Resurrecting the Extensive Margin

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Shimer, et. al.

- Recent debate about cyclical properties of labor market variables in MP models
  - Inelastic labor supply
  - **Quantitative** issues

- Once a participation margin is added to MP models
  - **Quantitative** issues
  - **Qualitative** issues

The Extensive Margin

- Previous authors found that combining
  - Technology shocks
  - MP labor search frictions
  - Elastic labor supply along extensive (participation) margin

→ Procyclical unemployment

→ Positively sloped Beveridge curve [unemployment and vacancies positively correlated]

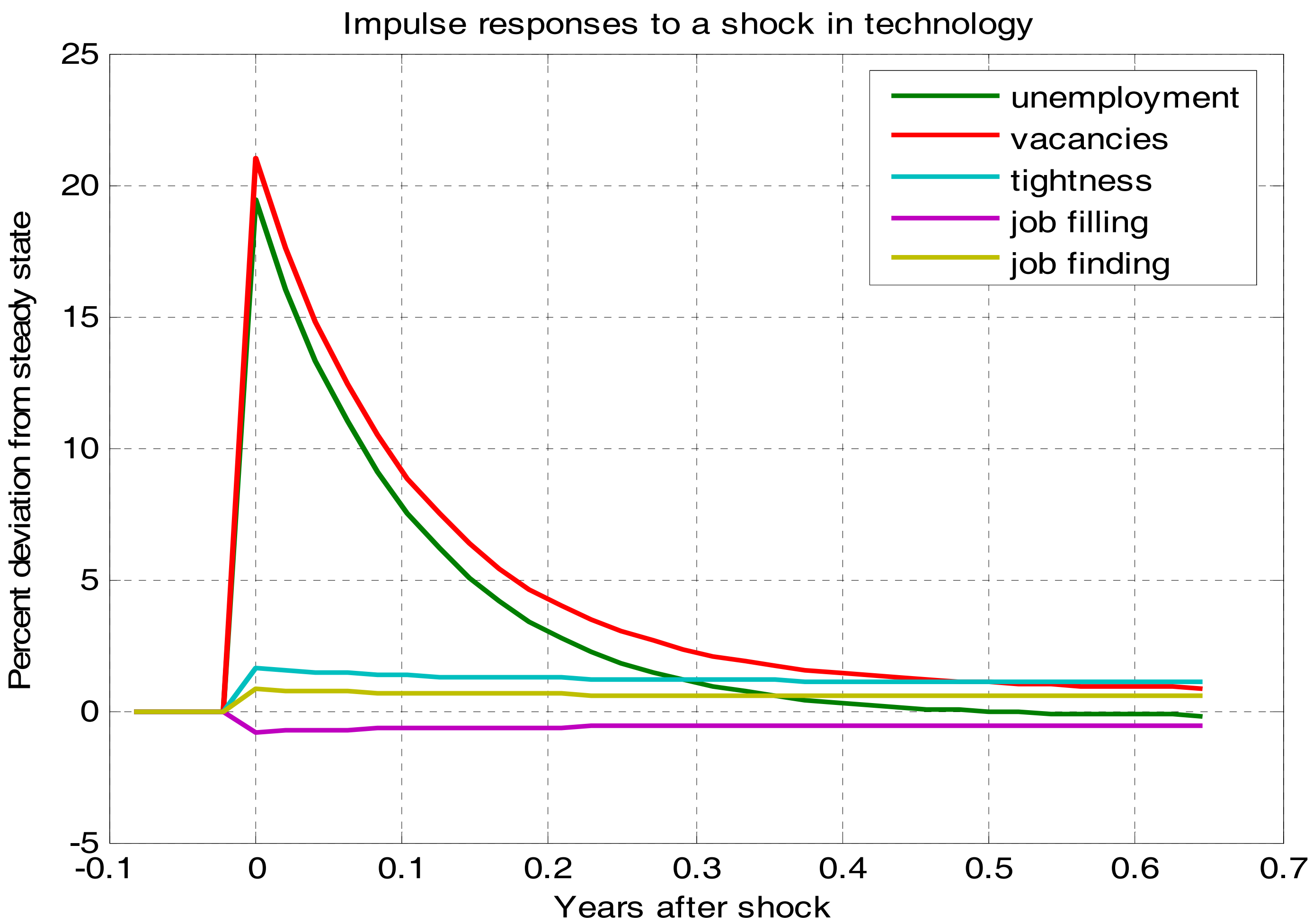
→ Limits use of search frictions in business cycle models.

