Supplement to:
Licensing Process Innovations when Losers' Bids Determine Royalty Rates

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1 Introduction

In this technical supplement we show that

- our analysis in Fan, Jun, and Wolfstetter (2011) is robust and can be extended to more than two firms
- our restriction of royalty rates is compelling due to antitrust concerns.

We consider model II.

2 Robustness: the case of three firms

With more than two firms, a new issue comes up: the innovator must decide how many fixed-fee licenses to supply. The choice of the number of fixed-fee licenses has been at center stage in the classical literature on patent licensing under complete information, but is more challenging in the present framework of incomplete information.

Specifically, we consider the case of three firms and allow the innovator to award either one or two fixed-fee licenses, whichever is more profitable for him.

All other assumptions are maintained.

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We proceed as follows: first, we consider the case when one fixed-fee license is offered and show that adding the royalty scheme is profitable; second, we consider the case of two fixed-fee licenses and show that adding the royalty scheme is profitable; third, we show that offering one fixed-fee license together with the royalty scheme is optimal.

Of course, it is never profitable to offer three fixed-fee licenses to three firms because in that case there is no competition.

2.1 One fixed-fee license

One fixed-fee license is offered to three firms. The firm that obtains the fixed-fee license is called "winner"; the other two firms are called "losers".

The innovator employs a direct revelation mechanism. There, firms are asked to simultaneously report their cost reduction, x_i, and, for each profile of reported cost reductions, the innovator awards licenses and collects payments according to the direct revelation mechanism (k, t, R).

We denote the message vector of firms other than firm i by x_{-i}, and the highest and the second highest cost reductions of the two other firms by $y := \max\{x_{-i}\}$ and $v := \min\{x_{-i}\}$.

If all three firms participate in the mechanism game, the rules (k, t, R) are as follows for all i (where $\mathbb{1}_{x_i>y}$ is the indicator function),

$$k_i(x_i, x_{-i}) = \mathbb{1}_{x_i>y} \tag{1}$$

$$t_i(x_i, x_{-i}) = \mathbb{1}_{x_i>y}\beta(y) + \mathbb{1}_{y>x_i} x_i q_{i_L} \tag{2}$$

$$\beta(x_i) \geq R. \tag{3}$$

And if either one or two firms participate, the rules are as stated in the paper.

After licensing, all messages are revealed to all players.

2.1.1 Cournot "subgames"

Consider one firm, say firm 1, that has drawn cost reduction x but reports cost reduction $z \geq x$. The following "subgames" occur, depending upon the observed outcome of the mechanism game.

Case 1): All three firms participated and firm 1 won ($\mathbf{z > y}$ and $\mathbf{x, y, v \geq r}$) In that case firms 2 and 3 believe to play a Cournot game with the profile of unit costs $(c_1, c_2, c_3) = (c - z, c, c)$. Denote the associated equilibrium strategies of the game the losers believe to play by $(q_W(z), q_{L_2}(z), q_{L_3}(z))$.

However, firm 1 privately knows that its cost reduction is x rather than z. Therefore, firm 1 plays the best reply to $(q_{L_2}(z), q_{L_3}(z))$:

$$q_{W_1}(x, z) = \arg\max_q \big(P(q + q_{L_2}(z) + q_{L_3}(z)) - c + x\big)\, q. \tag{4}$$

The reduced form profit function of firm 1 is

$$\pi_W(x, z) := \big(P(q_{W_1}(x, z) + q_{L_2}(z) + q_{L_3}(z)) - c + x\big)\, q_{W_1}(x, z). \tag{5}$$

Case 2): All three firms participated and firm 1 lost ($\mathbf{y > z}$ and $\mathbf{x, y, v \geq r}$) In that case firms 2 and 3 believe to play a Cournot game with the profile of unit costs $(c_1, c_2, c_3) = (c, c - y, c)$. Denote the associated equilibrium strategies of the game firms 2 and 3 believe to play by $(q_{L_1}(y), q_{W_2}(y), q_{L_3}(y))$.

