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Low price equilibrium in multi-unit auctions: the GSM spectrum auction in Germany

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Abstract

The second–generation (GSM) spectrum auction in Germany is probably the most clear cut example of a low price outcome in a simultaneous ascending-bid multi-unit auction. The present paper gives an account of the events, describes the auction rules and market conditions, and provides a game theoretic explanation of low price equilibrium in simultaneous, ascending-bid multi-unit auctions. In particular, it is shown that in the unique equilibrium that survives iterated elimination of dominated strategies, the efficient allocation is reached at minimum bids.

JEL classification: D44; D45

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1. Introduction

On October 28, 1999, the German regulatory authority for telecommunications (Regulierungsbehörde für Telekommunikation und Post) opened the auction of

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second-generation GSM radio frequencies in the 1800 MHz range. Altogether ten nationwide frequencies (nine identical, the tenth somewhat larger) were auctioned to the four incumbent operators of mobile phone services in a simultaneous ascending-bid auction.

The first round of bidding started at 10:15 a.m.; the minimum bid was set to the nominal level of just DM 1 million per (paired) 1 MHz bandwidth. Bidders had 30 minutes to make their first bids. When the results of the first round of bidding were shown on the screen, already after 8 min, the press room was filled with murmurs and whistles: Mannesmann (M) had topped the minimum bid by apparently surprising jump bids in the order of DM 36.36 million for frequencies 1–5, DM 40 million on frequencies 6–9, and DM 56 million on (the larger) frequency 10 (see Table 1, where rows represent the rounds of bidding).[1] In the second round, T-Mobil (T), a subsidiary of Deutsche Telekom, raised bids on the first five frequencies by slightly more than the minimum bid increment, and thus reduced bidding rights to five. As a result, it outbid the two smaller incumbents, Viag Interkom and E-Plus, who subsequently withdrew from the auction. In round three no bids were placed, and the auction was over, before it had gained momentum.

While this is by far the most clear cut example of a low price outcome in a simultaneous ascending-bid spectrum auction, similar incidents were observed before in spectrum auctions in the US.

In the present paper, we show that the low price equilibrium observed in the GSM auction in Germany, is a plausible outcome of simultaneous ascending auctions when the objects are mutual substitutes and valuations are common knowledge. Specifically, we show that the unique equilibrium strategies of the two dominant bidders involve immediate demand reduction, to one half of the available objects, together with a defense of these objects up to their marginal valuations. 'Naive' bidding—where bidders bid their true demand at the actual

Table 1
GSM spectrum auction in Germany, October 1999

	Frequency #									
	1	2	3	4	5	6	7	8	9	10
1	36.36 M	36.36 M	36.36 M	36.36 M	36.36 M	40.00 M	40.00 M	40.00 M	40.00 M	56.00 M
2	40.01 T	40.01 T	40.01 T	40.01 T	40.01 T	40.00 M	40.00 M	40.00 M	40.00 M	56.00 M
3	40.01 T	40.01 T	40.01 T	40.01 T	40.01 T	40.00 M	40.00 M	40.00 M	40.00 M	56.00 M

(Frequencies 1–9 were endowed with a bandwith of 2×1 MHz; frequency 10 with 2×1.4 MHz).

[1] The *Frankfurter Allgemeine Zeitung*, 29-10-1999 wrote: ''Ein Raunen ging durch den Saal . . . , als das an die Wand geworfene Display das Ergebnis der ersten Bietrunde zeigt . . . . Wie würde T-Mobil auf die Überrumpelungsstrategie reagieren?''

price—is not an equilibrium that survives iterated elimination of strategies. Thus, ‘naive’ bidding is not a credible strategy in some subgames.

There are several other contributions that deal with low price equilibria in multi-unit auctions. The major part of that literature shows that equilibrium prices range from the minimum bid to the competitive level. It is then suggested that, by open or tacit collusion, bidders coordinate on one of the low price equilibria. Wilson (1979) and Back and Zender (1993) establish this result for sealed bid auctions of a continuously divisible object (‘share auctions’). Menezes (1996) obtains it for an ascending clock share auction. Engelbrecht-Wiggans and Kahn (1998) and Brusco and Lopomo (2002) show that this multiplicity holds also true for auctions of two heterogeneous objects under incomplete information. Finally, Ausubel and Schwartz (1999) show uniqueness, in a sequential bid share auction, and Cramton (2002) and Milgrom (2000) give an example of low price equilibrium for the case of two bidders and two objects.

