Equilibrium Product Variety and Market Structure in Successive Oligopolies

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Abstract

We develop a successive oligopoly model in which multi-product upstream manufacturers sell their products to consumers through downstream retailers. The product variety offered by each manufacturer and the entry in the upstream market are both endogenous. We show that the equilibrium configuration of the upstream market depends crucially on the economies of scope in the process of new product creation. When the economies of scope are weak the number of manufacturers increases and each manufacturer produces a single product. Manufacturers produce multiple products only if the economies of scope are sufficiently strong. Furthermore, we examine how a number of other market characteristics, such as the market size, the product substitutability and the number of retailers affect product variety, entry, firms' profits and welfare.

Keywords: multiproduct firms; products variety; successive oligopoly; vertical relations
JEL classification: L22; L25, L42

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1 Introduction

In real world markets most of the firms produce multiple products. Yet, the economic literature has traditionally assumed that firms are single-good producers (see e.g., Hotelling, 1929, Salop, 1979). In real world markets also most of the product manufacturers do not sell their products directly to consumers. They sell them instead through retailers. In other words, in most real world markets manufacturers produce multiple products, they sell their products to retailers and the latter sell them to consumers. A typical example is the food industry, where food processing firms produce a line of food products which they sell through food retailers and supermarkets. The literature that has studied product variety in markets characterized by successive oligopoly is scarce and has assumed that manufacturers are single-product firms (see e.g., Reisinger and Schnitzer, 2008).

This paper aims to fill the gap between real world markets and economic theory by analyzing successive oligopolies in which the manufacturers that operate in the market's upstream tier have the option to produce a line of goods. All the manufacturers' goods are imperfect substitutes and entry in the upstream market is endogenous. The number of retailers that operate in the market's downstream tier is exogenously fixed. Each retailer can buy and resell to final consumers all the manufacturers' goods. Each manufacturer incurs a cost that is increasing in the number of varieties that it produces.[1] There are thus economies of scope in the creation of new products. Reselling costs are assumed to be null. Consumers' preferences are described by a quadratic Dixit (1979)-type utility function defined over all the varieties of the substitute goods offered in the market. A multi-stage game with observable actions is analyzed. In the beginning of the game, manufacturers decide whether or not they will enter in the upstream market. In the following stage, each manufacturer chooses the number of its products, i.e., product variety, and the wholesale price of each of its products. In the final stage, retailers buy the manufacturers goods and resell them in the final market by setting their quantities.

Using the above described framework, we study the equilibrium market structure in the upstream tier, that is, the number of manufacturers and the product variety offered be each manufacturer. Moreover, we explore the role of a number of market characteristics such as

[1] A similar approach is used in Alanson and Montagna (2005), Ottaviano et al. (2002), Chemla (2003), Feenstra and Ma (2007)

the economies of scope, the degree of product substitutability and the number of downstream retailers for market outcomes (i.e., number of upstream manufacturers, product variety, wholesale prices). Their impact on consumers' surplus and total welfare is also investigated. A comparison with the benchmark case of single-product manufacturers is conducted.

We demonstrate that the equilibrium number of manufacturers as well as the number of goods produced by each manufacturer depend crucially on the economies of scope. When the economies of scope are weak, the number of manufacturers increases and each manufacturer is single-product. When instead the economies of scope are too strong, a single manufacturer produces all the goods. Intuitively, the strong economies of scope translate into a lower cost of introducing an additional product in the market. Clearly, a manufacturer has higher incentives to introduce more products in the market when the cost of introducing them is lower. However, the higher product variety offered by a manufacturer increases the competition in the upstream tier and decreases in turn the entry incentives.

We also demonstrate that the product variety offered by each manufacturer is higher when there are more retailers in the downstream tier, as well as when the market is large. Both of these results are quite intuitive since when the market size is large and there are more downstream customers the manufacturers enjoy higher demand for their products. The impact however of the market size and the downstream concentration on the equilibrium number of manufacturer is negligible.

Comparing the case of multi-product manufacturers with the benchmark case in which all the manufacturers are single-product, we find that the wholesale prices are lower in former latter case. This occurs simply because a multi-product manufacturer internalizes the positive effect of an increase in the wholesale price of one of its products on the demand of the rest of its products. It turns outs that the total number of products, the total industry's output and the retailers' profits are higher, in the case of multi-product manufacturers than the respective ones in the case of single-product manufacturers. Regarding welfare, numerical simulations indicate that both the consumers' surplus and the total welfare increase with the intensity of the economies of scope, the product differentiation and the number of downstream firms.

The existing theoretical literature on product diversity suggests that product diversity may be excessive or insufficient depending on the relative strength of various effects. Studies of product diversity have been traditionally conducted using two alternative families of models. On the one hand, spatial models of localized competition, similar to those proposed by Hotelling

(1929) and Salop (1979), have been extensively used. On the other hand, a large literature has followed Spence (1976) and Dixit and Stiglitz (1977) and assumed the existence of a representative consumer with well defined preferences over all possible varieties. In this setup neighboring effects are absent and each firm competes against "the market". Both of this type of studies were made based typically on the assumption that an individual firm produces one good only. However, as mentioned above the single-good producing firm assumption is in stark contrast to reality where multi-product lines are a commonplace.

A number of more recent papers has started to investigate theoretically the behavior of multi-product firms in the industrial organization literature, as well as in the literature on international trade. Helpman (1985) has analyzed how a multinational firm will expand over multiple product lines. He has constant-elasticity demands (CES preferences) in his analysis and for this reason his model has not taken into account the implied effect on the markups of the firms, i.e., it has ignored the interaction of multiple products in demand. Instead, Helpman has relied on diseconomies of scope to limit firms' expansion into new product lines. Different versions of Helpman (1985) there exist in more recent literature dealing with CES preferences (see e.g., Allanson and Montagna, 2005, Bernard et al., 2006, Brambilla, 2006).[2] Departing from CES preferences, Nocke and Yeaple (2006) have used a partial equilibrium inverse demand curve for every product produced by a firm. They likewise have not taken into account the effect of increases in a firm's varieties on the demand for its existing products but have assumed decreasing returns to the range of products. Endogenous markups have been introduced using alternative preferences. More specifically, Anderson and de Palma (1992 and 2006) have considered a nested logit demand function. Ottaviano at al. (2002) and Melitz and Ottaviano (2005) have assumed linear-quadratic preferences. Eaton and Schmitt (1994), Norman and Thisse (1999), Eckel (2006), and Eckel and Neary (2006) have analyzed multi-product firms in models of spatial product differentiation in a Salop-type circular market.[3] In their settings, marginal costs increase with the distance from a firms' core competence, such that diseconomies of scope limit firms' expansion over the product space in addition to the cannibalization effect. Doraszelski and Draganska (2006) have analyzed product differentiation strategies in a duopoly by assuming that firms can either produce general purpose products or

[2] Erkel-Rousse (1997) has considered vertical product differentiation with multi-product firms and CES-preferences.

[3] Blanchard at al. (2007) have analyzed product differentiation in a Hotelling's model with a linear market.

products that are targeted to a certain market segment. Finally, Hansen and Jurgensen (2001) and Hansen and Nielsen (2007) have considered a linear demand function. In their model production strategies of multi-product firms are determined by the influence of the number of goods or the number of plants on fixed and variable costs. All of these papers have considered one-tier industries.