If no royalty scheme is used, the equilibrium play of firm 1 depends only on the winner's cost reduction y. The associated reduced form profit function of firm 1 is

$$\pi_{L_n}(y) := \big(P(q_{L_1}(y)+q_{W_2}(y)+q_{L_3}(y))-c\big)\,q_{L_1}(y). \tag{6}$$

Whereas if the royalty scheme is adopted, firm 1 privately knows that its cost reduction is x and that it pays a royalty rate equal to $z \geq x$. Therefore, firm 1 plays the best reply to $(q_{W_2}(y), q_{L_3}(y))$:

$$q_{L_1}(x,z,y) = \arg\max_q \big(P(q+q_{W_2}(y)+q_{L_3}(y))-c+x-z\big)\,q. \tag{7}$$

The associated reduced form profit function of firm 1 is

$$\pi_L(x,z,y) := \big(P(q_{L_1}(x,z,y)+q_{W_2}(y)+q_{L_3}(y))-c+x-z\big)\,q_{L_1}(x,z,y). \tag{8}$$

On the equilibrium path, i.e., for $z=x$, the payoff of firm 1 in the event when it loses is only a function of the highest cost reduction of other firms, y; therefore, we denote the on-the-equilibrium path output and profit of firm 1 when it loses by

$$q_L^*(y) := q_{L-1}(x,z,y)|_{z=x}, \quad \pi_L^*(y) := \pi_L(x,z,y)|_{z=x} = \pi_{L_n}(y). \tag{9}$$

Case 3): At least one firm did not participate If no one participated, the equilibrium profit of firm 1 is equal to π_{nn}. If firm 1 was the only participant, its equilibrium profit is $\pi_W(x,z)$, and if the rival firm with cost reduction y was the only participant, the equilibrium profit of firm 1 is the same as in the game without royalty scheme, i.e. $\pi_{L_n}(y)$.

Example Suppose inverse demand is linear, $P(Q) = 1-Q$, $Q = q_1+q_2+q_3$. Then, the payoffs of the relevant Cournot equilibrium subgames are:

$$\pi_W(x,z) = \frac{(1-c+2x+z)^2}{16}, \quad \pi_L(x,z,y) = \frac{(1-c+2x-2z-y)^2}{16} \tag{10}$$

$$q_L^*(y) = \frac{1-c-y}{4}, \quad \pi_{L_n}(y) = \pi_L^*(y) = \frac{(1-c-y)^2}{16}, \quad \pi_{nn} = \frac{(1-c)^2}{16}. \tag{11}$$

2.1.2 Licensing "subgame"

We now characterize the transfer rule β that induces truth–telling as a Bayesian Nash equilibrium. For this purpose, we first solve the relationship between the reserve price R and the cutoff value of firms' cost reductions, r, which characterizes firms' participation decision.

Lemma 1. *The reserve price R induces a unique cutoff value of cost reductions, r, below which firms do not participate in the mechanism. This cutoff value is implicitly defined as:*

$$R = \pi_W(r,r) - \pi_{nn}. \tag{12}$$

Proof. Consider the marginal firm, with cost reduction $x = r$. If that firm participates, and all other firms tell the truth if they also participate, that firm's payoff is $\Pi_p(r)$, whereas if it does not participate, its payoff is $\Pi_{np}(r)$:

$$\Pi_p(r) = F(r)^2(\pi_W(r,r)-R) + \int_r^c \int_0^r \pi_L^*(y) f_{(12:2)}(y,v)\,dv\,dy + \int_r^c \int_r^y \pi_L^*(y) f_{(12:2)}(y,v)\,dv\,dy$$

$$\Pi_{np}(r) = F(r)^2\pi_{nn} + \int_r^c \int_0^r \pi_{L_n}(y) f_{(12:2)}(y,v)dvdy + \int_r^c \int_r^y \pi_{L_n}(y) f_{(12:2)}(y,v)dvdy,$$

where $f_{(12:2)} = 2f(y)f(v)$ is the joint density of the highest and lowest order statistics in a sample of two *iid* random variables.