What Goes Wrong?

- Elastic labor supply along extensive margin
  - Workers enter labor force via search in response to changes in wages and job-finding rates

- Positive technology shock leads to:
  - Workers flowing into search (unemployment) from out-of-labor-force
  - Increase in unemployment on impact
  - Firms increase vacancies
  - Little increase in job-finding rates

Impulse-Response: High Elasticity



How to Get it Right

- Previous authors [Veracierto (2002), Tripier (2003), Ravn (2005)] either
  - assume utility linear in labor/leisure time
  - choose labor supply elasticity to **target employment volatility**

- This paper
  - chooses labor supply elasticity to **target participation volatility**

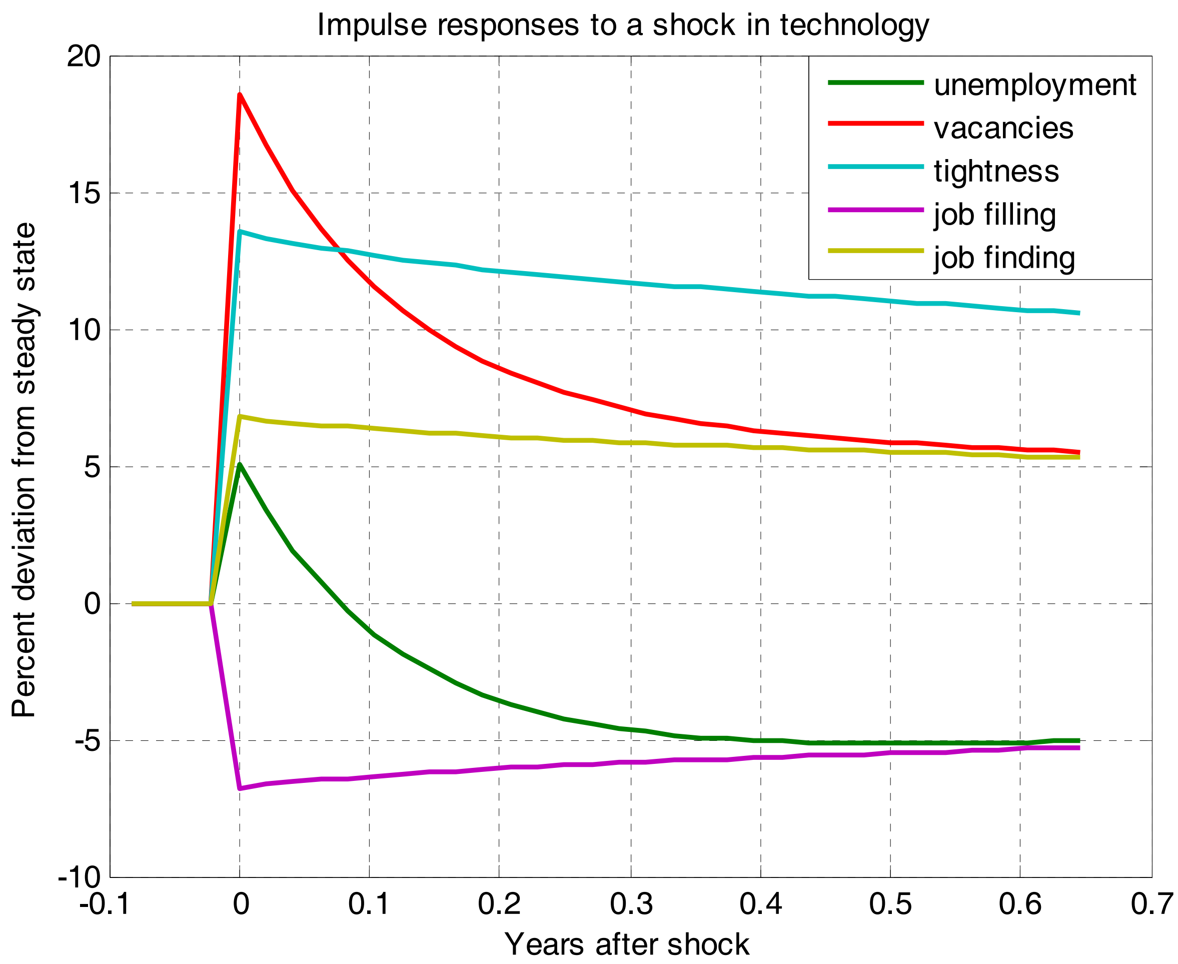
How to Get it Right

- Participation $p_t = u_t + h_t$
- Employment vs. Participation Volatility

$$\boxed{p^2\sigma_p^{\ 2}} = u^2\sigma_u^{\ 2} + \boxed{h^2\sigma_h^{\ 2}} + 2hu\,\mathrm{cov}(u,h)$$

- The two targets would only be equivalent if model generated equal
  - Unemployment volatility σ_u YES
  - Correlation between Unemployment and Employment $\rho_{u,h}$ NO