A recent paper by Reisinger and Schnitzer (2008) has analyzed product variety in vertically related industries. They have developed a model of successive oligopolies with endogenous market entry, allowing for varying degrees of product differentiation and entry costs in both the upstream and the downstream market. They have analyzed how different forms of vertical restraints influence the endogenous market structure and show when they are welfare enhancing. Although this paper has dealt with product variety in successive oligopolies, in contrast to ours, it has assumed that upstream manufacturers are single-product.

Summing up, the existing literature has not provided a general model with multi-product firms in vertically related oligopolies. One of the reasons for this lack is that such models quickly become very complicated. A contribution of our paper is to provide such a model that can be used for addressing a variety of issues that arise in vertically related industries.

The rest of the paper is organized as follows. In Section 2, we describe our model. In Section 3, we characterize the equilibrium when the number of manufacturers is given. In Section 4, we endogenize the upstream market structure, that is, the number of manufacturers. We conclude in Section 5. All the proofs are included in the Appendix.

2 The Model

We consider a two-tier industry. The industry's upstream tier consists of $M \geq 1$ product manufacturers, each denoted by m, with $m = 1, ..., M$. The manufacturers sell their products to consumers through $R \geq 1$ retailers that operate in the industry's downstream tier. Each retailer is denoted by r, with $r = 1, ..., R$.

Each manufacturer m produces $n_m \geq 1$ different products. The total number of products produced by all manufacturers is $N = \sum_{m=1}^{M} n_m$. The total cost faced by each manufacturer depends on the number of its products. More specifically, the total cost of manufacturer m is given by $TC_m = c(n_m) = bn^{\alpha}$, where $\alpha > 0$ determines the rate of economies of scope in the products creation process and b, with $0 < b < 1$ defines the scale of the cost function. This cost can be thought as the cost of investing in R&D for the creation of new products. Obviously, $c'(n_m) > 0$ and $c(1) > 0$. The last condition implies that entry in the upstream tier is costly.

Each manufacturer m sells its products to the retailers through linear wholesale price contracts. That is, it sets a wholesale price, w_i^m, per unit of product i, $i = 1, 2, ..., n_m$, that it sells. We denote by $\{w_1^m, ..., w_{n_m}^m\}$ the vector of wholesale prices of manufacturer m. The respective total vector of wholesale prices of all the products produced by all the manufacturers is denoted by $\{w_1, ..., w_{n_1}, w_{n_1+1}, ..., w_{n_1+n_2}, ..., w_N\} \equiv \{w_1^1, ..., w_{n_1}^1, w_1^2, ..., w_{n_2}^2, ..., w_1^M, ...w_{n_N}^M\}$, where the first n_1 numbers are the wholesale prices of the products of manufacturer 1, the next n_2 numbers are the wholesale prices of the products of manufacturer 2, etc.

Each retailer r faces no other cost than the cost of obtaining the products from the manufacturers, i.e., the wholesale price w_i per unit of product i. Each retailer may buy and resell any number of products that are produced by the manufacturers and may choose any quantity of each product.[4] We assume that each retailer will trade all products in equilibrium. We denote by $\{q_1^r, ..., q_{n_1}^r, q_{n_1+1}^r, ...q_{n_1+n_2}^r, ..., q_N^r\}$ the vector of quantities that retailer r trades. The retailer r buys quantities $\{q_1^r, ..., q_{n_1}^r\}$ from manufacturer 1, quantities $\{q_{n_1+1}^r, ..., q_{n_1+n_2}^r\}$ from manufacturer 2, etc. We denote by Q_i the total quantity of product i sold in the market by all retailers, i.e., $Q_i = \sum_{r=1}^{R} q_i^r$, where $i = 1, 2, ..., N$.

The representative consumer has the following quadratic utility function:

$$U = A\sum_{i=1}^{N} Q_i - \frac{1}{2}(\sum_{i=1}^{N} Q_i^2 + \gamma \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} Q_i Q_j) + L, \tag{1}$$

where L is the income spend on outside goods, A, with $A > 0$, is the size of the market, and γ, with $0 < \gamma < 1$, is the degree of product substitutability. Namely, the higher is γ the closer substitutes the products are. Note that for simplification reasons, γ denotes the degree of product substitutability both among the products of different manufacturers and among the products of the same manufacturer.

[4] This setting is referred to as "multilateral transaction" in Fauli-Oller and Mesa-Sanchez (2007) because there is no restriction in the products that retailers can buy.

$>$From (1), we obtain the demand function for each product variety i sold by any retailer:

$$p_i = A - Q_i - \gamma \sum_{\substack{j=1 \\ j \neq i}}^{N} Q_j.$$

It follows from the above that the profit function of each retailer r is given by:

$$\pi_r^D = \sum_{i=1}^{N} (p_i - w_i) q_i^r.$$

Respectively, the profit function of each manufacturer m is given by:

$$\pi_m^U = \sum_{i=1}^{n_m} w_i^m Q_i^m - c(n_m).$$

Competitive interactions are modeled as a two-stage game with observable actions. In stage one, each manufacturer m chooses how many goods it will produce n_m and sets the wholesale prices of its products, $\{w_1^m, ..., w_{n_m}^m\}$. In the following stage, stage two, each retailer r buys the manufacturers' products and chooses the quantities of each product $\{q_1^r, ..., q_N^r\}$ that it sells to the final consumers.

The solution concept that we use is the subgame perfect Nash Equilibrium in pure strategies which we obtain using backward in duction.

Note that in what follows we consider two different scenarios regarding entry in the upstream tier. In the first scenario, we assume that the number of manufacturers is exogenous, i.e., it is fixed and equal to M. In the second scenario, we endogenize the number of manufacturers using the free-entry condition. More specifically, we add one stage on the above described game, stage zero, where manufacturers decide whether or not they will enter in the upstream market.

Throughout, we use as a benchmark for comparisons the case in which each manufacturer produces a single product. Manufacturer m's cost function in this case is $TC_m = c(1)$. If N^s is the equilibrium number of manufacturers under the free-entry condition in the benchmark case then clearly N^s is also the equilibrium number of products then.

3 Equilibrium with M Manufacturers

In the last stage of the game, each retailer r chooses the quantity of each product in order to maximize its profits, taking as given wholesale prices:

$$\max_{q_1^r,...,q_N^r} \pi_r^D = \sum_{i=1}^{N}([A - Q_i - \gamma \sum_{\substack{j=1 \\ j\neq i}}^{N} Q_j] - w_i)q_i^r, \quad r = 1, ..., R; \quad i = 1, ..., N, \tag{2}$$

where q_i^r is the quantity of product i sold by retailer r and w_i is the wholesale price of good i.