The marginal bidder with $x = r$ must be indifferent between participating and not participating, $\Pi_p(r) = \Pi_{np}(r)$, which implies (12). □

Now suppose β is the transfer rule that induces truth-telling as a Bayesian Nash equilibrium. As a working hypothesis assume β is strictly monotone increasing (which we will need to confirm). Consider a firm, say firm 1, that has drawn cost reduction x, but deviates from equilibrium by reporting cost reduction $z \geq x$,[1] while firms 2 and 3 report their true cost reductions. Then, the payoff function of firm 1 in the mechanism with royalty scheme is

$$\Pi(x,z) = F(r)^2(\pi_W(x,z) - R) + \int_r^z (\pi_W(x,z) - \beta(y)) f_{(1:2)}(y)dy + \int_z^c \pi_L(x,z,y) f_{(1:2)}(y)dy,$$

where $f_{(1:2)}(y) = 2F(y)f(y)$ (the density of the largest order statistic in a sample of two *iid* random variables).

In the mechanism without royalty scheme, the payoff function is the same, except that in the last term $\pi_L(x,z,y)$ must be replaced by $\pi_{L_n}(y)$.

Proposition 1. *The transfer rules β, β_n that induce truth-telling as a Bayesian Nash equilibrium in the games with and without royalty scheme are, for all $x \geq r$:*

$$\beta(x) = \beta_n(x) + \frac{1}{f_{(1:2)}(x)} \int_x^c \partial_z \pi_L(x,z,y)|_{z=x} f_{(1:2)}(y)dy \tag{13}$$

$$\beta_n(x) = \pi_W(x,x) - \pi_{L_n}(x) + \frac{F(x)}{2f(x)} \partial_z \pi_W(x,z)|_{z=x}. \tag{14}$$

Proof. It is straightforward to derive the two transfer rules and to confirm that β, β_n are strictly monotone increasing. However, one also needs to assure that second order conditions for best replies are satisfied. Like in the case of two firms, this requires that the cutoff value r is sufficiently large. The exact condition will be stated when we compute an example below. □

2.1.3 The innovator's expected revenue

The expected revenue of the innovator in the mechanism with and without royalty scheme, $G(r)$ and $G_n(r)$, are:

$$G_n(r) = 3(1 - F(r))F(r)^2(\pi_W(r,r) - \pi_{nn}) + \int_r^c \beta_n(y) f_{(2:3)}(y)dy$$

$$G(r) = G_n(r) + \int_r^c (\beta(y) - \beta_n(y)) f_{(2:3)}(y)dy + 6\int_r^c \int_r^x \int_r^y (v+y) q_L^*(x) f(x) f(y) f(v) dvdydx,$$

where $f_{(2:3)}(y) = 6(1 - F(y))F(y)f(y)$ is the density of the second highest order statistic in a sample of three *iid* random variables.

[1] The "downward" deviations, $z \leq x$ yield the same differential equation.

In Figure 1 we plot the functions $G(r), G_n(r)$, assuming linear inverse demand, as in (10)-(11), $c = 0.49$, and $F(x) := x/c$ (uniform distribution). As one can easily confirm, in the game with royalty scheme, second-order conditions (pseudoconcavity of $\Pi(x,z)$ in z) are satisfied only if $r \geq r_{\min} = 0.2089$. Therefore, the dashed portion of the G function must be ignored. Comparing G and G_n, this example shows that adding the royalty scheme is profitable also in the case of three firms and one fixed-fee license.

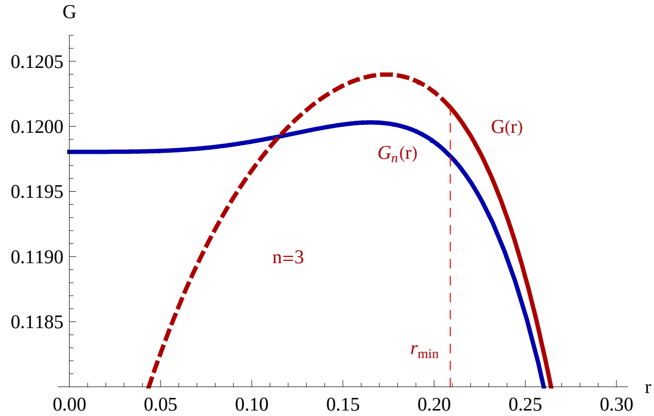

Figure 1: Profitability of the royalty scheme with 3 firms and 1 fixed-fee license

2.2 Two fixed-fee licenses

When there are three firms, the innovator may also award two fixed-fee licenses. Therefore, in order to determine whether the royalty scheme is profitable when there are three firms, we also need to choose the optimal number of fixed-fee licenses. For this purpose we now solve the game for the case when two fixed-fee licenses are offered. As we will see later, in our example it is actually optimal for the innovator to offer only one license together with the royalty scheme.