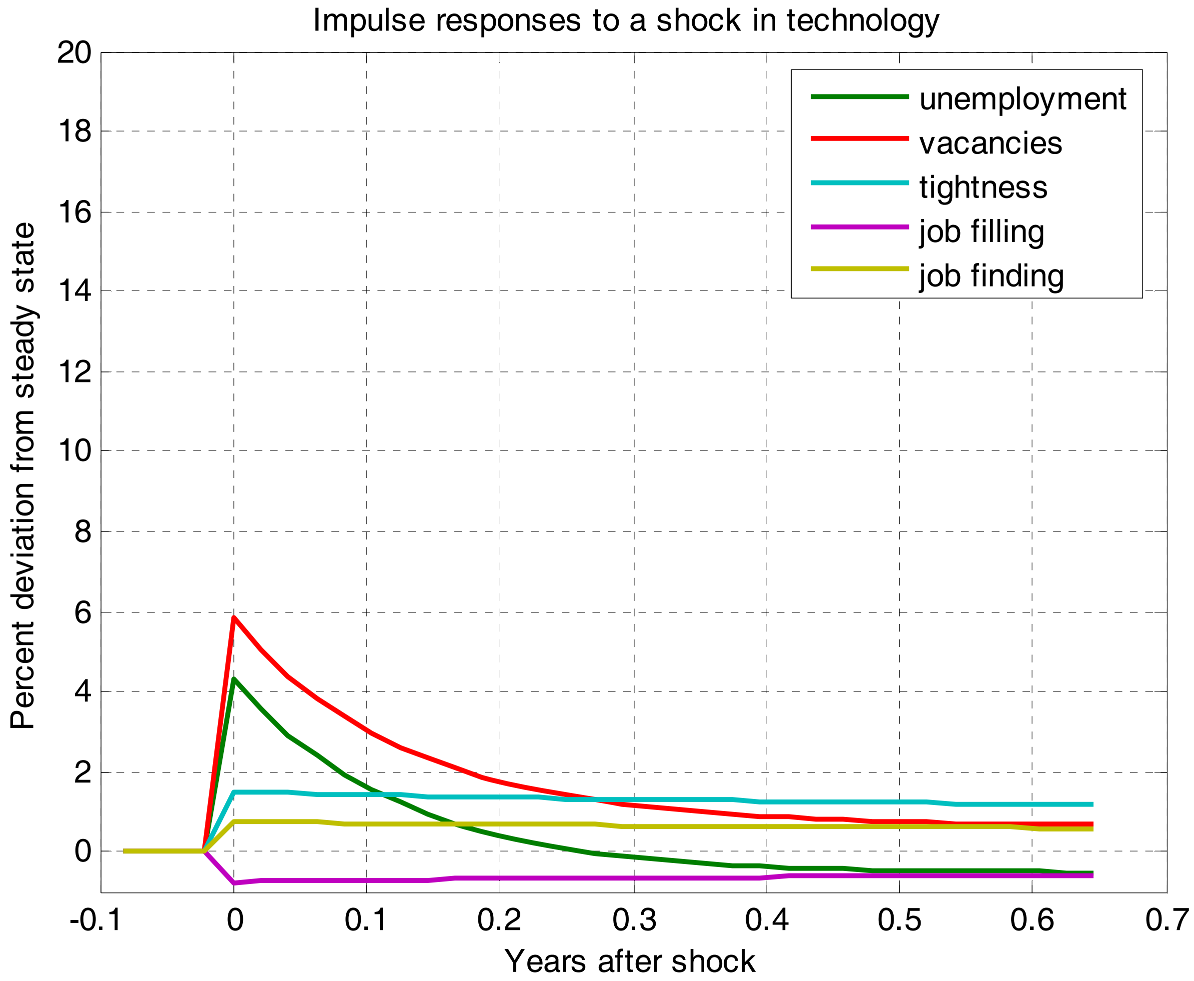
Impulse-Response: Target σ_p



How to Get it Right, 2

- Wage volatility
  - Shimer vs. Hagedorn/Manovskii
  - If wages too elastic w.r.t. productivity, then vacancies increase about as much as unemployment → little increase in job-finding rates
  - Here, target wage volatility to match data
- Time Aggregation
  - weekly, not quarterly calibration
  - unemployment has procyclical impact at weekly frequency, but not quarterly

Wage Volatility High, Target p



Outline

1. Introduction
2. Model
3. Calibration
4. Quantitative Results

Households' Problem

- Large family solves

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left[\ln c_t + \phi \frac{l_t^{1-\frac{1}{\nu}}}{1-\frac{1}{\nu}} \right]$$

subject to budget and time constraints:

$$w_t h_t + r_t k_{t-1} + \pi_t \geq c_t + i_t$$

$$1 = h_t + u_t + l_t$$

$$k_t = (1-\delta) k_{t-1} + i_t$$

$$h_{t+1} = (1-\chi) h_t + f_t u_t$$

Households' Euler Equations

- Participation

