Bounded Rationality and Macroeconomics

Winter term 2005/06
Week 13

Holger Gerhardt

E-MAIL gerhardt@wiwi.hu-berlin.de
PHONE 030 2093-1680
OFFICE HOURS Tuesday, 2:15–3:45 p.m., room 334

Overview

Review of rational expectations

1. "Weak-form" rational expectations
2. "Strong-form" rational expectation

Review of LEIJONHUFVUD*'s (1993) arguments*

1. Computer simulations vs. analytical solutions
2. "Modelling the elements": focus on individual behaviour

ARTHUR ET AL. *(1996): "Asset Pricing Under Endogenous Expectations in an Artificial Asset Market"*

1. Indeterminacy of expectations under heterogeneity
2. Description of the asset market and the rational-expectations equilibrium
3. Learning in the framework of a "classifier system"
4. Simulation results

Review of rational expectations

Weak-form RE (Tesfatsion, 2004, pp. 1–2):

Suppose an economy consists of N agents, $i = 1, \ldots, N$, and that $I_{t,i}$ denotes the information set available to agent i at the end of some period t. Let $\mathrm{E}_{t,i}v_{t+k}$ denote agent i's subjective (personal) expectation formed at the end of period t regarding the value that some variable v will take on in period $t + k$ for some $k \geq 1$. Also, let $\mathrm{E}[v_{t+k} \,|\, I_{t,i}]$ denote the objectively true expectation for v_{t+k} conditional on $I_{t,i}$. Then the agents are said to have *weak-form rational expectations* for v_{t+k} if the following condition holds: For each $i = 1, \ldots, N$, $\mathrm{E}_{t,i}v_{t+k} = \mathrm{E}[v_{t+k} \,|\, I_{t,i}] + \varepsilon_{t,i}$ where the $\varepsilon_{t,i}$ are serially and mutually independent finite-variance error terms that satisfy $\mathrm{E}[\varepsilon_{t,i} \,|\, I_{t,i}] = 0$.

Strong-form RE (Tesfatsion, 2004, p. 3):

Agents in a theoretical model of a multi-agent economy will be said to have *strong-form RE* if they have weak-form RE and, in addition, their information sets at the end of each period t contain the following information:
(a) the true structural equations and classification of variables for the model, *including the actual decision rules used by each private and public (government) agent to generate actions and/or expectations*;
(b) the true values for all deterministic exogenous variables for the model;
(c) the true probability distributions governing all exogenous stochastic terms;
(d) Realized values for all endogenous variables observed by the modeller through the end of period t.

An example of strong-form RE

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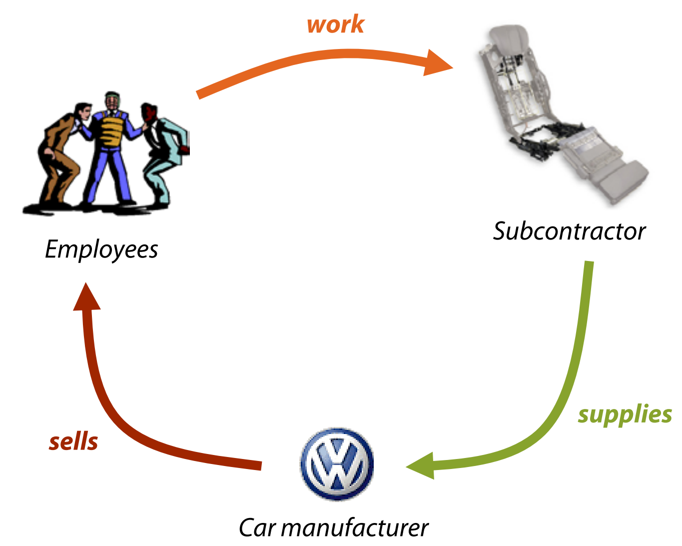