When two fixed-fee licenses are offered to three firms, the mechanism (k,t,R) takes the following form, where we again use the definition $v := \min\{x_{-i}\}$.

If all three firms participate,

$$k_i(x_i, x_{-i}) = \mathbb{1}_{x_i > v} \quad \text{and} \quad t_i(x_i, x_{-i}) = \mathbb{1}_{x_i > v}\beta(v) + \mathbb{1}_{v > x_i} x_i q_{i_L}, \quad \beta(x_i) \geq R. \tag{15}$$

If only tow firms (i and j) participate, the mechanism simplifies to $k_i(x_i,x_j) = 1$ and $t_i(x_i,x_j) = R$; similarly, if only one firm, i, participates: $k_i(x_i) = 1, t_i(x_i) = R$.

2.2.1 Cournot "subgames"

Consider one firm, say firm 1, that has drawn cost reduction x but reports cost reduction $z \geq x$. The following "subgames" occur, depending upon the observed outcome of the mechanism game.

Case a) Only firm 1 participated ($\mathbf{x \geq r}$ and $\mathbf{y, v < r}$) In this case, the equilibrium profit of firm 1 is the same as that in case 1) of the game if only one fixed-fee license is offered, i.e., $\pi_{W_a}(x,z) = \pi_W(x,z)$.

Case b) At least two firms participated and firm 1 won (either $\mathbf{z > v}$ and $\mathbf{x, y, v \geq r}$ or $\mathbf{x, y \geq r}$ and $\mathbf{v < r}$) In that case, firms 2 and 3 believe to play a Cournot game with the profile of unit costs $(c_1, c_2, c_3) = (c - z, c - y, c)$. Denote the associated equilibrium strategies of the game firms 2 and 3 believe to play by $q_{W_1}(z, y), q_{W_2}(z, y), q_{L_3}(z, y)$.

However, firm 1 privately knows that its cost reduction is x rather than z, and plays the best reply to $(q_{W_2}(z, y), q_{L_3}(z, y))$:

$$q_W(x, z, y) := \arg\max_q \big(P(q + q_{W_2}(z, y) + q_{L_3}(z, y)) - c + x\big) q. \tag{16}$$

The associated reduced form profit function of firm 1 is then

$$\pi_{W_b}(x, z, y) := \big(P(q_W(x, z, y) + q_{W_2}(z, y) + q_{L_3}(z, y)) - c + x\big) q_W(x, z, y). \tag{17}$$

Case c): All three firms participated and firm 1 lost ($\mathbf{v > z}$ and $\mathbf{x, y, v \geq r}$) In that case, both firms 2 and 3 won and believe to play a Cournot game with the profile of unit costs $(c_1, c_2, c_3) = (c, c - y, c - v)$. Denote the associated equilibrium strategies of the game the winners believe to play by $(q_{L_1}(y, v), q_{W_2}(y, v), q_{W_3}(y, v))$.

If no royalty scheme is used, the equilibrium strategy of firm 1 depends only on the winners' cost reductions, y and v. The reduced form profit function of firm 1 is

$$\pi_{L_{2n}}(y, v) := \big(P(q_{L_1}(y, v) + q_{W_2}(y, v) + q_{W_3}(y, v)) - c\big) q_{L_1}(y, v). \tag{18}$$

Whereas if the royalty scheme is adopted, firm 1 privately knows its cost reduction is x, yet pays a royalty rate equal to $z \geq x$. Therefore, firm 1 plays the best reply to $(q_{W_2}(y, v), q_{W_3}(y, v))$:

$$q_L(x, z, y, v) := \arg\max_q \big(P(q + q_{W_2}(y, v) + q_{W_3}(y, v)) - c + x - z\big) q. \tag{19}$$