$$\underbrace{u_{l.t}}_{\phi l^{-\frac{1}{\nu}}} = f_t \beta E_t \left\{ w_{t+1} u_{c,t+1} - u_{l,t+1} + u_{l,t+1} \frac{1-\chi}{f_{t+1}} \right\}$$

- Consumption

$$u_{c,t} = \beta E_t \left\{ u_{c,t+1} \left[r_{t+1} + 1 - \delta \right] \right\}$$

Labor Search

- Standard MP search framework
- CRS Matching Function $m(U_t, V_t) = U_t^{\eta} V_t^{1-\eta}$
- Job finding rate $f(\theta_t) = \frac{m(U_t, V_t)}{U_t} = \theta_t^{1-\eta}$
- Job filling rate $q(\theta_t) = \frac{m(U_t, V_t)}{V_t} = \theta_t^{-\eta}$
- Labor market tightness $\theta_t \equiv \frac{V_t}{U_t}$

Worker's Values

- Value of Employment

$$V^E{}_t = w_t - u_l\left(c_t, l_t\right) + E_t\left\{\tilde{\beta}_{t+1}\left[\left(1-\chi\right)V^E{}_{t+1} + \chi V^U{}_{t+1}\right]\right\}$$

- Value of Unemployment

$$V^U{}_t = b - u_l\left(c_t, l_t\right) + E_t\left\{\tilde{\beta}_{t+1}\left[f_t V^E{}_{t+1} + \left(1-f_t\right)V^U{}_{t+1}\right]\right\}$$

→ Worker's Surplus

$$V^W{}_t = w_t - b + \left(1-\chi-f_t\right)E_t\left\{\tilde{\beta}_{t+1}V^W{}_{t+1}\right\}$$

