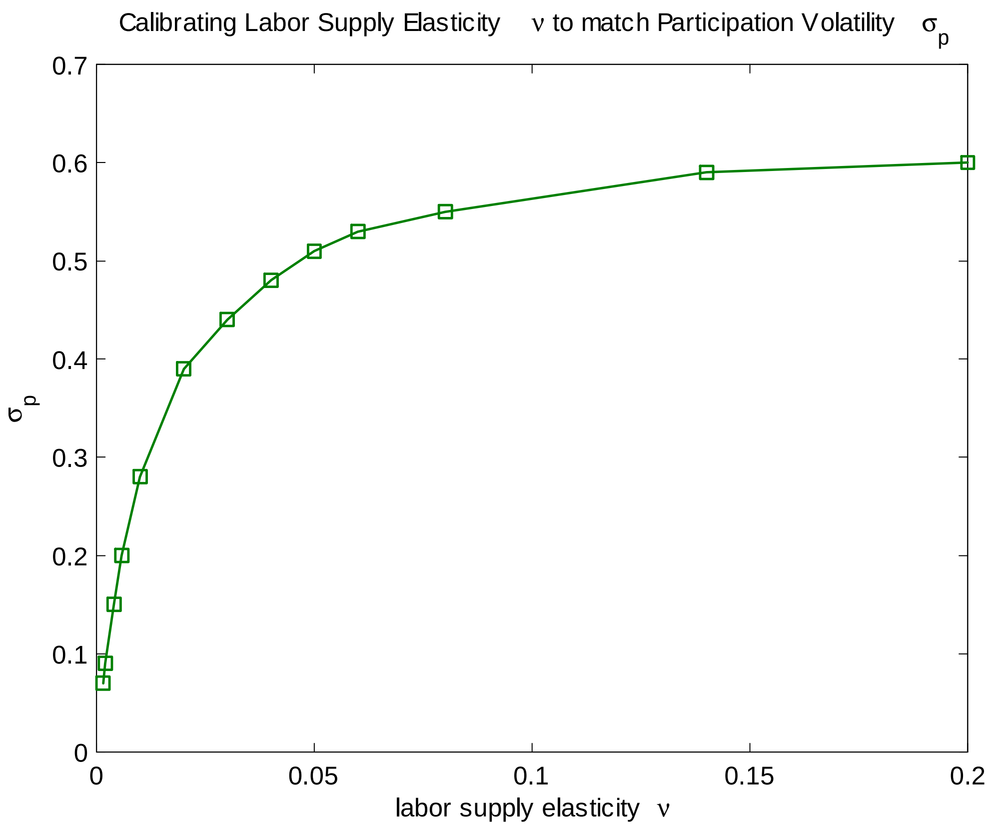

Figure 1: This figure plots the values for participation volatility σ_p implied by each value of participation elasticity ν. The graph was generated by creating a grid over labor supply elasticity values $\nu \in [0.0015, 2.00]$, recalibrating to all other targets for each gridpoint, and calculating the implied participation volatility.