The associated reduced form profit function of firm 1 is

$$\pi_L(x, z, y, v) := \big(P(q_L(x, z, y, v) + q_{W_2}(y, v) + q_{W_3}(y, v)) - c + x - z\big) q_L(x, z, y, v). \tag{20}$$

On the equilibrium path, i.e., for $z = x$, the equilibrium output strategy and reduced form profit function of firm 1 when it is the loser is only a function of the winners' cost reductions, y and v; and we write

$$q_L^*(y, v) := q_L(x, z, y, v)|_{z=x}, \quad \pi_L^*(y, v) := \pi_L(x, z, y, v)|_{z=x} = \pi_{L_{2n}}(y, v). \tag{21}$$

Case d) If firm 1 did not participate ($\mathbf{x < r}$) If firm 1 did not participate while both rival firms participated, the equilibrium profit of firm 1 depends only on the rivals' cost reductions, $\pi_{L_{2n}}(y, v)$. If firm 1 did not participate and only the rival firm with higher cost reduction (y) participated, the equilibrium profit of firm 1 is the same as in the game without royalty scheme, i.e. $\pi_{L_n}(y)$. If no firm participated, the equilibrium profit of firm 1 is equal to π_{nn}.

Example Suppose inverse demand is linear, as stated in the first example of this supplement, Then, the equilibrium payoffs of the relevant Cournot subgames are:

$$
\begin{aligned}
\pi_{W_a}(x,z) &= \frac{(1-c+2x+z)^2}{16}, \quad \pi_{W_b}(x,z,y) = \frac{(1-c+2x+z-y)^2}{16}, \\
\pi_L(x,z,y,v) &= \frac{(1-c+2x-2z-y-v)^2}{16}, \quad \pi_L^*(y,v) = \pi_{L_{2n}}(y,v) = \frac{(1-c-y-v)^2}{16}, \\
q_L^*(y,v) &= \frac{1-c-y-v}{4}, \quad \pi_{L_n}(y) = \frac{(1-c-y)^2}{16}.
\end{aligned}
$$

2.2.2 Licensing "subgame"

To characterize the transfer rule β that induces truth–telling as a Bayesian Nash equilibrium, we first derive the relationship between the reserve price R and the cutoff value of firms' cost reduction r:

Lemma 2. *The reserve price R induces a unique cutoff value of cost reductions, r, below which firms do not participate in the mechanism. The cutoff value is implicitly defined as:*

$$
R(r) = \frac{1}{2-F(r)}\Big(\big(\pi_{W_a}(r,r)-\pi_{nn}\big)F(r) + 2\int_r^c \big(\pi_{W_b}(r,r,y)-\pi_{L_n}(y)\big)f(y)dy\Big). \tag{22}
$$

Proof. Consider the marginal firm with cost reduction $x=r$. If that firm participates, and all other firms tell the truth if they also participate, that firm's payoff is $\Pi_p(r)$, whereas if it does not participate, its payoff is $\Pi_{np}(r)$:

$$
\begin{aligned}
\Pi_p(r) &= F(r)^2\big(\pi_{W_a}(r,r)-R\big) + \int_r^c\int_0^r \big(\pi_{W_b}(r,r,y)-R\big) f_{(12:2)}(y,v)dvdy \\
&\quad + \int_r^c\int_r^y \pi_L^*(y,v)f_{(12:2)}(y,v)dvdy \\
\Pi_{np}(r) &= F(r)^2\pi_{nn} + \int_r^c\int_0^r \pi_{L_n}(y)f_{(12:2)}(y,v)dvdy + \int_r^c\int_r^y \pi_{L_{2n}}(y,v)f_{(12:2)}(y,v)dvdy,
\end{aligned}
$$

where $f_{(12:2)}(y,v) = 2f(y)f(v)$.