Firm's Problem

- Firms solve:

$$V^J\left(z_t, h_t\right) = \max_{v_t, i_t} y_t - w_t h_t - r_t k_{t-1} - \kappa v_t + E_t\left\{\tilde{\beta}_{t+1} V^J\left(z_{t+1}, h_{t+1}\right)\right\}$$

subject to

production function: $y_t = Ae^{z_t} k_{t-1}{}^{\alpha} h_t^{1-\alpha}$

transition h_t: $h_t = \left(1-\chi\right) h_{t-1} + q_t v_t$

wage curve: $w_t = w\left(h_t, z_t\right)$

technology shock: $z_t = \rho z_{t-1} + \varepsilon_t$

Firm's Optimality

- Capital $$r_t = f_k\left(z_t, k_{t-1}, h_t\right)$$
- Vacancies $$\frac{\kappa}{q_t} = E_t\left\{\tilde{\beta}_{t+1} V_h^{\,J}\left(h_{t+1}, z_{t+1}\right)\right\}$$
- Envelope h_t $$V^J{}_h\left(z_t, h_t\right) = f_h\left(z_t, h_t, k_{t-1}\right) - w_t + \left(1-\chi\right)\frac{\kappa}{q_t}$$

→ *Euler Equation Labor*

$$\frac{\kappa}{q_t} = E_t\left\{\tilde{\beta}_{t+1}\left[f_h\left(z_{t+1}, h_{t+1}, k_t\right) - w_{t+1} + \left(1-\chi\right)\frac{\kappa}{q_{t+1}}\right]\right\}$$

Individual Wage Bargaining

- Individual bargaining with multi-worker firm
- Firm's surplus is marginal value of worker

- Nash bargaining problem

$$\max_{w_t} \mu \ln V^{W}{}_{t} + (1-\mu) \ln \frac{\partial V^{J}(h_t, z_t)}{\partial h_t}$$

→ Wage curve

$$w_t = (1-\mu) b + \mu \left[f_h(k_{t-1}, h_t, z_t) + \kappa \theta_t \right]$$

Equilibrium

- Sequence of wages and tightnesses $\{\theta_t, w_t\}_{t=0}^{\infty}$ that satisfy
  - HH's Euler equations
  - Firm's optimality conditions
  - Firm's and HH's constraints
  - Wage curve
  - Transition for aggregate unemployment
  - Market Clearing
- 13 log-linear equations in the 13 unknowns $(h_t, l_t, k_t, f_t, q_t, \theta_t, u_t, v_t, w_t, y_t, c_t, i_t, z_t)$

Calibration

- Key element: Use participation volatility σ_p to pin down participation elasticity ν

- Otherwise: Similar to Hagedorn and Manovskii (2006)