The first order conditions are:

$$\frac{\partial \pi_r^D}{\partial q_i^r} = -q_i^r + A - Q_i - \gamma \sum_{\substack{j=1 \\ j\neq i}}^{N} Q_j - w_i - \gamma \sum_{\substack{j=1 \\ j\neq i}}^{N} q_j^r. \tag{3}$$

Looking for the symmetric equilibrium, we assume that each retailer sells the same amount of each product, that is, $q_i^1 = q_i^2 = ... = q_i^R = \overline{q}_i$. Therefore, $Q_i = R\overline{q}_i$. Given this, the first order conditions (3) can be rewritten in the following way:[5]

$$\frac{\partial \pi_r^D}{\partial q_i^r} = -\overline{q}_i + A - R\overline{q}_i - \gamma \sum_{\substack{j=1 \\ j\neq i}}^{N} R\overline{q}_j - w_i - \gamma \sum_{\substack{j=1 \\ j\neq i}}^{N} \overline{q}_j.$$

Rearranging terms we have:

$$(R+1)[\overline{q}_i + \gamma \sum_{\substack{j=1 \\ j\neq i}}^{N} \overline{q}_j] = A - w_i. \tag{4}$$

The above system of first order conditions (4) determines the equilibrium quantities as functions of the wholesale prices and of the total number of goods N:

$$q_i(w_1, ..., w_N) = \frac{(1 - \gamma + \gamma(N-1))(A - w_i) - \gamma \sum_{\substack{k=1 \\ k\neq i}}^{N} (A - w_k)}{(1+R)(1-\gamma)(1-\gamma+\gamma N)}. \tag{5}$$

The respective total demand for manufacturer's product i is $Q_n(w_1, ..., w_N) = Rq_i(w_1, ..., w_N)$.

[5] We discuss the conditions that ensure the existence of interior solutions in the model later on.

In the previous stage, stage two, each manufacturer m chooses the wholesale prices of its products, as well as the number of its products in order to maximize its profits:

$$\max_{w_1^m,\ldots,w_{n_m}^m,\; n_m} \pi_m^U = \sum_{i=1}^{n_m} w_i^m Q_i^m - c(n_m). \tag{6}$$

To derive the first order conditions for the manufacturers' problems for a symmetric equilibrium it is convenient to assume that all firms but firm 1 choose the same product variety, $n_2 = n_3 = ... = n_M = n$ and set the same prices for all their goods ($w_{n_1+1} = w_{n_1+2} = ... = w_N = w$) while manufacturer 1 chooses its variety n_1 and sets the price w_1^1 for its first good and the price w^1 for the rest of its goods, $w_2^1 = ... = w_{n_1}^1 = w^1$. Given this, the total number of goods may be written as $N = (M-1)n + n_1$. Now by (5) we have that:

$$\begin{aligned}
\overline{q}_1^1(w_1^1, w^1, n_1, w, n) &= \frac{(1-\gamma)(A-w_1^1) + \gamma(M-1)n(w-w_1^1) + \gamma(n_1-1)(w^1-w_1^1)}{(1+R)(1-\gamma)(1-\gamma+\gamma((M-1)n+n_1))} \\
\overline{q}^1(w_1^1, w^1, n_1, w, n) &= \frac{(1-\gamma)(A-w^1) + \gamma(M-1)n(w-w^1) + \gamma(w_1^1-w^1)}{(1+R)(1-\gamma)(1-\gamma+\gamma((M-1)n+n_1))},
\end{aligned}$$

where $\overline{q}_1^1$ is the demand for good 1 of manufacturer 1 and $\overline{q}^1$ is the demand for the rest of its goods in terms of w_1^1, w^1, n_1, w and n.

Abstracting from the fact that n_1 is integer-valued we rewrite manufacturer 1's profit as:

$$\pi_1^U(w_1^1, w^1, n_1, n) = w_1^1 R\overline{q}_1^1 + (n_1-1)w^1 R\overline{q}^1 - c(n_1). \tag{7}$$

Differentiating (7) with respect to w_1 and n_1 we obtain:

$$\begin{cases}
\frac{\partial \pi_1^U}{\partial w_1^1} = \frac{R}{1+R} \frac{A(1-\gamma)+\gamma(M-1)nw-2\gamma(n_1-1)w_1-2(1-\gamma+\gamma((M-1)n+n_1-1))w_1^1}{(1-\gamma)(1-\gamma+\gamma((M-1)n+n1)} \\
\frac{\partial \pi_1^U}{\partial n_1} = \frac{R}{1+R} \frac{(A(1-\gamma)+\gamma(M-1)n(w-w^1)-w^1+\gamma w_1^1))((1+\gamma(M-1)n)w^1-\gamma w_1^1)}{(1-\gamma)(1-\gamma+\gamma((M-1)n+n1)^2} - abn_1^{-1+a}
\end{cases}. \tag{8}$$

The system of the first order conditions for the symmetric equilibrium is:

$$\begin{cases}
\frac{\partial \pi_1^U}{\partial w} = \frac{R}{1+R} \frac{A(1-\gamma)-(2(1-\gamma)+\gamma(M-1)n)w}{(1-\gamma)(1-\gamma+\gamma Mn)} \leq 0 \\
\frac{\partial \pi_1^U}{\partial n} = \frac{R(A-w)w}{1+R} \frac{1-\gamma+\gamma(M-1)n}{(1-\gamma+\gamma Mn)^2} - abn^{-1+a} \leq 0
\end{cases}. \tag{9}$$

Whenever there exist an internal solution to (9), it is determined implicitly by the following system of equations:

$$\begin{cases} w^* = \frac{A(1-\gamma)}{2(1-\gamma)+(M-1)n^*\gamma} \\ \frac{R(A-w^*)w^*}{1+R}\frac{1-\gamma+\gamma n^*(M-1)}{(1+\gamma(Mn^*-1))^2} = -abn^{*-1+a} \end{cases}. \tag{10}$$

Note that the derivative of the manufacturer's profits with respect to w_1^1 is positive at point $w_1^1 = 0$:

$$\left.\frac{\partial \pi_1^U}{\partial w_1^1}\right|_{n_1=n; w_1=w; w_1^1=0} = \frac{R}{1+R}\frac{A(1-\gamma)+\gamma w(Mn+n-2)}{(1-\gamma)(1-\gamma+\gamma Mn)} > 0.$$

Therefore, the optimal w^* is always positive. It follows that if there exists a corner solution of (9) then it is given by $w^* = \frac{A(1-\gamma)}{2(1-\gamma)+(M-1)\gamma}$ and $n^* = 1$.

Next, we discuss the second order conditions and the existence of a symmetric equilibrium. It is easy to see that the second derivative of manufacturer 1's profits function with respect to any of its wholesale prices is negative: $\frac{\partial^2 \pi_1^U}{\partial w_i^2} = -\frac{2(1-\gamma+\gamma((M-1)n+n_1-1))}{(1-\gamma)(1-\gamma+\gamma((M-1)n+n1)} < 0$, for any $M \geq 1, n \geq 1, n_1 \geq 1$ and $i = 1, ..., n_1$. Thus, the manufacturer's profits function is strictly concave in every of its wholesale prices for any n_1 and therefore there exists a unique point of maximum with respect to the wholesale prices. From this we conclude that, whenever a symmetric equilibrium exists, the wholesale price $w^* = \frac{A(1-\gamma)}{2(1-\gamma)+(M-1)n^*\gamma}$ is indeed the *maximizer* of the manufacturer's profits function for any n_1.

Remark 1. Thus the internal symmetric equilibrium exists for any fixed n^* with equilibrium price $w^* = \frac{A(1-\gamma)}{2(1-\gamma)+(M-1)n^*\gamma}$. In particular this implies that in the case of single-good producers we have that $w^* = \frac{A(1-\gamma)}{2(1-\gamma)+(M-1)\gamma}$.