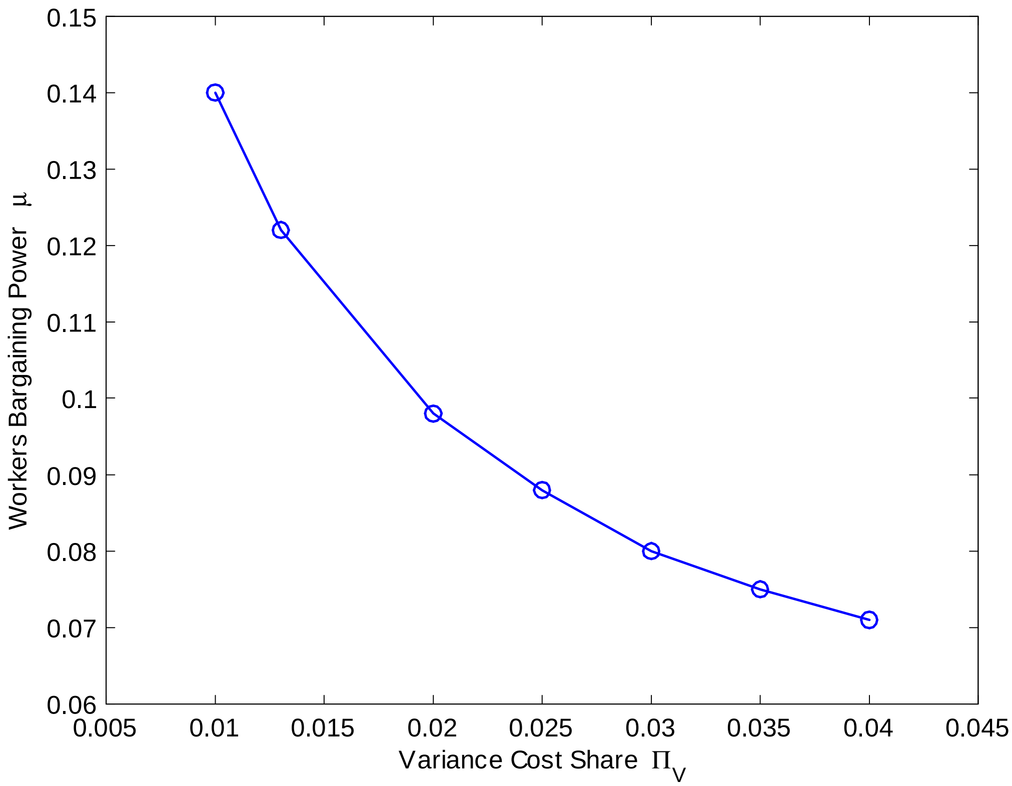

Figure 2: Each data point represents a pair of worker's bargaining power and vacancy cost share (μ, Π_V) which results in wage volatility which matches its target value of $\sigma_{w/y} = 0.42$. The graph was generated by creating a two-dimensional grid over $\mu \in [0.01, 0.99]$ and $\Pi_{\mathrm{v}} \in [0.01, 0.08]$, recalibrating to all other targets for each gridpoint, calculating the implied wage volatility for each gridpoint and isolating the appropriate (μ, Π_v) pairs.