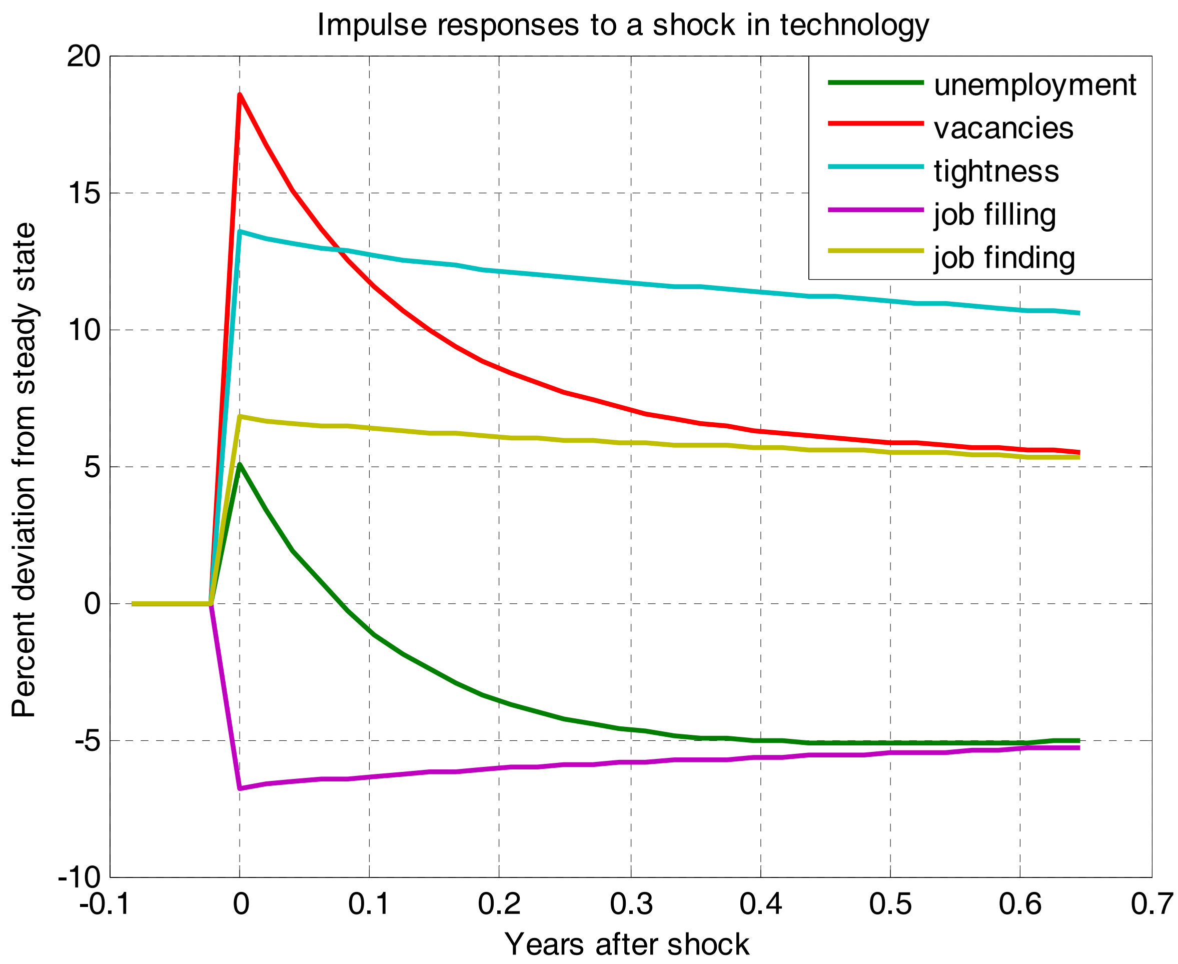
Baseline Calibration: Weekly

Parameter	Value	Inelastic	Target
A	1.0	1.0	normalization
ν	0.0058	—	$\sigma_{p/y} = 0.20$
ρ	0.9895	0.9895	data
σ_{ε}	0.34	0.34	data
χ	0.0081	0.0081	$u = 5.5\ \%, f = 0.139$
$\widetilde{\beta}$	$0.988^{1/12}$	$0.988^{1/12}$	$\widetilde{r} = 4.0\ \%$ annually
δ	0.0026	0.0026	$\frac{i}{y} = 0.25$
η	0.50	0.50	data range
μ	0.122	0.087	$\sigma_{w/y} = 0.42$
α	0.347	0.347	$\Pi_k = 34.74\ \%$
κ	0.29	0.29	data
m	0.175	0.175	$\theta = 1.8, f = 0.139$
$\frac{b}{w}$	0.956	0.970	$\Pi_l = 64.00\ \%$