The paper most closely related in terms of the uniqueness result is Ausubel and Schwartz (1999). However, by assuming that bidding follows a given order of moves, this paper analyzes a finite, sequential bargaining problem with declining discount factors, rather than a typical auction.[2] Adopting a backward induction argument in the spirit of Ståhl (1972) and Rubinstein (1982), yields uniqueness of low price equilibrium. In our model, bids are submitted simultaneously. This makes it impossible to invoke results from bargaining theory to establish uniqueness, because in simultaneous bargaining even strong refinements do not imply uniqueness (see Chatterjee and Samuelson, 1990). Another difference is that they consider two distinct bidders with flat valuations.

The plan of the paper is as follows. In Section 2 we present some stylized facts about the mobile phone market in Germany, which motivates some of our assumptions. In Section 3, we present a model of simultaneous ascending-bid multi-unit auctions, which fits very closely the actual GSM auction in Germany, and then show that this auction game has a unique equilibrium that survives iterated elimination of dominated strategies: in that equilibrium, the two dominant firms share the available licenses equally, at minimal prices. This analysis is supplemented, by a sketch of several extensions. The paper ends with some conclusions in Section 5.

2. The German GSM spectrum auction

In mobile communications, the scarce resource is the spectrum required for radio transmission between users and base stations. The first generation analogue systems used the spectrum inefficiently, so that only few customers could be

[2] In the last rounds of an open, ascending auction, bids are often made in repeating sequence. However, a bidder can always break that sequence by overbidding his own bid. Therefore, bidding is never trapped in a fixed sequence of moves.

served and few providers could be licensed. A fundamental improvement in the utilization of the radio spectrum occurred with the transition from analogue to digital technology, and the introduction of the European GSM (global system for mobiles) standard. This technology used the spectrum much more efficiently, and it was able to accommodate four to five times as many customers. Thus, a mass market of mobile communications could develop.

In Germany, the mobile phone industry started with commercial services in 1992 as a duopoly, served by T-Mobil (a subsidiary of the former state monopoly, Deutsche Telekom), and Mannesmann Mobilfunk. When the second generation GSM 1800 technology became available, the regulator used this opportunity to license two additional providers: E-Plus, in 1994, and Viag Interkom, in 1998. In order to give them a kick start, the regulator endowed them quite generously with second generation GSM frequencies that operate in the 1800 MHz range. Indeed, while Mannesmann and T-Mobil had to live with 2×12.5 MHz in the 900 MHz spectrum each, both E-Plus and Viag were endowed with 2×22.5 MHz in the 1800 MHz spectrum. Nevertheless, the two early entrants became dominant firms, with almost equal market shares (each roughly 40%), whereas E-plus (14.9%), and in particular Viag Interkom (5.3%), remained remarkably insignificant.[3]

By the time of the GSM auction in 1999, when more second-generation GSM 1800 spectrum became available, the two dominant providers were subject to severe spectrum capacity constraints, which inhibited their growth, whereas the two latecomers were sitting on idle capacity, except in a few urban areas. Altogether, the two dominant firms were eager to gain access to the GSM 1800 technology, whereas the small providers had only a limited interest in these frequencies because they could remove the bottlenecks by building additional radio stations in urban centers, instead of buying more nationwide frequencies. Moreover, it was already known that third-generation (UMTS) frequencies would be auctioned in the following year 2000.

On this background, the two dominant firms had a pretty good idea of each others' valuations for the new frequencies. Moreover, they could easily assess the valuations of the two small providers, by computing the cost of building a more closely knit network of radio stations in urban centers, which was an obvious alternative to buying more nationwide frequencies.