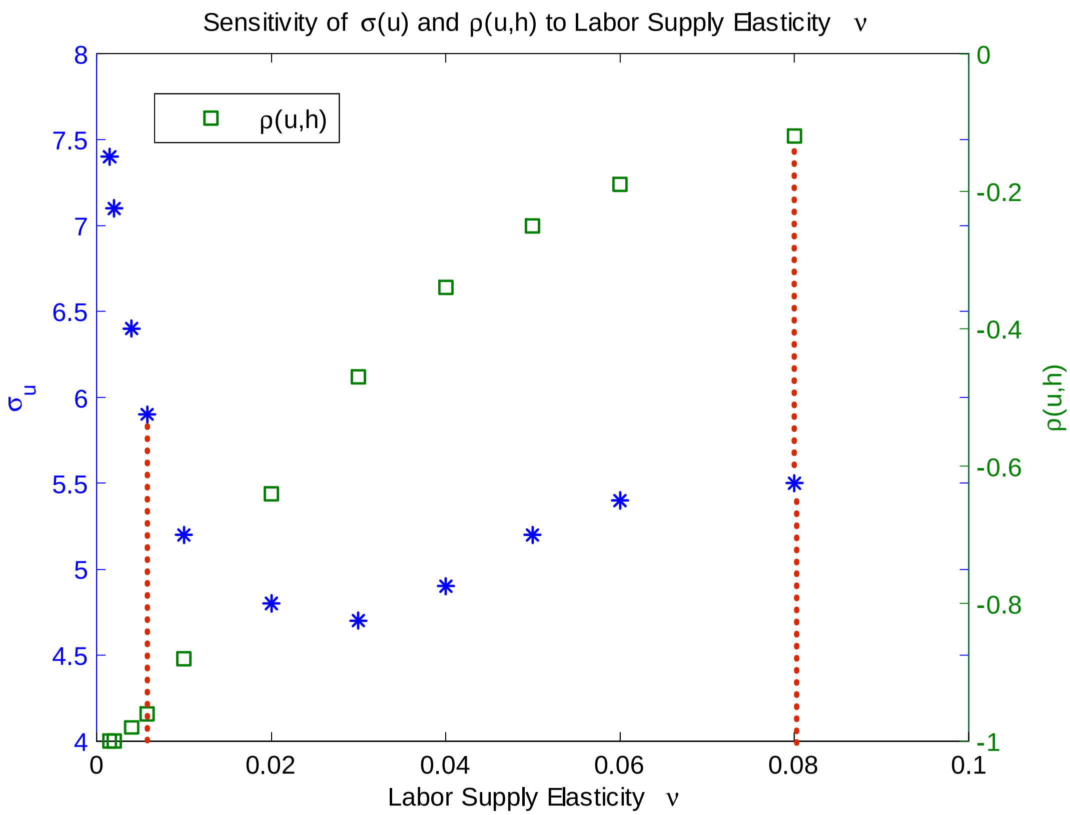

Figure 3: Plot of $\rho\,(u,h)$ and σ_u versus labor supply elasticity. Matching $\sigma_{p/y} = 0.20$ implies a labor supply elasticity value of $\nu = 0.0058$. To match $\sigma_{h/y} = 0.56$, one needs to set $\nu = 0.08$.

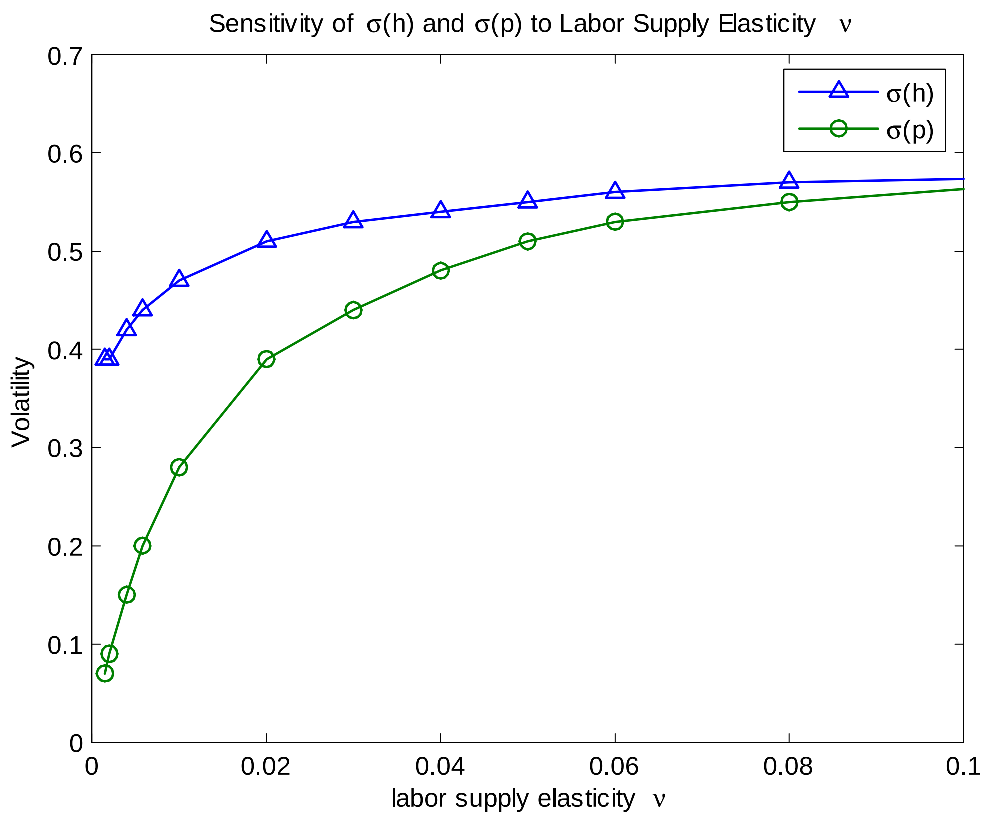

Figure 4: Sensitivity of implied values for employment and participation volatility when labor supply elasticity is varied.