Plugging $w^* = \frac{A(1-\gamma)}{2(1-\gamma)+(M-1)n^*\gamma}$ into (7) we get the profits of manufacturer 1 as a function of n_1 and n^* :

$$\left.\pi_1^U(n_1, n^*)\right|_{w_1^1=w^1=w^*(n^*)} = \frac{A^2R}{1+R}n_1\frac{(1-\gamma)(1-\gamma+\gamma n^*(M-1))}{(2(1-\gamma)+\gamma(M-1)n^*)^2(1-\gamma+\gamma(M-1)n^*+n_1)} - bn_1^a. \tag{11}$$

Note that manufacturer 1's profit function (11) in general is neither concave nor quasiconcave in n_1. Therefore, it is not necessary that the solution of the first order conditions (w^*, n^*) provides a point of maximum. Indeed, the solution of (9), (w^*, n^*), is the symmetric solution of the manufacturer's problem (6) if and only if $n_1 = n^*$ is the *maximizer* of (11).

Example 1 *Let's consider the set of parameters:* $M = 2, \gamma = 0.6, A = 10, b = 0.1, R = 4$ *and* $a = \{0.43; 0.55; 0.7\}$.

- $a = 0.43$ *(Figure 1):* $\pi_1^U(n_1, n)$ *is neither concave nor quasiconcave; the solution of the FOCs (10) is* $\{n^* = 181.7, w^* = 0.08\}$ *and it is a point of local maximum of* $\pi_1^U(n_1, n)$*. The point of global maximum is* $\{n_1^* = 1, w^* = 0.08\}$*.*

- $a = 0.55$ *(Figure 2):* $\pi_1^U(n_1, n)$ *is neither concave nor quasiconcave; the solution of the FOC (10) is* $\{n^* = 102.3, w^* = 0.14\}$ *and it is a point of global maximum of* $\pi_1^U(n_1, n)$*.*

- $a = 0.7$ *(Figure 3):.* $\pi_1^U(n_1, n)$ *is quasiconcave; the solution of the FOC (10) is* $\{n^* = 57.9, w^* = 0.25\}$ *and it is a point of global maximum of* $\pi_1^U(n_1, n)$*.*

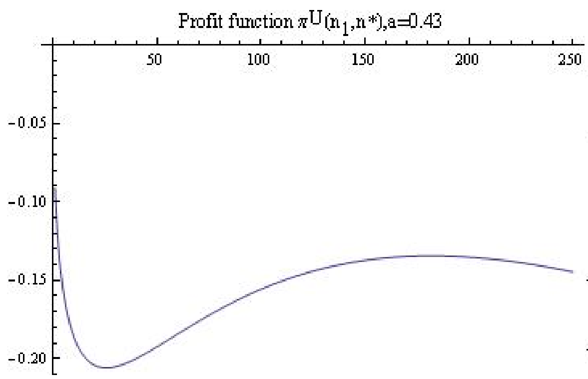


Figure 1

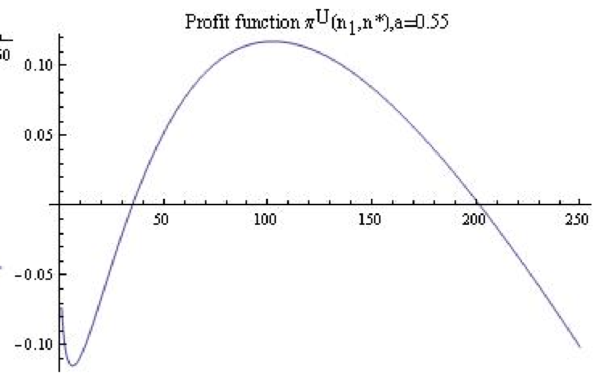


Figure 2

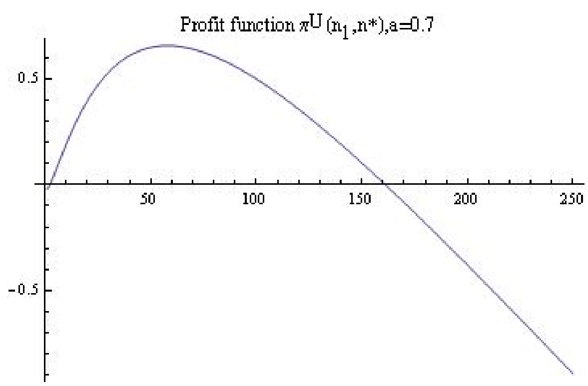


Figure 3

The system (10) is equivalent to the following whenever there exists an internal solution to the manufacturer's maximization problem (6):

$$\frac{A^2 R(1-\gamma)(1-\gamma+\gamma(M-1)n^*))^2}{(1+R)(2(1-\gamma)+\gamma(M-1)n^*)^2(1+\gamma(Mn^*-1))^2} = abn^{*a-1}, \tag{12}$$

where the left and the right sides are the marginal revenue and the marginal cost (in terms of the variety n^*) of each manufacturer in the symmetric equilibrium.

Assumption 1. *Suppose that a set of parameters* $\{A, R, M, a, b, \gamma\}$ *is such that*

$$\frac{A^2 R(1-\gamma)(1-\gamma+\gamma(M-1))^2}{(1+R)(2(1-\gamma)+\gamma(M-1))^2(1+\gamma(M-1))^2} > ab.$$

The condition implies that the marginal revenue of each manufacturer is greater then its marginal cost (and thus the profit of each manufacturer increases in n) at the point $n = 1, w^*|_{n=1} = \frac{A(1-\gamma)}{2(1-\gamma)+(M-1)\gamma}$.

Lemma 1 *The system of the first order conditions (9) has an unique internal solution on* $n^* \in [1, +\infty)$ *if and only if Assumption 1 holds.*

Proof of Lemma. First, let's show that the equation (12) has no more than two roots on $n^* \in R_+$. Taking into account that all expression in brackets with power two are positive, that is $(2(1-\gamma)+\gamma(M-1)) > 0$, $1-\gamma+\gamma(M-1) > 0$, $1+\gamma(M-1) > 0$, on $n^* \in R_+$ we may rewrite (12) w.l.g. in the form of

$$(\sqrt{\frac{R(1-\gamma)}{1+R}}Aab)n^{*\frac{a-1}{2}}(1-\gamma+\gamma(M-1)n^*)) = (2(1-\gamma)+\gamma(M-1)n^*)(1+\gamma(Mn^*-1))$$

where the left side is a strictly increasing function and the right side is quadratic function. It is obvious that there can not exist more than two roots of the last equation on $n^* \in R_+$ and therefore (12) has also not more than two roots on $n^* \in R_+$.

Sufficiency. Suppose Assumption 1 holds. Then the left side of (12) is greater than its right side at $n^* = 1$. Obviously the left side of (12) is smaller than its right side for big n^* and thus there are odd number of roots of (12) on $n^* \in (1, +\infty)$. Given that there (12) has no more than two roots on $n^* \in R_+$ we conclude that if Assumption 1 holds then there exists an unique root on $n^* \in (1, +\infty)$.

Necessity. Suppose that (12) has unique root on $n^* \in (1, +\infty)$. Given that the left side of (12) is smaller than its right side for n^* big enough it must be that the left side is bigger than the right side at $n^* = 1$ and thus Assumption 1 must hold. *QED*.