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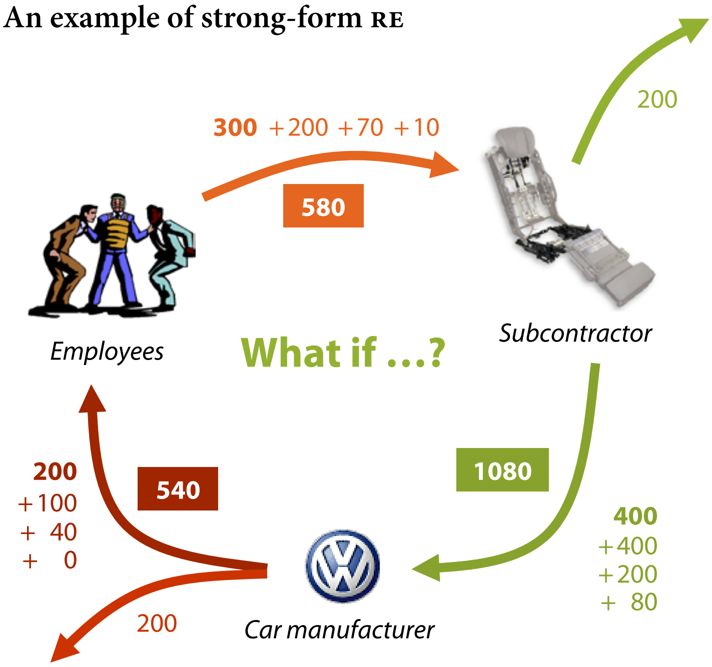

Review of Leijonhufvud's (1993) arguments

"[A]nalytical methods will not take us very far. The properties of more complex systems can *only* be investigated through computer simulations.

"Computable Economics will not only mean the study of more complex systems; it will also bring a different orientation towards the modelling of the elements of those systems, that is towards the representation of individual behavior" (p. 1).

"The economy is best conceived of as a network of interacting processors, each one with less capability to process information than would be required of a central processor set to solve the overall allocation problem for the entire system" (p. 4).

Arthur et al. (1996)

Inductive (instead of *deductive*) reasoning (see p. 5):

Assume econometrician/economist is faced with available shared information I_t, consisting of past prices, past dividends, trading volumes, economic indicators, rumors, news, etc.

Without knowing the true data-generating process, *there are many different, perfectly defensible statistical ways,* based on different assumptions and error criteria, to use the information to predict future dividends.

Thus, there is no objectively laid-down expectational model which differing agents can coordinate upon. Hence, no objective means for agents to know [deduce] other agents' expectations of future dividends exists.

Forming expectations by inductive reasoning

Consequence (see pp. 6 and 7):

"[S]o, perfect rationality in the market can not be well-defined. Infinitely intelligent agents cannot form expectations in a determinate way."

Instead, "[t]hey may derive expectational models by sophisticated, subjective reasoning. But in the end all such models will be—can only be—hypotheses. There is no objective way to verify them, except by observing their performance in practice. Thus agents, in facing the problem of choosing appropriate predictive models, face the same problem that statisticians face when choosing appropriate predictive models given a specific data set, but no objective means by which to choose a functional form."

Forming expectations by inductive reasoning

"Of course, the situation here is made more difficult by the fact that the expectational models investors choose affect the price sequence, so that our statisticians' very choices of model affect their data and so their choices of model.

"… [W]e assume that each agent acts … continually creates multiple 'market hypotheses'—subjective, expectational models—of what moves the market price and dividend. … Some of these will perform well in predicting market movements. These will gain the agent's confidence and be retained and acted upon in buying and selling decisions. Others will perform badly. They will be dropped" (p. 7).

Forming expectations by inductive reasoning

"This type of behavior—coming up with appropriate hypothetical models to act upon, strengthening confidence in those that are validated, and discarding those that are not—is called *inductive reasoning*. It makes excellent sense where problems are ill-defined. It is, in micro-scale, the scientific method. Agents that act use inductive reasoning we will call inductively rational" (p. 7).

The model setup

Analysis uses the Santa Fe Artificial Asset Market

$N = 25$ heterogeneous agents:
Each agent i forms individual expectations of next period's dividend and price, given market information I_t: $E_i[p_{t+1} \mid I_t]$.

Apart from expectations, purely neoclassical assumptions:
—agents maximize utility;
—every possibility of arbitrage exploited;
—market clears at single price.

Agents' utility function is of CARA type:
$U(c) = -\exp(\lambda c)$.

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The model setup

Two assets:

—risk-free asset: paying r units of consumption per period (in unlimited supply);

—risky asset: fixed supply of N units, each paying a stochastic dividend sequence $\{d_t\}_{t=1}^{\infty}$.