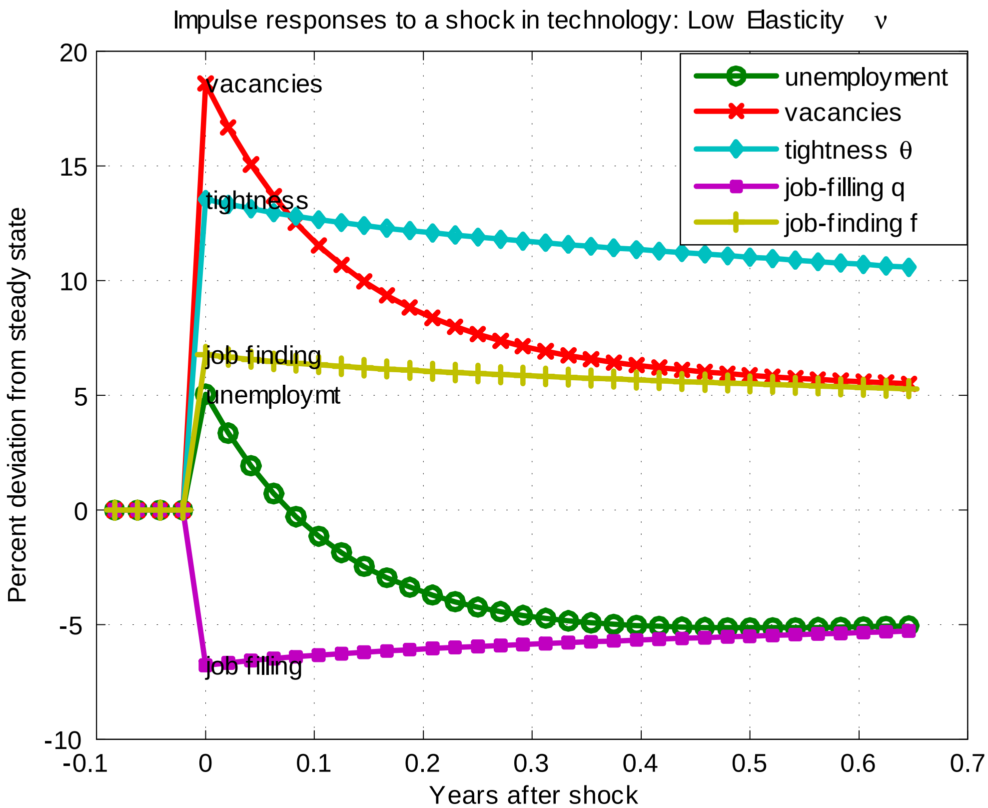

Figure 5: Low labor supply elasticity scenario, labor supply elasticity $\nu = 0.0058$ to match participation volatility $\sigma_{p/y} = 0.20$.