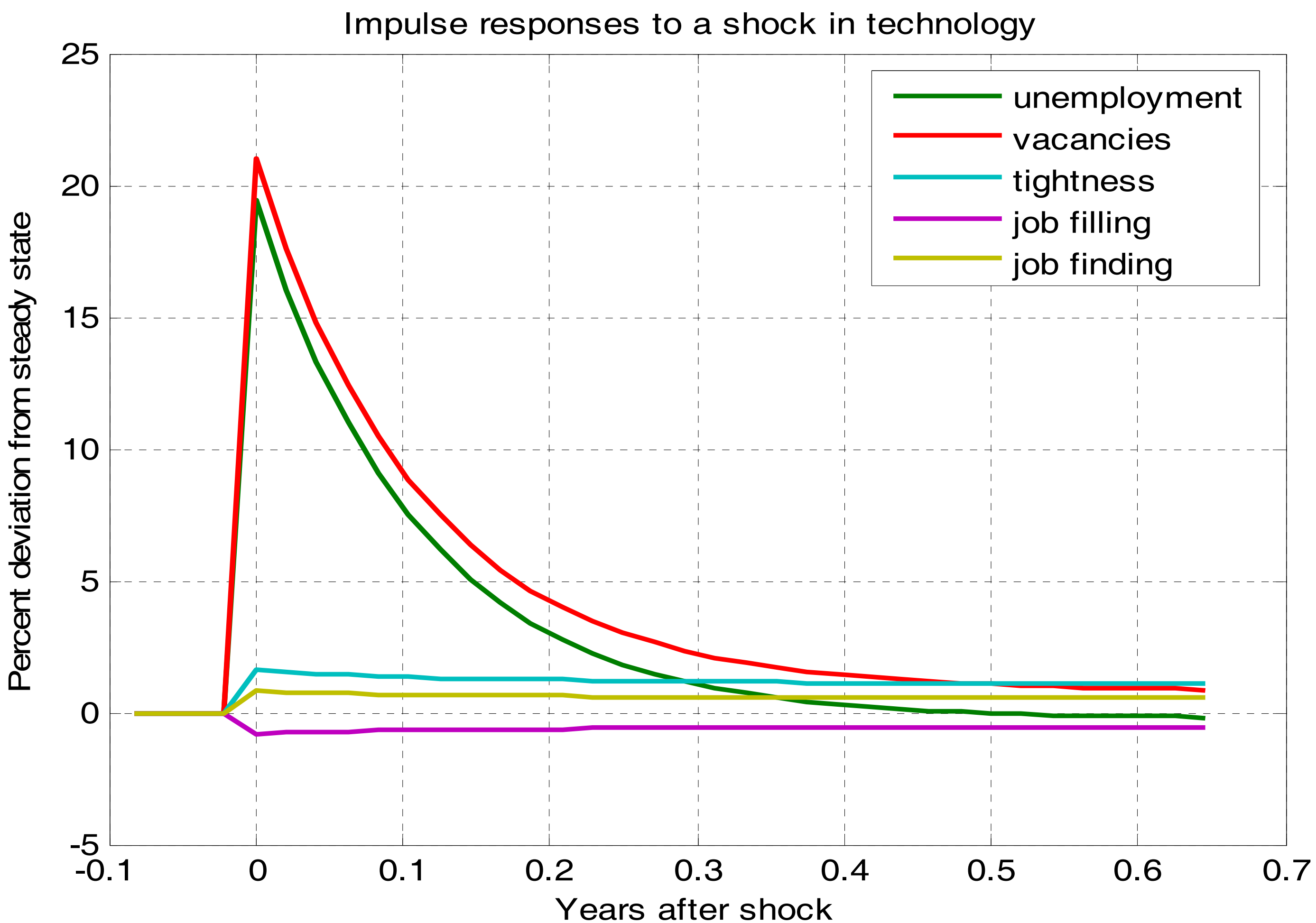
Baseline Results: Correlations

Variable	Data	Target p	Target h	Veracierto
$\rho(u,y)$	-0.88	-0.85	0.26	0.22
$\rho(u,v)$	-0.97	-0.49	1.00	–
$\rho(u,h)$	-0.95	-0.96	-0.07	–
$\sigma_{w/y}$	0.42	0.42	0.42	–
$\sigma_{p/y}$	0.20	0.20	0.62	0.58
$\sigma_{h/y}$	0.60	0.44	0.60	0.56
μ	—	0.122	0.737	–
ν	—	0.0058	0.38	—

Impulse-Response: Target σ_p



Impulse-Responses: Target σ_h



Baseline Results: Volatilities

Variable	Data	Target p	Target h	Veracierto
$\rho(u,v)$	-0.97	-0.49	1.00	–
$\sigma_{\theta/z}$	19.1	13.2	0.7	–
$\sigma_{u/z}$	9.5	5.9	6.6	4.9[1)]
$\sigma_{v/z}$	10.1	9.2	6.9	–
$\sigma_{c/y}$	0.61	0.29	0.33	0.31
$\sigma_{i/y}$	3.79	3.08	3.13	4.50
$\sigma_{h/y}$	0.60	0.44	0.60	0.56
μ	—	0.122	0.737	–
$\frac{b}{w}$	—	0.956	0.970	—

Sensitivity: Volatilities

Impact of Participation Margin

Variable	Data	Target $\sigma_{p/y}$	Inelastic	HM
$\rho(u,y)$	-0.88	-0.85	-0.96	–
$\rho(u,v)$	-0.97	-0.49	-0.81	-0.97
$\rho(u,h)$	-0.95	-0.96	-1.00	–
$\sigma_{\theta/z}$	19.1	13.2	18.1	22.5
$\sigma_{u/z}$	9.5	5.9	8.1	11.2
$\sigma_{v/z}$	10.1	9.2	10.9	13.0
$\sigma_{w/y}$	0.42	0.42	0.42	–
$\sigma_{p/y}$	0.20	0.20	–	–
μ	–	0.122	0.087	0.052
$\frac{b}{w}$	–	0.956	0.970	0.955