Price of risky asset at time t denoted by p_t.

Assume that agent i's predictions at time t of next period's price and dividend are normally distributed with mean $\mathrm{E}_{i,t}(p_{t+1}+d_{t+1})$ and variance $\sigma^2_{p+d,i,t}$.

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Solving the model

In combination with CARA utility, this yields:

$$x_{i,t} = \frac{\mathrm{E}_{i,t}(p_{t+1}+d_{t+1}) - p_t(1+r)}{\lambda \sigma^2_{p+d,i,t}}.$$

Closing the model: demand equals supply,

$$\sum_{i=1}^{N} x_{i,t} = N.$$

This delivers the market-clearing price,

$$p_t = \frac{1}{1+r} \sum_{j=1}^{N} w_{j,t} \mathrm{E}_j[p_{t+1}+d_{t+1} \mid I_t],$$

$$\text{where } w_{j,t} = \frac{1}{\sigma^2_{p+d,j,t}} \Bigg/ \left(\sum_{k=1}^{N} \frac{1}{\sigma^2_{p+d,k,t}} \right).$$

Solving the model

Under rational expectations:

$$p_t = \sum_{k=1}^{\infty} \left(\tfrac{1}{1+r}\right)^k \mathrm{E}[d_{t+k} \mid I_t].$$

Assume that the dividends follow an AR(1) process which is known to the agents:

$$d_t = \bar{d} + \rho(d_{t-1} - \bar{d}) + \varepsilon_t.$$

where $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$.

This determines p_t. Furthermore,

$$\mathrm{E}_t[p_{t+1} + d_{t+1}] = (1+r)p_t + \frac{\lambda(2+r)\sigma_\varepsilon^2}{1+r-\rho}.$$

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Modelling heterogeneous expectations

How the agents form their expectations:

$$\mathrm{E}_i[d_{t+1} + p_{t+1}] = a_i(d_t + p_t) + b_i.$$

Initial values of a_i and b_i are assigned to agents; they are centred around the rational-expectation equilibrium values.

However, the values of a_i and b_i are not constant across different market situations. Instead, agents can distinguish between various *market states* via a *classifier system.*

Classifier system: collection of bits (0 or 1) indicating whether certain criterion is fulfilled or not.

Modelling heterogeneous expectations

In this case: $J = 12$ bits for recognising different "market states". Meaning of the bits:

1–6 (Current price × interest rate) / dividend > 0.25, 0.5, 0.75, 0.875, 1.0, 1.125

7–10 Current price > 5-period moving average of past prices (MA), 10-period MA, 100-period MA, 500-period MA

11 Always on (1)

12 Always off (0)

Agents observe the market state. Based on their observation, they come up with a (subjective) prediction, i.e. values for a_i and b_i.

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Modelling heterogeneous expectations

Example of a market state:

$0\ 1\ 1\ 0\ \boxed{1}\ 0\ 0\ 1\ 0\ \boxed{0}\ 1\ 0$

5th position: *equals 1 if (current price × interest rate) / dividend > 1*

10th position: *equals 1 if price is larger than its 500-period moving average*

Modelling heterogeneous expectations

Example of a market state:

$$0\ 1\ 1\ 0\ \boxed{1}\ 0\ 0\ 1\ 0\ \boxed{0}\ 1\ 0$$

Example of a predictor:

$$[(\#\ \#\ \boxed{\#}\ \#\ \boxed{1}\ \#\ \#\ \#\ \#\ \boxed{0}\ \#\ \#),\ (0.96, 0)]$$

"#" means: Don't take this market state into account for prediction.

When market descriptor features 1 and 0 *here forecast* $a_i = 0.96$ *and* $b_i = 0$, i.e. $E_i[d_{t+1} + p_{t+1}] = 0.96\,(d_t + p_t) + 0$.

Modelling heterogeneous expectations

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During simulation they discard useless predictors and replace them with new ones. Achieved by every predictor storing the moving average of its past squared forecast error.

If multiple predictors are active, only the one that was most accurate in the past is used.

The less accurate a predictor, the higher the chance that it is replaced through recombination.

Simultaneously, the more states are set, the higher the probability of recombination. This causes a weak drift towards the "all-#" configuration.