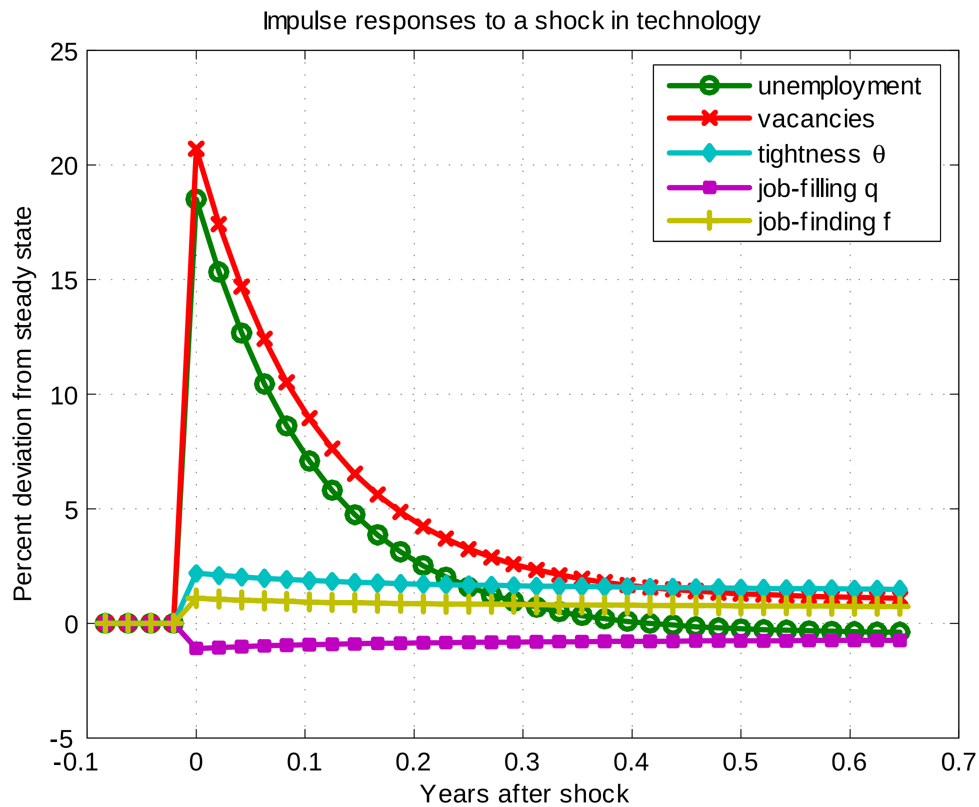

Figure 6: High labor supply elasticity scenario, labor supply elasticity $\nu = 0.08$ to match employment volatility $\sigma_{h/y} = 0.56$.