The marginal bidder with $x=r$ must be indifferent between participating and not participating, $\Pi_p(r) = \Pi_{np}(r)$, which implies (22). □

Now suppose β is the transfer rule that induces truth–telling as a Bayesian Nash equilibrium. As a working hypothesis, assume β is strictly monotone increasing, which we need to confirm later. Consider firm 1, it has drawn cost reduction x, but deviates from the equilibrium by reporting cost reduction $z \geq x$, while firms 2 and 3 report their true cost reductions. Then, the payoff function of firm 1 in the mechanism with royalty scheme is

$$
\begin{aligned}
\Pi(x,z) &= F(r)^2(\pi_{W_a}(x,z)-R) + \int_r^c\int_0^r (\pi_{W_b}(x,z,y)-R)f_{(12:2)}(y,v)dvdy \\
&\quad + \int_r^z\int_r^y (\pi_{W_b}(x,z,y)-\beta(v))f_{(12:2)}(y,v)dvdy + \int_z^c\int_r^z (\pi_{W_b}(x,z,y)-\beta(v))f_{(12:2)}(y,v)dvdy \\
&\quad + \int_z^c\int_z^y \pi_L(x,z,y,v)f_{(12:2)}(y,v)dvdy
\end{aligned}
$$

where $f_{(12:2)}(y,v) = 2f(y)f(v)$.

In the mechanism without royalty scheme, the payoff function is the same, except that in the last term $\pi_L(x,z,y,v)$ must be replaced by $\pi_{L_{2n}}(y,v)$.

Proposition 2. *The transfer rules $\beta(x)$ and $\beta_n(x)$, that induce truth–telling as a Bayesian Nash equilibrium in the games with and without royalty scheme are, for all $x \geq r$:*

$$\beta(x) = \beta_n(x) + \int_x^c \int_x^y \partial_z \pi_L(x,z,y,v)\big|_{z=x} f_{(12:2)}(y,v)dvdy.$$

$$\beta_n(x) = \frac{1}{2f(x)(1-F(x))} \Bigg(F(r)^2 \,\partial_z \pi_{W_a}(x,z)\big|_{z=x} + 2F(x) \left(\int_x^c \partial_z \pi_{W_b}(x,z,y)\big|_{z=x} dF(y) \right) + \int_r^x \partial_z \pi_{W_b}(x,z,y)\big|_{z=x} f_{(1:2)}(y)dy + 2f(x) \int_x^c (\pi_{W_b}(x,x,y) - \pi_{L_n}(y,x)) f(y)dy \Bigg).$$

Proof. The derivation of the equilibrium transfer rules β, β_n and the proof of their monotonicity are similar to that in the case of two firms. Also similar to other cases with royalty scheme, the second order conditions for best replies are satisfied only when the cutoff value r is sufficiently large. ☐

2.2.3 The innovator's expected revenue

The expected revenue of the innovator without and with royalty scheme are (where $R(r)$ is defined in (22)),

$$G_n(r) = 3(1-F(r))F(r)^2 R(r) + 3(1-F(r))^2 F(r) 2R(r) + 2\int_r^c \beta_n(v) f_{(3:3)}(v)dv$$

$$G(r) = G_n(r) + \int_r^c 2(\beta(v) - \beta_n(v)) f_{(3:3)}(v)dv + 6\int_r^c \int_r^x \int_r^y v q_L^*(x,y) f(x) f(y) f(v) dvdydx$$

where $f_{(3:3)}(v) = 3\left(1 - F(v)^2\right) f(v)$ is the density of the lowest order statistic in a sample of three *iid* random variables.

In Figure 2 we plot the innovator's expected revenue with one fixed-fee license, $G(r)$ (one license case with royalty scheme), with two fixed-fee licenses, $G(r)$ (two license case with royalty scheme), and $G_n(r)$ (two license case without royalty scheme). Like before, we assume linear inverse demand, $c = 0.49$, and $F(x) = {}^x/_c$ (uniform distribution). Comparing the plots in Figures 1-2 we conclude that awarding one fixed-fee license, combined with the royalty scheme, is strictly better than all other mechanisms.[2]

[2]Note, the different shapes of $G(r)$ (one license case with royalty scheme) in Figures 1 and 2 are due to the different scales we use in these two figures.