Bidding and selling procedure

Each agent is made available all market information, i.e. current and past prices and dividends (however, not other agents' expectations and intentions).

Each agent forms individual expectations and decides on that basis what holding the risky asset is worth to her.

All bids and offers are passed to a "specialist" who sets the market-clearing price.

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Which bits are acted upon?

Sequential structure of predictors allows "mutation"—i.e., from time to time parts of successful predictors are combined randomly into a new one.

One can observe in the course of the simulation which digits of the predictors are currently successful.

The simulations' results

Two preliminary tests:

First: All agents equipped with identical predictors, such that a_i and b_i correspond to the rational-expectations values. Are rational-expectations-equilibrium price and allocation reached? ✓ *Yes!*

Second: Given dividend and rational-expectations-equilibrium price series. Do agents learn the correct forecasting parameters a_i and b_i? ✓ *Yes!*

The simulations' results

Initial predictors are always set such that they are close to rational expectations.

With a *low* rate of mutation, the price converges rapidly to the rational-expectations equilibrium.

Reason: Most agents forecast something close to the rational-expectations equilibrium. If some agents forecast differently, they won't be successful in the market and discard the predictors they used.

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The simulations' results

With a *high* rate of mutation, the market behaves in a significantly different way:

Technical trading (forecasting future prices from past ones) becomes profitable, bubbles occur, followed by crashes, GARCH behaviour can be observed.

Reason: Bigger fraction of predictors deviates from rational expectations. For example, subpopulation predicts price increase. This becomes self-fulfilling.

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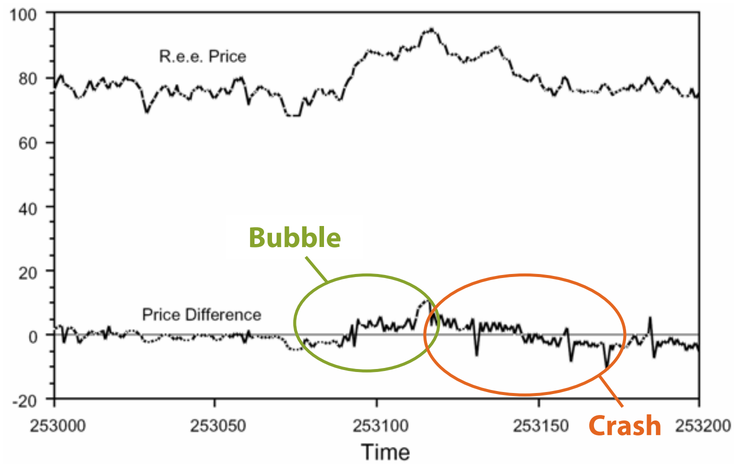
The simulations' results



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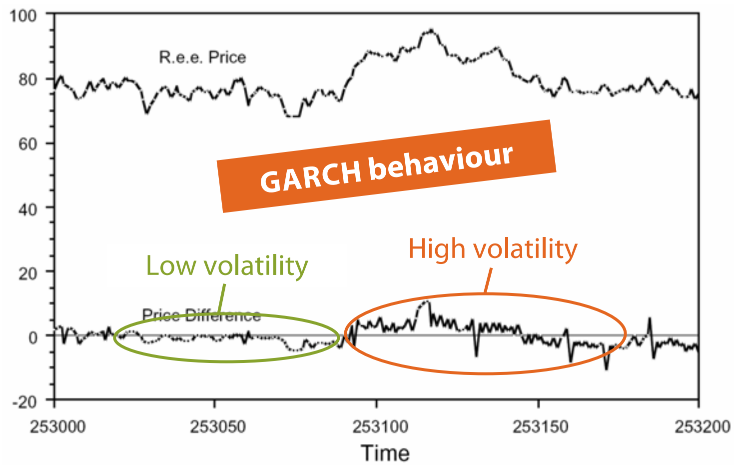