Based on this assessment, the two dominant firms could figure out, with a reasonable degree of certainty, at which critical bid level the two smaller providers would quit the auction. So they had to decide whether it was worthwhile to

[3] One could argue that the second generation GSM technology unduly burdened the new entrants with high capital cost. Operating in the higher 1800 MHz range requires a closer network of base stations, even at low levels of capacity utilization. This combination is not the most attractive for a new entrant with a small customer base. On the other hand, the late entrants were subject to dramatically lower investment costs (roughly one half of what it was in the early 1990s), and benefitted from an already mature technology.

displace the smaller providers, by raising the price to that critical level, or to accommodate, and share the frequencies with the smaller providers, at a lower price. Given the extreme asymmetry of the endowments with GSM 1800 spectrum at the time of the auction—the dominant firms did not have any, while the latecomers had more than the auction made available altogether—it was clear that it would not take much to displace the two small bidders, and therefore the right decision was obvious.

Mannesmann started with a jump bid on all frequencies (see Table 1) for two reasons: to bring the price uniformly to the critical level at which the smaller providers would quit the auction, and to coordinate efficiently with T-Mobil on how to divide the frequencies numbered from 1 to 10. And indeed, this strategy worked smoothly. While the two small providers still made a last-minute bid on some of the first five frequencies, they did not acquire a high bid, and quit immediately thereafter.[4]

3. Low price equilibrium

We now present a general model of multi-unit auctions for homogeneous objects that is inspired by the German GSM auction. Our analysis focuses on the relevant game played between the two dominant firms, after the two insignificant bidders have dropped out from the auction. We show that the game is dominance solvable, and that the solution explains the observed play.

We also abstract from the following idiosyncracies of that auction: (1) The units were not entirely homogeneous (one of the ten abstract frequencies had 2×1.4 MHz; all others had 2×1 MHz). (2) The auctioneer employed a simultaneous, ascending price auction rather than an ascending price clock auction which we assume here. (3) Only high bids, and not all bids, were published; hence, the game was not exactly one with observable actions. (4) The minimum bid increment was fixed as a percentage of the previous bids, and the auctioneer had some discretion to reduce it during the auction.

3.1. The model

There are two bidders, M and T who bid for $2n$, $n \in \mathbb{N}$, units of a homogeneous good. Bidders are risk neutral, and have identical valuations that are common knowledge among them. Their marginal valuations, $v(k)$, are strict monotone

[4] The *Frankfurter Allgemeine Zeitung*, 29-10-1999, reports: ‘‘Die Uhr sprang auf die letzte Minute, als der Computer noch Gebote von E-Plus ... registriert.’’ (The clock approached the last minute when the computer registered bids by E-Plus ... .) They could have bid the same as T-Mobil, since the rules of the auction prescribed that in case of a tie, the earlier bidder is the high bidder.

decreasing in the number of units $k \in \{1,2,...,2n\}$. $w_k := \sum_{j=1}^{k} v(j)$ is the absolute valuation of both bidders for k objects.

The auction is a finite, open, ascending bid (English) 'clock' auction. There, the price clock goes up by the fixed increment Δ, starting at 0, until there is no excess demand or the final price $\mathcal{S}\Delta$ is reached. In each round $t = 0,1,2,...,\mathcal{S}$, bidders simultaneously submit a bid $B_i \in \{0,1,2,...,2n\}$, $i = M,T$, which states how many units they demand at the given price $t\Delta$. If the sum of bids does not exceed the supply $2n$, the game ends in round t and bidders pay $t\Delta$ for each of the $B_i(t)$ objects they get. If there is excess demand in a round $t < \mathcal{S}$, i.e. $B_M(t) + B_T(t) > 2n$, the game continues to the next round. And if excess demand persists until the final round $\mathcal{S}$, every bidder gets a payoff of 0.[5] After each round, the auctioneer announces all previous bids. Once a bidder has bid on $k < 2n$ units, he cannot later bid on more than k units; therefore, the sequence of bids must be nonincreasing (activity rule).