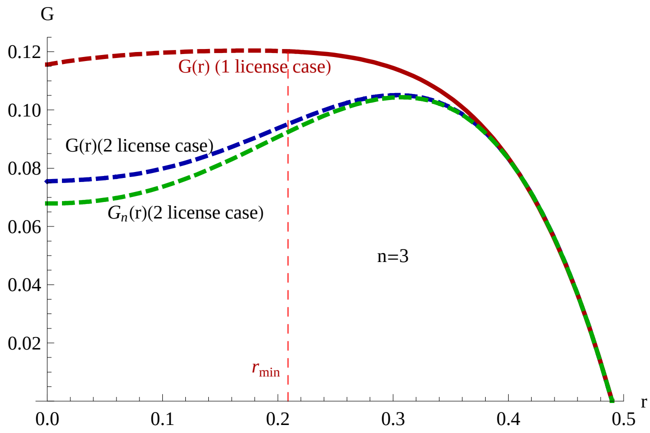

Figure 2: Profitability of 1 *vs* 2 fixed-fee licenses when there are 3 firms

3 Restrictions on royalty rates due to antitrust concerns

One feature of the proposed mechanism is that we restrict royalty rates to be equal to the cost reduction reported by the loser. One may wonder why we do not consider either lower or higher royalty rates.

We now show that our restriction on royalty rates is justified, in two steps:

1. if the innovator were permitted to charge the loser a royalty rate that exceeds the loser's cost reduction, he would benefit from charging an excessively high royalty rate that crowds out the loser. Of course, such anticompetitive conduct is not tolerated by antitrust authorities

2. We show that if the royalty rate were smaller than the loser's cost reduction, the innovator would benefit from raising it. Therefore, royalty rates that are lower than the reported cost reduction are not optimal for the innovator.

It follows that our restriction on royalty rates is justified. This also suggests that an economically meaningful theory of optimal licensing must incorporates restrictions that capture antitrust concerns.

3.1 Why we exclude royalty rates that exceed the loser's cost reduction

Suppose the innovator sets a high royalty rate that crowds out the loser from the duopoly market. Then, the licensing mechanism reduces to a second-price license auction in which bidders compete for an exclusive license. Whenever both firms participate, the winner becomes a monopolist; however, if only one firm participates, that firm wins while the other firm is still active in the duopoly market. Signalling strength towards the rival still matters, because it confers a strategic advantage in the event when the rival does not participate and hence is not crowded out.

Altogether, the game induces equilibrium bids that are higher than the monopoly profit $\pi_M(x)$; however, in order to induce high participation the innovator may have to set a low and possibly negative reserve price:

Proposition 3. *Assume the innovator charges a royalty rate that crowds out the bidder who loses the auction. Then,*

$$\beta(x) = \pi_M(x) + \frac{F(r)}{f(x)}\, \partial_z \pi_W(x,z)|_{z=x} > \pi_M(x) \tag{23}$$

$$R = \pi_W(r,r) - \pi_{nn} - \frac{1}{F(r)} \int_r^c \pi_L(y) dF(y). \tag{24}$$

Proof. As a working hypothesis suppose β is strictly increasing. Then, the payoff function of a bidder whose cost reduction is $x \geq r$ and bids as if his cost reduction is $z \geq x$ is

$$\Pi(x,z) = F(r)\,(\pi_W(x,z) - R) + \int_r^z (\pi_M(x) - \beta(y))\, dF(y). \tag{25}$$

Invoking the equilibrium requirement $x = \arg\max_z \Pi(x,z)$, for all x, one finds the asserted β function and the assumed monotonicity of β confirms).

To confirm the asserted relationship between r and R, consider the marginal bidder with cost reduction $x = r$. That bidder must be indifferent between bidding and not bidding, which gives the condition

$$F(r)\,(\pi_W(r,r) - R) = F(r)\pi_{nn} + \int_r^c \pi_L(y) dF(y), \tag{26}$$

and the asserted equation (24) follows immediately. □

Note that one cannot induce a cutoff value $r = 0$, since R goes to minus infinity as r goes to zero.