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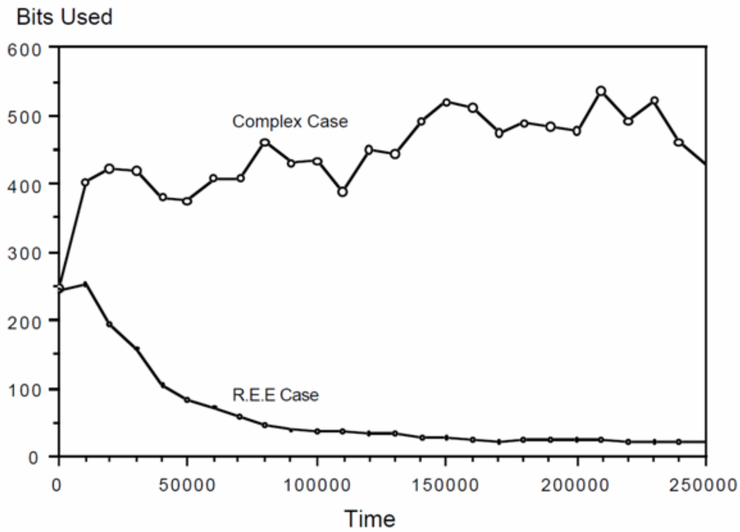

Number of "technical trading" bits set in the course of the simulation (median over 25 experiments each in the two regimes). *(Source: Fig. 3 from Arthur et al., 1996).*

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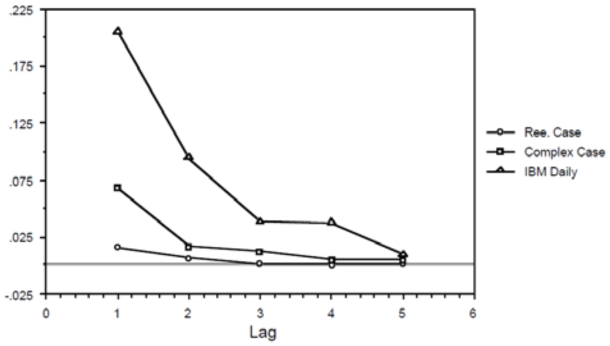

Fig. 4. Autocorrelation of volatility in rational-expectations and complex regimes, and in IBM daily returns

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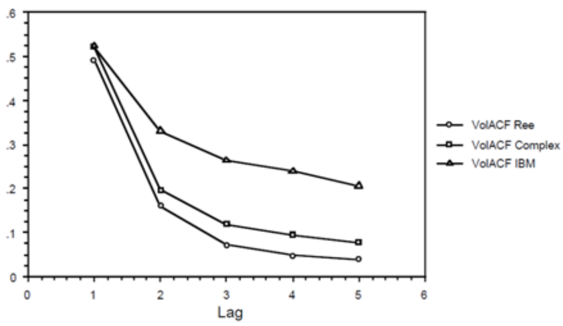

Fig. 5. Autocorrelation of trading volume in the rational-expectations and complex regimes, and in IBM daily returns

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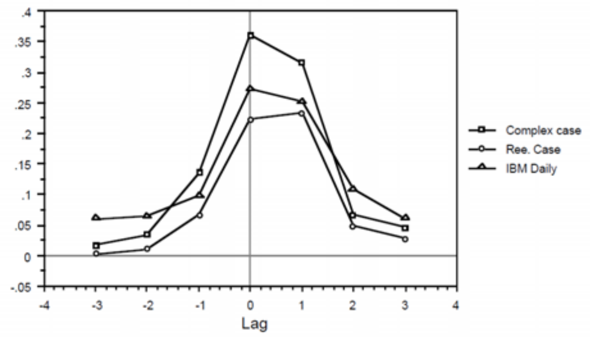

Figure 6. Crosscorrelation of trading volume with volatility, in the rational-expectations and complex regimes, and in IBM daily returns

Literature

Arthur, W. Brian, John H. Holland, Blake LeBaron, Richard G. Palmer and Paul Tayler (1996): "Asset Pricing Under Endogenous Expectations in an Artificial Stock Market". Santa Fe Institute Working Paper no. 96-12-093. Available online: *http://www.santafe.edu/s/publications/Working-Papers/96-12-093.ps*. Also published in: W. Brian Arthur; Steven N. Durlauf; David Lane (eds.) (1997): *The Economy as an Evolving Complex System II*. Addison-Wesley: Reading.

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Tesfatsion, Leigh (2004): "Introduction to Rational Expectations", lecture notes, Iowa State University, Ames. Available online: *http://www.econ.iastate.edu/tesfatsi/reintro.pdf*.