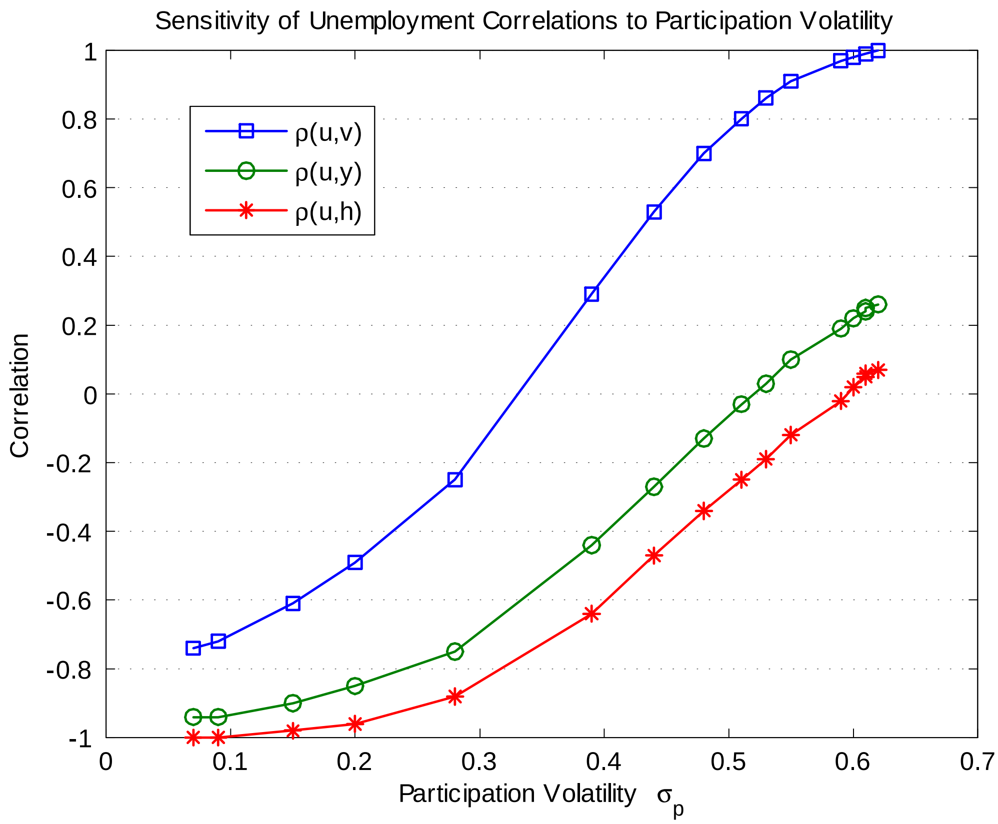

Figure 7: Sensitivity of unemployment correlations with output, vacancies and employment to the targetted participation volatility. Weekly calibration, all remaining calibration targets maintained.

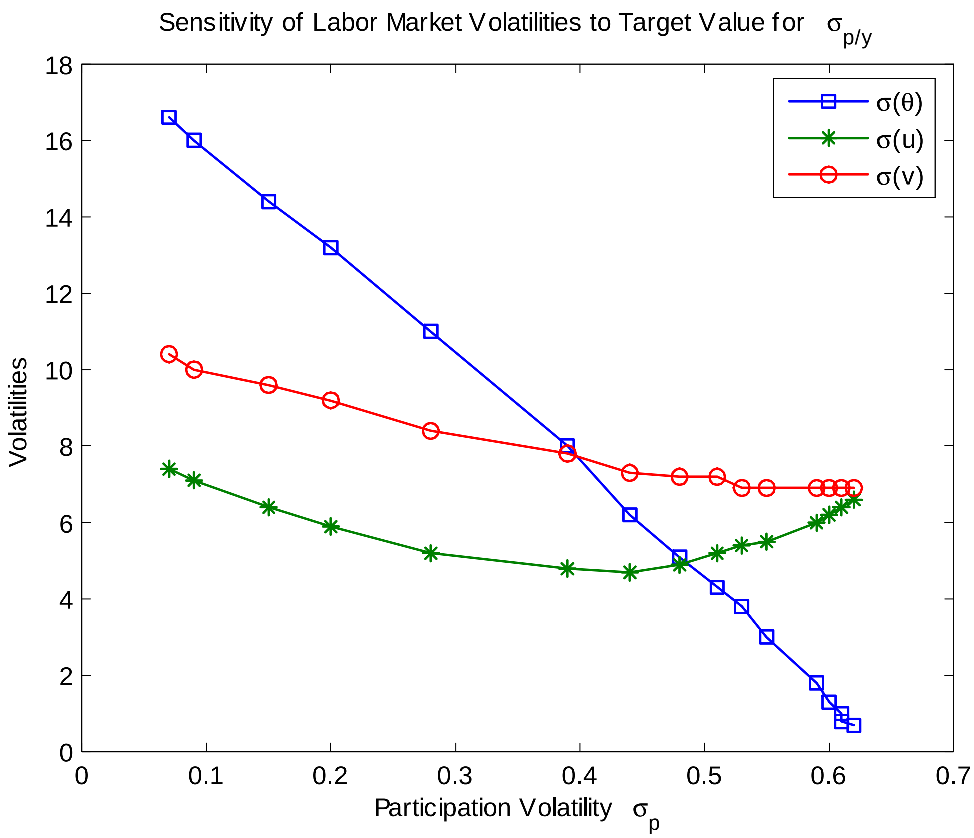

Figure 8: Sensitivity of volatilities of key labor market variables unemployment, vacancies and tightness to the targetted participation volatility. Weekly calibration, all remaining calibration targets maintained.