For later use, we denote the earliest period in which the current price exceeds the valuation $v(n+1)$ by T_{n+1}, the demand function by $d(t)$, and the restricted demand function (constrained by available bidding rights) by $\hat{d}_i(t)$:

$$T_{n+1} := \min\{t \in \{0,...,\mathcal{S}\} \mid t\Delta \geq v(n+1)\}, \tag{1}$$

$$d(t) := \max\{k \mid v(k) \geq t\Delta\}, \tag{2}$$

$$\hat{d}_i(t) := \min\{d(t), B_i(t-1)\}. \tag{3}$$

$T_{n+1}\Delta$ can be interpreted as the 'competitive price' which is reached if bidders engage in truthful bidding. Of course, if a bidder assumes that the auction will be over at the current price $t\Delta$, $d(t)$ maximizes that bidder's payoff, and $\hat{d}_i(t)$ maximizes it under the given bidding rights constraint.

A formal description of the game is as follows. A history up to round $t \geq 1$ is given by the sequence of past bids, $h_t = \left((B_M(s), B_T(s))_{s=0,...,t-1}\right)$ and the set of all histories up to $t \geq 1$ is denoted by H_t; with $B_i(-1) := 2n$.

A strategy for player $i = M,T$ is a sequence $(\beta_{it})_{t=0,...,\mathcal{S}}$ of functions $\beta_{it}: H_t \to \{0,...,2n\}$, satisfying the activity rule $\beta_{it}(h_t) \leq B_i(t-1)$ for all $h_t \in H_t$. The strategy sets are denoted by $S_i, i = M,T$.

A pair of strategies (β_M, β_T) induces a play or sequence of actions A which is defined recursively by $A_i(0) := \beta_{i0}$, and

$$A_i(s+1) := \begin{cases} \beta_{i,s+1}\left((A_M(r), A_T(r))_{r=0,...,s}\right) & \text{if } A_M(s) + A_T(s) > 2n \\ \varnothing & \text{otherwise} \end{cases}$$

Given the sequence of actions A, denote by $\tau := \max\{t : A_M(t) \neq \varnothing\}$ the last round

[5] This assumption could be replaced by any rule that assures that it is not profitable to buy one or more units in the final round $\mathcal{S}$.

of the play if the game ends without excess demand, and set $\tau = \infty$ otherwise. The payoff function of bidder i is then

$$\pi_i(\beta_M, \beta_T) = \begin{cases} w_{A_i(\tau)} - A_i(\tau)\tau\Delta & \text{if } \tau < \infty \\ 0 & \text{otherwise} \end{cases}$$

3.2. Uniqueness of equilibrium

As in other dynamic games, the concept of a Nash equilibrium is too weak, because all relevant outcomes can be supported as an equilibrium by using appropriate threats. For example, $(B_M(0) = k,\ B_M(t) = \min\{k,\ B_M(t-1)\}, \forall t > 0,$ $B_T(0) = 2n - k,\ B_T(t) = \min\{2n - k,\ B_T(t-1)\}, \forall t > 0)$ is such a Nash equilibrium, for all $k \in \{0, 2n\}$. However, these strategies are not credible, since they involve the threat to maintain a demand for k resp. $2n - k$ units at all prices.

Truthful bidding—bidding one's true demand—is also a Nash equilibrium; however, it is not credible, and does not survive iterated elimination of weakly dominated strategies.

Assumption 1. (a) The price increment is small: $n\Delta < v(n) - v(n+1)$, and (b) the game has many rounds: $\mathcal{S}\Delta > v(1)$.

Assumption 1(a) is equivalent to $w_n - n(v(n+1) + \Delta) > w_{n-1} - (n-1)v(n+1)$, i.e., bidders prefer obtaining n units at a price equal to one increment above the competitive equilibrium price, $v(n+1) + \Delta$, to obtaining $n-1$ units at price $v(n+1)$. Assumption 1(b) says that if the game is played until the last round, $\mathcal{S}$, the price is higher than the highest marginal valuation.

The main result of the paper for the two bidder case is the following.

Theorem 1. (Low price equilibrium) *The game is solvable by iterated elimination of weakly dominated strategies. In that solution both bidders get n objects, and the game ends immediately, in round* 0, *at price* 0.