Assumption 2. *Suppose that* $\{A, R, M, a, b, \gamma\}$ *and* $n^* > 1$ *determined by (12) are such* *that*

$$\frac{\partial \pi_1^U}{\partial n_1}(n_1,n^*)\Big|_{\substack{w_1^1=w^*(n^*)\\ w^1=w^*(n^*)\\ n_1=1}} = \frac{A^2R(1-\gamma)(1-\gamma+\gamma(M-1)n^*))^2}{(1+R)(2(1-\gamma)+\gamma(M-1)n^*)^2(1+\gamma(M-1)n^*)^2} - ab > 0.$$

The condition $\frac{\partial \pi_1^U}{\partial n_1}(n_1,n^*)\Big|_{\substack{w_1^1=w^1=w^*(n^*)\\ n_1=1}} > 0$ means that the the profit of the manufacturer 1 increases in n_1 at the point $n_1 = 1$ given that $n^* > 1$, where n^* determined by (12).

Lemma 2 *The profit function (11) is quasiconcave and has an unique internal maximum on* $n_1 \in [1,+\infty)$ *if and only if Assumption 2 holds.*

Proof of Lemma 2. First, let's show that the equation $\frac{\partial \pi_1^U}{\partial n_1}(n_1,n^*)\Big|_{\substack{w_1^1=w^*(n^*)\\ w^1=w^*(n^*)}} = 0$ has not more than two roots on $n_1 \in R_+$.

On $n_1 \in R_+$ the number of roots of the equation

$$\frac{\partial \pi_1^U}{\partial n_1}(n_1,n^*)\Big|_{\substack{w_1^1=w^*(n^*)\\ w^1=w^*(n^*)}} = \frac{A^2R(1-\gamma)(1-\gamma+\gamma(M-1)n^*))^2}{(1+R)(2(1-\gamma)+\gamma(M-1)n^*)^2(1-\gamma+\gamma(M-1)n^*+\gamma n_1)^2} - abn_1^{a-1} = 0$$

is not bigger than the maximum number of roots of

$$\frac{A^2R(1-\gamma)(1-\gamma+\gamma(M-1)n^*))^2}{(1+R)(2(1-\gamma)+\gamma(M-1)n^*)^2}\frac{1}{ab}n_1^{1-a} = (1+\gamma(M-1)n^*+n_1)^2$$

where the left side is a strictly increasing function and the right side is a quadratic function. Thus it is obvious the maximum number of roots is two. Second, let's not that $\frac{\partial \pi_1^U}{\partial n_1}(n_1,n^*)\Big|_{w_1^1=w^1=w^*(n^*)} < 0$ for a big enough n_1.

Sufficiency. Suppose Assumption 2 holds. Then $\frac{\partial \pi_1^U}{\partial n_1}(n_1,n^*)\Big|_{w_1^1=w^1=w^*(n^*)} > 0$ at $n_1 = 1$ and therefore the equation $\frac{\partial \pi_1^U}{\partial n_1}(n_1,n^*)\Big|_{w_1^1=w^1=w^*(n^*)} = 0$ has an odd number of roots on $n_1 \in (1,+\infty)$. Because there can not exist more than two roots we conclude that there exist an unique root on $n_1 \in (1,+\infty)$. By Assumption 1 we have also that $\pi_1^U(n_1,n^*)$ increases at $n_1 = 1$ and therefore its unique point of extremum is the point of maximum. Therefore (11) is quasiconcave function.

Necessity. Suppose that (11) is a) quasiconcave and b) has an unique internal maximum on $n_1 \in [1,+\infty)$. The part b) rules out the case when (11) is strictly decreasing function because in this case there is no internal maximum. Thus (a) and (b) implies that (11) has one and only one extremum and it is a point of maximum. Hence (11) must be an increasing function in n_1 at $n_1 = 1$ which implies Assumption 2. *QED*.

Combining Assumption 1 and Assumption 2 we obtain the following result.

Proposition 1 *Suppose that Assumption 1 and Assumption 2 hold. Then the system of the first order conditions (9) determines the unique internal symmetric equilibrium.*

Proof of Proposition. If Assumption 1 holds then there exists one root of (12) on $n^* > 1$. If Assumption 2 holds then $\frac{\partial \pi_1^U}{\partial n_1}(n_1, n^*)\Big|_{w_1^1 = w^1 = w^*(n^*)} = 0$ have the unique maximum on $n_1 > 1$. Taking into account that at $n_1 = n^*$ the equation $\frac{\partial \pi_1^U}{\partial n_1}(n_1, n^*)\Big|_{w_1^1 = w^1 = w^*(n^*)} = 0$ is equivalent to (12) we conclude that if both Assumptions 1 and Assumption 2 hold then $n_1 = n^*$ is unique symmetric equilibrium. *QED*

Let's note that an unique internal solution of the manufacturers' maximization problem (6) may exists even if Assumptions 1-2 do not hold and manufacturer's profit functions are not quasiconcave. Thus Assumption 2 is necessary and sufficient for quasiconcavity of the manufacturers' profit functions and Assumption 1 and 2 together are sufficient for uniqueness of the internal solution of the problem (6).

Plugging w^* and n^* into (5), (6) and (2), we obtain the equilibrium quantities, as well as the equilibrium profits and the total industry's output:

$$q^* = \overline{q}_n = \frac{A}{1+R}\frac{1-\gamma+\gamma n^*(M-1))}{(2(1-\gamma)+\gamma(M-1)n^*)(1-\gamma+\gamma M n^*)}; \quad (13)$$

$$\pi^{U*} = \pi_m^U = \frac{A^2 R}{1+R} n^* \frac{(1-\gamma)(1-\gamma+\gamma n^*(M-1))}{(2(1-\gamma)+\gamma(M-1)n^*)^2(1-\gamma+\gamma M n^*)} - c(n^*); \quad (14)$$

$$\pi^{D*} = \pi_r^D = \frac{A^2}{(1+R)^2}\frac{n^*(1-\gamma+\gamma n^*(M-1))^2}{(2(1-\gamma)+\gamma(M-1)n^*)^2(1-\gamma+\gamma M n)}; \quad (15)$$

$$TQ^* = MR\overline{q} = \frac{AR}{1+R}\frac{1-\gamma+\gamma n^*(M-1))}{1-\gamma+\gamma M n^*} M. \quad (16)$$

The respective final price is:

$$p^* = \frac{A((2+R)(1-\gamma)+\gamma(M-1)n^*))}{(1+R)(2(1-\gamma)+\gamma(M-1)n^*)}. \quad (17)$$

Finally, one could obtain the equilibrium consumers' surplus using (1), as well as the total welfare defined as the sum of the consumers surplus and the firms' profits:

$$CS^* = \frac{A^2MR^2n^*}{2(1+R)^2}\frac{(1-\gamma+\gamma n^*(M-1))^2}{(2(1-\gamma)+\gamma n^*(M-1))^2(1-\gamma+\gamma Mn^*)}, \tag{18}$$

$$W^* = \frac{A^2MR(2+R)n^*}{2(1+R)^2}\frac{(1-\gamma+\gamma n^*(M-1))^2}{(2(1-\gamma)+\gamma n^*(M-1))^2(1-\gamma+\gamma Mn^*)} + M\pi^{U^*}. \tag{19}$$

Setting $n^* = 1$ in the equilibrium expressions (13)-(19), we obtain the respective equilibrium expressions for the benchmark case with single-product manufacturers. In particular, the equilibrium wholesale price and the manufacturer's profits in the benchmark are:

$$w^{*s} = \frac{A(1-\gamma)}{2(1-\gamma)+\gamma(N^s-1)}; \tag{20}$$

$$\pi^{U*s} = \frac{RA^2(1-\gamma)(1+\gamma(N^s-2))}{(1+R)((2+\gamma(N^s-3))^2(1+\gamma(N^s-1))} - c(1). \tag{21}$$

The next Proposition compares the multi-product manufacturer's wholesale prices with the ones of a single-product manufacturer for the same level of total product variety.