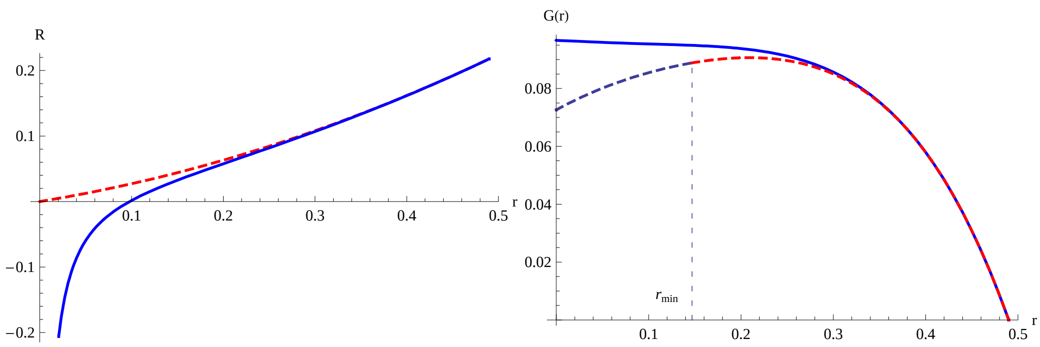

Figure 3: Left: relationship between r and R; right: superiority of implementing monopoly

Now consider the linear model with a uniform distribution. Then it is easy to compute β, R, $G(r)$, and the relationship between r and R. In Figure 3 we plot the relationship between r and R and $G(r)$ when monopoly is implemented (solid curves) and when the proposed royalty scheme is adopted (dashed curves), again assuming $c = 0.49$. Comparing these curves, we conclude that implementing monopoly through adopting an excessively high royalty rate is more profitable for the innovator than the proposed royalty scheme. And if monopoly is implemented, it is profitable to choose a negative reserved price that essentially subsidizes bidding (albeit with low probability).

We conclude that one must exclude royalty rates that exceed the loser's cost reduction because, without this restriction, the innovator would implement monopoly, in violation of antitrust rules.

3.2 Why we exclude royalty rates lower than the loser's cost reduction

Suppose the innovator is free to set a royalty rate equal to a multiple ρ of the cost reduction, with $\rho \in [0, 1]$. Again, consider the linear model with uniformly distributed cost reductions. Then, one can show that the expected profit of the innovator is an increasing function of the parameter ρ, as illustrated in Figure 4. Therefore, the innovator would always like to raise the royalty rate up to $\rho x = x$.

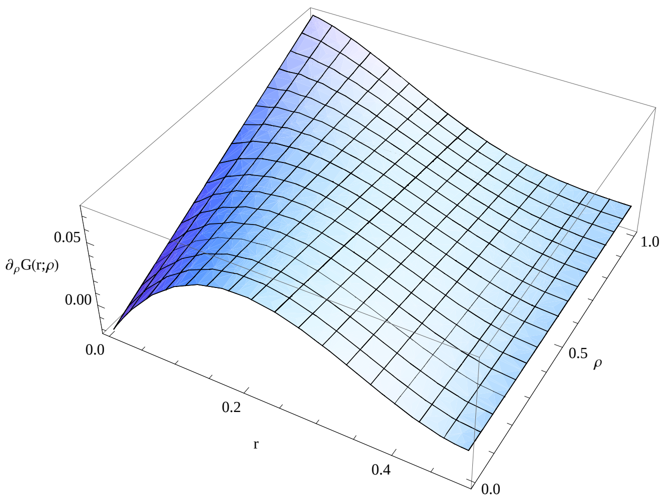

Figure 4: Plot of $\partial_\rho G(r; \rho)$ for all $(r, \rho) \in [0, c] \times [0, 1]$

3.3 Conclusion

Combining both results we conclude that setting the royalty rate equal to the loser's cost reduction is justified, because the innovator would never choose a royalty rate below that cost reduction, whereas if the innovator were free to choose any royalty rate, he would set a royalty rate that exceeds the loser's cost reduction and implements monopoly, in violation of antitrust rules.

References

Fan, C., B. Jun, and E. Wolfstetter (2011). *Auctioning Process Innovations when Losers' Bids Determine Royalty Rates*. Working Paper. Department of Economics, Humboldt University at Berlin.