The associated equilibrium strategy is: $\beta_{it}(h_t) = \hat{d}_i(t), \forall t \geq T_{n+1}$, *and for all* $t < T_{n+1}$:

$$\beta_{it}(h_t) = \begin{cases} n & \text{if } B_M(t-1), B_T(t-1) \geq n \\ B_i(t-1) & \text{if } B_i(t-1) < n \\ \min\{\hat{d}_i(t), 2n - B_j(t-1)\} & \text{if } B_j(t-1) < n, B_i(t-1) > n. \end{cases}$$

The proof is broken down into three successive lemmas.

In the first lemma we prove that truthful bidding is a dominant strategy if the price is already at or above the competitive price. At such prices, no one should ever bid more than his (restricted) demand; therefore, the auction must end immediately at the current price, and bidders are effectively price takers; hence, by definition of a demand function, truthful bidding is a dominant strategy.

Lemma 1. *Consider subgames starting at* $t \in \{T_{n+1}, \ldots, \mathcal{S}\}$. *Iterated elimination of dominated strategies yields the unique equilibrium*:

$$\beta_{it}(h_t) = \hat{d}_i(t), \quad i \in \{M,T\}, \tag{4}$$

and the games ends immediately at t.

Proof. By induction.

1) The assertion is true in all subgames that start at $t = \mathcal{S}$, because $B(\mathcal{S}) = d(\mathcal{S}) = 0$ is a weakly dominant strategy for all $h(\mathcal{S})$.
2) Suppose the assertion is true for all subgames between rounds $s + 1 > T_{n+1}$ and $\mathcal{S}$. Then, Eq. (4) is played effective s.

Bidding $k > \hat{d}_i(s)$ makes a difference if and only if either (a) the auction ends in round s, or (b) it does not end, but would have ended with the lower bid $\hat{d}_i(s)$. In case (a) it is better to bid the payoff maximizer at price s, which is $\hat{d}_i(s)$. And in case b), it is better to end the auction in round s rather than acquire a lower (or equal) quantity $\hat{d}_i(s + 1)$ at the higher price $(s + 1)\Delta$.

Having eliminated bids $k > \hat{d}_i(s)$, and using the fact that $\hat{d}_i(s) \leqslant n$, each bidder knows that the auction ends immediately. Hence, truthful bidding strictly dominates $k < \hat{d}_i(s)$. □

Lemma 2. *Consider subgames starting at* $t \leqslant T_{n+1}$ *in which* $B_T(t-1), B_M(t-1) \geqslant n$. *The unique action that survives iterated elimination of dominated strategies is* $\beta_{it}(h_t) = n$, $i \in \{M,T\}$, *and the game ends immediately at* t.

Proof. By induction.

1) The assertion is true for $t = T_{n+1}$, by Lemma 1.
2) Suppose the assertion is true for all subgames between rounds $s + 1$ and T_{n+1}. We show that it also holds true for round s.

Consider the subgame that begins in round s. If a bidder bids $k < n$ units in round s, he gets at most $w_k - ks\Delta \leqslant w_{n-1} - (n-1)s\Delta =: \pi'$, by the activity rule. Whereas, if he bids n units, he gets at least $\pi := w_n - n(s+1)\Delta$, by the induction hypothesis. However,

$$\begin{aligned} \pi - \pi' \quad &= (v(n) - s\Delta) - n\Delta \\ &\geqslant (v(n) - v(n+1) - n\Delta \ \text{ since } s\Delta \leqslant v(n+1) \\ &> 0 \ \text{ by Assumption 1(a).} \end{aligned} \tag{5}$$

Therefore, $k < n$ is strictly dominated by $k = n$.

After eliminating these dominated strategies, each bidder knows that $\beta_{is}(h_s) \geqslant n$.

Therefore, if a bidder bids $k > n$ in round s, the game continues to round $s+1$ and ends there, by the induction hypothesis. That bidder then gets n objects at price $(s+1)\Delta$. Whereas, $\beta_{is}(h_s) = n$ assures the same or a higher payoff. □

Lemma 3. *Consider subgames starting at* $t \leqslant T_{n+1}$, *in which* $B_T(t-1) < n$ *and* $B_M(t-1) > n$. *Iterated elimination of dominated strategies yields the equilibrium*: $\beta_{Tt}(h_t) = B_T(t-1)$ *and* $\beta_{Mt}(h_t) = \min\{\hat{d}_M(t),\ 2n - B_T(t-1)\}$, *and the game ends immediately at t*. (*A similar statement applies if* $B_M(t-1) < n$, $B_T(t-1) > n$.)