Proposition 2 *Suppose that Assumption 1 and Assumption 2 hold. When the number of manufacturers in the benchmark case N^s is such that $N^s = Mn^*$ then the equilibrium wholesale price in the benchmark case is lower than the equilibrium wholesale price in the case of multi-product manufacturers, $w^{*s} < w^*$.*

Proof of Proposition. $N^s = Mn^* \Rightarrow N^s - 1 = Mn^* - 1 > Mn^* - n^* = (M-1)n^* \Rightarrow w^s < w^*$. *QED*

According to Proposition, the equilibrium wholesale prices are higher when the manufacturers are multi-product than when they are single-product. This finding is driven by the impact of a change in the wholesale price of a product on the demand for the rest of the products. More specifically, if a multi-product manufacturer increases the wholesale price of one of its products then there will be an increase in the demand for the products of its rival manufacturers, as well as in the demand for the rest of its own products. The latter is a positive effect. The multi-product manufacturer internalizes this effect and increases its wholesale prices.

Next, we analyze the role of a number of market characteristics for the market equilibrium. We start by examining the impact of concentration in the upstream market sector.

Proposition 3 *Suppose that Assumption 1 and Assumption 2 hold for some $M_1, M_2 \in \mathbb{N}$ with $1 < M_1 < M_2$. Then the product variety offered by each manufacturer n^*, as well as the equilibrium profits of each manufacturer decrease in M in the sense that $n^*(M_1) > n^*(M_2) > 1$ and $\pi_m^{U*}(M_1) > \pi_m^{U*}(M_2)$.*

Proof of Proposition. First, let's show that the left side of (12) is a decreasing function in M for any $M > 2$ and any *fixed* n. Let's $LS(n, M) = \frac{A^2R(1-\gamma)(1-\gamma+\gamma(M-1)n))^2}{(1+R)(2(1-\gamma)+\gamma(M-1)n)^2(1+\gamma(Mn-1))^2}$. Then it is easy to see that

$$\frac{\partial LS}{\partial M} = -\frac{2A^2R(1-\gamma)}{(1+R)} \frac{\gamma n(1-\gamma+\gamma n(M-1))((1-\gamma)^2+\gamma((1-\gamma)(2M-3)n+(M-1)^2n^2))}{(2(1-\gamma)+\gamma n(M-1))^3(1-\gamma+\gamma Mn)^3} < 0$$

for any $M > 2$.

Also it may be shown that $LS(n, M)|_{M=1} = \frac{A^2R(1-\gamma)}{4(1+R)(1-\gamma+\gamma n)^2} > \frac{A^2R(1-\gamma)(1-\gamma+\gamma n)^2}{(1+R)(2-2\gamma+\gamma n)(1-\gamma+2\gamma n)^2} = LS(n, M)|_{M=2}$. This implies that the graph of $LS(n, M)$ shifts downward as M increases for all $M \in \mathbb{N}$. Therefore a point of intersection of $LS(n, M)$ and abn^{*a-1} shifts left as M increases. Thus the equilibrium variety $n^*(M)$ decreases in M.

Using the FOC (12) the manufacturer's profits (14) can be rewritten as:

$$\begin{aligned} \pi^U &= nc'(n)\frac{1-\gamma+\gamma Mn}{1-\gamma+\gamma Mn-\gamma n} - c(n) = \\ &= nc'(n)[1 + \frac{\gamma n}{1-\gamma+\gamma Mn-\gamma n}] - c(n). \end{aligned}$$

By the envelope theorem we have

$$\frac{d}{dM}\pi^U(M, n^*(M)) = \frac{\partial}{\partial M}\pi^U(M, n^*(M)) = -\frac{nc'(n)(\gamma n)^2}{(1-\gamma+\gamma Mn-\gamma n)^2} < 0.$$

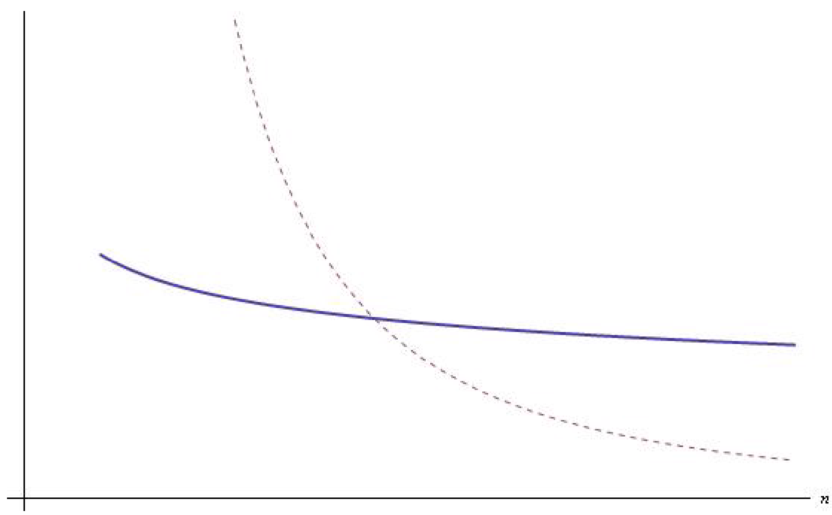
QED.

Remark 2. Let's note that the result of the Proposition holds for $M \in \mathbb{Z}$. Actually for a fixed n the left side of (12) is not monotone in $M \in [1; +\infty)$, it has one point of maximum at $\widetilde{M} : 1 < \widetilde{M} < 2$ and thus $LS(n, M)$ increases on the interval $[1; \widetilde{M}]$ and decreases on $[\widetilde{M}; +\infty]$.

Proposition 3 asserts that the more manufacturers are in the upstream market, and thus the less concentrated the upstream market is, the less is the product variety offered by each of them, as well as the lower are each manufacturer's profits. Intuitively, a higher number of manufacturers clearly means stronger competition among them. When competition is strong, the manufacturer's incentives to insert a new product in the market are reduced. This occurs because due to the intensity of competition the new product will not be so profitable.

Proposition 4 *Suppose that Assumption 1 and Assumption 2 hold on some set of* $\{A, R, M, a, b, \gamma\}$. *Then the equilibrium variety of each firm* n^* *increases in* R, A *and decreases in* α, b; *the equilibrium wholesale price* w^* *increases in* α, b *and decreases in* R *on this set.*

Proof of Proposition. On the picture bellow the dotted and the bold line represent the graphs of the left and the right side of (12) respectively. The intersection point is the point of the symmetric equilibrium. An increase in R or A shifts the graph of the left side upward and thus the equilibrium point shifts right while an increase in α or b shifts the graph of the right side upward and thus equilibrium point moves left. Therefore the equilibrium variety of each firm n^* increases in R, A and decreases in α, b. As $w^* = \frac{A(1-\gamma)}{2(1-\gamma)+(M-1)n^*\gamma}$ inversely depends on n^* and does not depends on α, b, R directly we have that w^* increases in α, b and decreases in R.