Consumption-Tightness Puzzle

- In a nearly identical model, Ravn (2006) derives a relationship between c_t and θ_t:

$$\theta_t = \frac{1-\mu}{\mu}\frac{1}{\kappa}\frac{u_{l,t}}{u_{c,t}}$$

but he assumes a utility function that is linear in leisure:

$$u(c_t, l_t) = c_t^{1-\eta}/(1-\eta) - \phi h_t$$

→ Linear relationship between volatilities of c_t and θ_t

High, 23.7 %

$$\sigma(\hat{\theta}_t) = \eta \cdot \sigma(\hat{c}_t)$$

Low, 1.23 %

Reducing the Puzzle

- Now take utility function that allows for finite elasticity of participation:

$$u\left(c_t, l_t\right) = \frac{c_t^{1-\eta}}{1-\eta} - \phi \frac{l_t^{1-\frac{1}{\nu}}}{1-\frac{1}{\nu}}$$

→ New relationship between volatilities of c_t and θ_t

244.1

31.3

23.7 %

$$\sigma\left(\hat{\theta}_t\right) = \sqrt{\eta \cdot \sigma^2\left(\hat{c}_t\right) + \frac{1}{\nu^2}\sigma^2\left(\hat{l}_t\right) - 2\frac{1}{\nu}\operatorname{cov}\left(\hat{c}_t, \hat{l}_t\right)}$$

Consumption-Tightness Puzzle?

	$\sigma\left(\widehat{\theta}_t\right)$
$\eta = 1$	9.5 %
$\eta = 2$	9.7 %
$\eta = 3$	10.1 %
$\eta = 4$	10.6 %
$\eta = 5$	11.2 %

Summary

1. Resurrected RBC with labor search and participation margin
   - Procyclical unemployment
   - Positively-sloped Beveridge curve
   - Decent cyclical volatilities

2. Participation elasticity matters for cyclical volatilities

3. Can contribute substantially to resolving the consumption-tightness puzzle.

Baseline Calibration: Quarterly

Parameter	Value	Target
A	1.0	normalization
ν	0.064	$\sigma_{p/y} = 0.20$
ρ	0.765	data
σ_{ε}	0.83	data
χ	0.10	$f = 0.83, u = 5.5\%$
$\widetilde{\beta}$	0.988	$\widetilde{r} = 4.0\ \%$ annually
δ	0.031	$\frac{i}{y} = 0.25$
η	0.50	data range
μ	0.271	$\sigma_{w/y} = 0.42$
α	0.347	$\Pi_k = 34.74$
κ	0.16	data
m	0.62	$\theta = 1.8, f = 0.83$
$\frac{b}{w}$	0.939	$\Pi_{\text{labor}} = 0.64$

Sensitivity: Correlations

Baseline Results

Variable	Data	Target $\sigma_{p/y}$	Target $\sigma_{h/y}$	Veracierto
$\rho(u,y)$	-0.88	-0.85	-0.96	0.22
$\rho(u,v)$	-0.97	-0.49	-0.81	–
$\rho(u,h)$	-0.95	-0.96	-1.00	–
$\sigma_{\theta/z}$	19.1	13.2	18.1	–
$\sigma_{u/z}$	9.5	5.9	8.1	4.9[1)]
$\sigma_{v/z}$	10.1	9.2	10.9	–
$\sigma_{c/y}$	0.61	0.29	0.27	0.31
$\sigma_{i/y}$	3.79	3.08	3.06	4.50
$\sigma_{h/y}$	0.60	0.44	0.36	0.56
$\sigma_{w/y}$	0.42	0.42	0.42	–
$\sigma_{p/y}$	0.20	0.20	—	0.58
μ	—	0.122	0.087	–
ν	—	0.0058	—	—
$\frac{b}{w}$	—	0.956	0.970	—