Proof. By induction.

1) The assertion is true for $t = T_{n+1}$, by Lemma 1.
2) Suppose the assertion is true for all subgames between rounds $s+1$ and T_{n+1}. We show that it also holds true for round s.

Consider the subgame that begins in round s. If T bids $k < B_T(s-1)$ in round s, he gets at most $w_k - ks\Delta \leqslant w_{B_T(s-1)-1} - (B_T(s-1)-1)s\Delta =: \pi'$, by the activity rule. Whereas, if he bids $B_T(s-1)$, he gets at least $\pi := w_{B_T(s-1)} - B_T(s-1)(s+1)\Delta$, by the induction hypothesis. However,

$$\begin{aligned} \pi - \pi' \quad = \quad & (v(B_T(s-1)) - s\Delta) - B_T(s-1)\Delta \\ & > (v(n) - v(n+1)) - B_T(s-1)\Delta \\ & > (v(n) - v(n+1)) - n\Delta \\ & > 0 \ \text{ by Assumption 1(a),} \end{aligned} \tag{6}$$

where the second inequality follows from the fact that $v(B_T(s-1)) > v(n)$ and $s\Delta \leqslant v(n+1)$. Therefore, for player T, $k < B_T(s-1)$ is strictly dominated by $k = B_T(s-1)$, and player M knows that $\beta_{Ts}(h_s) = B_T(s-1)$, and his best reply is $\beta_{Ms}(h_s) = \min\{\hat{d}_M(s), 2n - B_T(s-1)\}$. □

3.3. The role of the activity rule

The activity rule is simply the multi-unit auction analogue of the no-reentry restriction in the single-unit 'button' English auction. However, whereas, in a single-unit auction, this rule assures only that an exit from the auction is irreversible, in a multi-unit context it may also serve as a commitment device for a bidder to reliably tell his rival that he does not wish to compete on more than the currently demanded number of objects. One may thus be led to conjecture that this commitment device is valuable to achieve a low price through mutual demand reduction.

However, as we now show, in the present framework the activity rule is not important for uniqueness of the low price equilibrium. Indeed, if the activity rule is removed, and bidders are unconstrained by previous bids, the above analysis is actually simplified, and one obtains:

Theorem 2. (Equilibrium without activity rule) *Suppose the auction is played without activity rule. Then, the unique equilibrium that survives iterated elimination of dominated strategies is*

$$\beta_{it}(h_t) = \begin{cases} d_i(t), & \text{for} \quad t \geqslant T_{n+1} \\ n, & \text{for} \quad t \leqslant T_{n+1}. \end{cases} \tag{7}$$

The game ends immediately, and each bidder gets n objects at price 0.

Proof. Replace constrained demand $\hat{d}_i(t)$ by unconstrained demand $d(t)$ in the proof of Lemma 1. It follows immediately that $\beta_{it}(h_t) = d_i(t)$, for all subgames starting at $t \geqslant T_{n+1}$.

Now consider subgames starting at $t \leqslant T_{n+1}$. The assertion is true for $t = T_{n+1}$, by the above. Suppose it is true for all $t \in \{s+1, \ldots, T_{n+1}\}$; then, as we show, it is also true for $t = s$.