The dotted (bold) line is the graph of the left (right) side of (12)

QED.

In order to examine the role of the number of retailers, as well as of the economies of scope and of product substitutability we resort to numerical simulations. We do so because the system (12) that determines endogenously the equilibrium values of the wholesale prices and of the number of products (w^*, n^*) does not have a solution in closed form. Thus, it is not possible to perform a comparative statics analysis for all of the market characteristics analytically. We start by setting values for of parameters round the point: $R = 4, M = 2, \gamma = 0.6,\ A = 10,$ $b = 0.1$, and $\alpha = 0.7$. In order to examine the impact of the number of retailers R we allow for different values for R.

R	n^*	Mn^*	π^{U*}	w^*	TQ^*	π^{D*}	CS^*	W^*
2	25.43	50.87	.37	.25	10.69	17.38	34.75	70.26
3	27.35	54.69	.39	.23	12.06	9.81	44.18	74.41
4	28.45	56.90	.40	.22	12.88	6.29	50.38	76.37
5	29.16	58.37	.41	.22	13.43	4.37	54.74	77.12
6	29.67	59.35	.41	.21	13.82	3.22	57.97	78.11
50	32.21	64.42	.44	.20	15.85	.061	76.15	80.08
∞	32.6	65.20	.44	.19	16.17	0	79.27	80.16

Table 1: The impact of R for given M

It follows from Table 1 that the more retailers (higher R) are in the market, the higher is the product variety of each manufacturer and the total variety, the manufacturer's profits, the total output, the consumers surplus and the total welfare. These results are quite intuitive: the stronger is competition among retailers the lower are their prices and therefore the higher are quantities sold. Thus manufacturers' profits and consumer's surplus are higher.

Next, we examine the impact of the intensity of the economies of scope by considering different values of a.

α	n^*	Mn^*	π^{U*}	w^*	TQ^*	π^{D*}	CS^*	W^*
0.7	28.45	56.89	.400	.22	12.88	6.30	50.38	76.37
0.8	21.68	43.36	.675	.28	12.75	6.19	49.52	75.64
0.9	17.12	34.25	0.99	.36	12.61	6.08	48.60	74.88
1	13.92	27.84	1.32	.44	12.45	5.95	47.63	74.11
1.1	11.59	23.18	1.68	.51	12.29	5.83	46.63	73.32

Table 2: The impact of α for given M

As Table 2 indicates the smaller is α, and thus, the stronger are the economies of scope, the higher is product variety and the lower are the manufacturer's profits. The intuition is that the strong economies of scope translate into a lower cost of introducing an additional product in the market. Clearly, a manufacturer has higher incentives to introduce more products in the market when the cost of introducing them is lower. However, the higher product variety offered by each manufacturer increases the competition in the upstream level leading in turn to lower manufacturer's profits. As Table 2 also indicates, due to the higher product variety, the total output, the consumers' surplus and the total welfare are also higher when the economies of scope are stronger.

γ	n^*	Mn^*	π^{U*}	w^*	w^*	TQ^*	π^{D*}	CS^*	W^*
0.5	40.10	80.20	0.50	.23	.12	15.42	7.53	60.24	91.38
0.6	28.44	56.89	0.40	.22	.11	12.88	6.29	50.38	76.37
0.7	20.11	40.21	0.31	.20	.10	11.08	5.42	43.40	65.73
0.8	13.60	27.21	0.24	.17	.09	9.73	4.78	38.24	57.84
0.9	7.93	15.87	0.16	.13	.07	8.71	4.29	34.34	51.63

Table 3: The impact of γ for given M

In the above table, Table 3, we examine the role of product substitutability γ. As you can see, the smaller is γ and thus the more differentiated are the products, the higher is product variety, the higher are the manufacturer's profits as well as the consumers surplus and the total welfare. Intuitively, the manufacturers have stronger incentives to offer more products when these products do not compete fiercly among them. However, not only the manufacturers but also the consumers and the retailers benefit from the higher product variety. Thus, welfare is also higher when γ is low.

Considering the impact of the cost function parameter b we find the following:

b	n^*	Mn^*	π^{U*}	w^*	TQ^*	π^{D*}	CS^*	W^*
0.2	18.53	37.07	0.58	.33	12.65	6.12	48.93	74.56
0.3	14.35	28.71	.71	.42	12.47	5.97	47.79	73.11
0.4	11.94	23.88	.82	.5	12.32	5.85	46.80	71.85
0.5	10.33	20.66	.91	.57	12.18	5.74	45.93	70.73
0.6	9.16	18.33	.99	.63	12.04	5.64	45.13	69.69
0.7	8.27	16.54	1.06	.69	11.93	5.55	44.40	68.73
0.8	7.56	15.12	1.13	.75	11.81	5.46	43.71	67.83
0.9	6.98	13.95	1.19	.80	11.70	5.38	43.06	66.97

Table 4: The impact of b for given M

Table 4 indicate that the higher cost of production b the lower the total variety produced and it makes competition weaker that increases the manufacturers' profits while the consumers' surplus, the total welfare.

Finally, we analyze the impact of changes in M for the set of parameters $R = 4, \gamma = 0.6$, $A = 10$, $b = 0.1$, and $\alpha = 0.9$.

M	n^*	Mn^*	π^{U*}	w^*	TQ^*	π^{D*}	CS^*	W^*
2	17.12	34.25	.99	.36	12.60	6.08	48.6	74.88
3	11.27	33.82	.30	.28	12.71	6.18	49.42	75.03
4	8.35	33.40	.13	.25	12.74	6.21	49.68	75.04
5	6.61	33.04	.07	.24	12.56	6.22	49.80	75.02
6	5.46	32.75	.03	.23	12.76	6.23	49.86	75.00
7	4.64	32.49	.02	.22	12.77	6.24	49.90	74.98
8	4.03	32.27	.01	.23	12.77	6.24	49.92	74.96
9	3.56	32.07	.003	.22	12.77	6.24	49.94	74.94
10	3.19	31.90	-.0006	.22	12.77	6.24	49.95	74.92
11	2.98	31.74	-.003	.22	12.77	6.24	49.95	74.90
12	2.63	31.59	-.005	.22	12.77	6.24	49.96	74.89

Table 5: The impact of M.

The table demonstrates that while an increase in M has a strong negative effect on the variety of each firm n^* it has only week negative effects on the total variety, the total quantity and the final price and therefore the week (positive) effect on the consumers' surplus. The manufacturer's profit decreases in M while the total welfare in non-monotone in M. Also the table 5 shows that the highest variety is offered by a monopoly and that the socially optimal number of firms is such that all manufacturers get positive profit.

4 Endogenous Number of Manufacturers

In this Section, we do no longer treat the number of manufacturers M as exogenous. Instead, we endogenize it by imposing the free entry condition, that is, $\pi^U_m(M^*) = 0$.