Consider $t = s$. If a bidder bids $k < n$ units in round s, his payoff is *at most* equal to $\pi_k := \max\{w_k - ks\Delta, w_n - n(s+1)\Delta\}$. Whereas, if he bids n units, he gets *at least* $\pi_n := w_n - n(s+1)\Delta$. However,

$$\begin{aligned} \pi_n - \pi_k &= w_n - n(s+1)\Delta - \max\{w_k - ks\Delta, w_n - n(s+1)\Delta\} \\ &\geqslant w_n - n(s+1)\Delta - \max\{w_{n-1} - (n-1)s\Delta, w_n - n(s+1)\Delta\} \\ &= 0, \quad \text{by (5).} \end{aligned}$$

Therefore, $\beta_{is}(h_s) = k < n$ is dominated by $\beta_{is}(h_s) = n$. The remainder of the proof is precisely as in last paragraph of the proof of Lemma 2, and hence omitted. □

Of course, the activity rule is an important ingredient of an open, ascending bid auction. This auction format is generally advised on the ground that it is an attractive information system that reveals a maximum of information about those who, in the course of the auction, have reduced their demand. This can only work, if demand reductions are final, and cannot be altered or undone. In other words, the activity rule is not something that one would give up easily. Therefore, it is good news that the activity rule is not responsible for the low price equilibrium.

4. Extensions

These results can be extended in various directions.

A first modification concerns introducing a rationing rule to prevent that some objects are not allocated, in the event that after one price increment excess demand turns into excess supply. This would bring the ascending clock auction analyzed in the present paper closer to the actual GSM auction in Germany, where bidders could not shed their standing high bids. This modification poses no problems, and actually simplifies our proofs, essentially because it makes bidding more than $\hat{d}(t)$ even less attractive, and because it reduces the number of subgames. Nevertheless, we decided not to use it in order to keep our analysis free from unimportant idiosyncracies of spectrum auctions.

A second group of extensions concerns the introduction of an arbitrarily large number of bidders (greater than two), permitting asymmetries between bidders, and permitting an arbitrary number of objects that may not be evenly divisible among bidders. These extensions are feasible; indeed, the general auction game with arbitrarily many (potentially asymmetric) bidders and arbitrarily many objects can also be solved by iterated elimination of dominated strategies, and the equilibrium that survives this elimination implements the efficient allocation at minimum bids. Therefore, the results of the present model are robust. However, these generalizations require more complex notation, slightly more stringent assumptions, and more elaborate proofs.[6]

A third modification concerns replacing the clock auction format assumed here, by an ascending price format, where bidders are required to make bids on the numbered frequencies, as it was actually (although unnecessarily) the case in the German GSM auction.[7] Evidently, this format gives rise to a coordination problem, because even if bidders split the available frequencies, they still need to coordinate who gets which. This coordination problem may explain the attraction of jump bidding, as it occurred in the German GSM auction. Altogether, this auction format gives rise to more subgames, and it makes the dominance criterion less effective, because bids need not increase in small increments (see Avery (1998) and Kamecke (1998)). Therefore, one must replace solvability by elimination of dominated strategies by the less compelling subgame perfection.[8]

A fourth important modification concerns the introduction of incomplete information and market structure effects of the auction. In this case, bidders may have an incentive to test their rivals, and after some learning play similar low price strategies, as we show in our equilibrium analysis of the third generation (UMTS) spectrum auction in Germany (see Grimm et al. (2002)), where uncertainty about new entrants and predatory motives played a crucial role. However, the extension to general models of incomplete information is still an open problem.

Finally, we mention that in the GSM auction in Germany only high bids were observable. Therefore, the game was not exactly one of observable actions, and

[6] See our technical companion paper Riedel and Wolfstetter (2003).

[7] Van Damme (2002) also criticizes the use of an ascending price format as 'using a design that had been developed to auction heterogeneous lots.'

[8] In a previous version of this paper (which can be downloaded at http://papers.ssrn.com/) we covered this case, and showed that the ascending price auction has a low price equilibrium, using subgame perfection rather than iterated elimination of dominated strategies.

bidders had to draw inferences on their rival's bidding rights. However, in the case of two bidders, this inference could be drawn already after a few rounds of bidding.

5. Conclusions

The present paper has not only shown that low price outcomes may be an equilibrium in multi-unit auctions. We showed that the low price equilibria that implement the efficient allocation at the minimum bid is rationalizable as the only equilibrium that survives iterated elimination of dominated strategies. This lends support for predicting a low price outcome, precisely as it occurred in the German GSM auction. The strength of this prediction is that it explains an observed, seemingly collusive, outcome from a strictly noncooperative perspective, leaving practically no role for open or tacit collusion.

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