Whenever there exists the internal solution of FOCs (8), that is $n^* > 1$, it is determined by the system of (12) which may be written as

$$\frac{A^2R(1-\gamma)(1-\gamma+\gamma M^*n^*-\gamma n^*)n^*}{(1+R)(2-2\gamma+\gamma M^*n^*-\gamma n^*)^2(1-\gamma+\gamma M^*n^*)} = \alpha b n^{*\alpha}\frac{1-\gamma+\gamma M^*n^*}{1-\gamma+\gamma(M^*-1)n^*}.$$

Then the manufacturer's profit function (14) may be written as $\pi^U = bn^{*\alpha}(\alpha\frac{1-\gamma+\gamma Mn^*}{1-\gamma+\gamma(M-1)n^*}-1)$ and clearly it is positive for any $\alpha \geq 1$.

Therefore if $\alpha \geq 1$ then if the free-entry equilibrium exists it is such that $n^* = 1$. The following proposition asserts that such an equilibrium indeed exists for any $\alpha \geq 1$.

Proposition 5 *Suppose that $A^2 \geq 8b$. Under the free-entry condition for any $\alpha \geq 1$ the system (8) has a corner solution only, that is for any $\alpha \geq 1$, $n^* = 1$. Moreover the equilibrium number of manufacturers is the same as in the benchmark case, that is $M^* = N^{s*}$.*

Proof of Proposition. As we show before the internal equilibrium (with $n^* > 1$) cannot satisfy the free-entry condition. Thus we need to show that for any $\alpha \geq 1$ there exist $M^c > 1$ such that (i) (9) satisfied at point $\{M^c, n^* = 1\}$, (ii) it satisfies the free-enrty condition, that is $\pi^U|_{M^c,n^*=1} = 0$ and (iii) $\{M^c, n^* = 1\}$ indeed solves the manufacturer 1's maximization problem or, in other words, that $n_1 = 1$ maximizes (11) given $\{M^c, n^* = 1\}$.

First let's note that if n^* the manufacturers profit function (14),

$$\pi^U|_{M,n^*=1} = \frac{A^2R}{1+R}\frac{(1-\gamma)(1-\gamma+\gamma(M-1))}{(2(1-\gamma)+\gamma(M-1))^2(1-\gamma+\gamma M)} - b,$$

has the following properties: $\pi^U\big|_{M=1,n^*=1} \geq 0$, it monotonically decreases in M and it goes to $-b < 0$ as M goes to infitity. Therefore there exists unique $M = M^c$ such that $\pi^U\big|_{M=M^c,n^*=1} = 0$.

Second, it is easy to see that given when $M = M^c, n^* = 1, w^* = \frac{A(1-\gamma)}{2(1-\gamma)+(M^c-1)\gamma}$ then the system (9) satisfied with $\frac{\partial\pi^U}{\partial w} = 0$ and $\frac{\partial\pi^U}{\partial n} < 0$.

Next, let's note that manufacturer profit $\pi_1^U(n_1, n^*)\big|_{n^*=1}$ described by (11) is concave in n_1 for any $\alpha \geq 1$. Therefore it has unique extremum point which is the point of maximum. Combining these facts we conclude that for any $\alpha \geq 1$ under free entry condition there exists unique solution and it is the corner solution with $n^* = 1$.

Finally as $\pi^U(M^*, 1) = 0$ we have that $M^* = N^{s*}$ by definition of N^{s*}. *QED*

The Proposition says that if the production function exhibits diseconomy of scope then each manufacturer produces one good only and the equilibrium is the same as in the case of singe-good producers. The intuition for this result is the following. A cost of entering market (b) is lower than the cost of creation of one additional product (bn^α) by existing producers. Thus each manufacturer produces one good only and the number of firms coincide with equilibrium number of firm in the benchmark case.

Now let's consider a case $\alpha < 1$. An internal solution of FOCs which satisfied the free-entry condition(12) determined by the system:

$$\left\{\begin{array}{rcl} \pi^U & = & \frac{A^2R(1-\gamma)(1-\gamma+\gamma(M^*-1)n^*)n^*}{(1+R)(2(1-\gamma)+\gamma(M^*-1)n^*)^2(1-\gamma+\gamma M^*n^*)} - bn^{*\alpha} = 0 \\ \frac{\partial\pi^U}{\partial n^*} & = & \frac{A^2R(1-\gamma)(1-\gamma+\gamma M^*n^*-\gamma n^*)^2}{(1+R)(2-2\gamma+\gamma M^*n^*-\gamma n^*)^2(1-\gamma+\gamma M^*n^*)^2} - \alpha bn^{*\alpha-1} = 0 \end{array}\right. , \tag{22}$$

where the first equation is the free entry condition and the second is the first order condition in terms of n^*. Equivalently the system (22) may be written in the form of:

$$\left\{\begin{array}{c} M^* = \frac{1}{1-\alpha} - \frac{1-\gamma}{\gamma n^*} \\ \frac{(1-\alpha)^2\alpha A^2R(1-g)n}{(1+R)((1-g)(1-\alpha)+\alpha gn^*)^2} = bn^{*\alpha} \end{array}\right. , \tag{23}$$

and the equilibrium wholesale price as a function of n^* is given by:

$$w^* = \frac{(1-\alpha)(1-\gamma)A}{(1-\alpha)(1-\gamma)+\alpha\gamma n^*}. \tag{24}$$

Using $M^* = \frac{1}{1-\alpha} - \frac{1-\gamma}{\gamma n^*}$ we may rewrite Assumptions 1 and 2 as the following.

Assumption 1'. *Suppose that a set of parameters* $\{A, R, a, b, \gamma\}$ *is such that*

$$\frac{(1-\alpha)^2 \alpha A^2 (1-\gamma) R}{(1+R)((1-\gamma)(1-\alpha)+\alpha\gamma)} > b.$$

Assumption 2'. *Suppose that* $\{A, R, a, b, \gamma\}$ *and* $n^* > 1$ *determined by the second equation in (23) are such that*

$$\frac{(1-\alpha)^2 \alpha A^2 (1-\gamma) R n^{*2}}{(1+R)(1-\alpha+\alpha n^*)^2((1-\alpha)(1-\gamma)+\alpha\gamma n^*)^2} \geq b.$$

It is straightforward corollary of the Proposition 1 that Assumptions 1' and 2' ensure that manufacturers' profit functions are quasiconcave and there exists the unique internal solution. Let's note that in the benchmark case each manufacturer produces one good only and thus the equilibrium number of producers is determined by

$$\frac{A^2 R(1-\gamma)(1-\gamma+\gamma(N^{s*}-1))}{(1+R)(2(1-\gamma)+\gamma(N^{s*}-1))^2(1-\gamma+\gamma N^{s*})} = b$$

while the equilibrium wholesale price is $w^{s*} = \frac{A(1-\gamma)}{2(1-\gamma)+\gamma(N^s-1)}$.

In order to obtain additional results we need an assumption that garantees that α and γ are not close to zero together.

Assumption 3. *Suppose* $\gamma + \alpha > 1$.

Lemma 3 *Under Assumption 3 the free entry condition* $\pi^U(M, n) = 0$ *determined implicitly the function* $M(n)$ *with* $\frac{dM}{dn} < 0$ *and* $\frac{d(Mn)}{dn} > 0$.

Proof of Lemma. By the implicit function theorem $\frac{dM}{dn} = -\frac{d\pi^U/dn}{d\pi^U/dM}$. Calculating derivatives, using $\pi^U(M, n) = 0$ and rearranging terms we obtain that $\frac{dM}{dn} = \frac{\theta(M,n;\gamma,\alpha)}{\varphi(M,n;\gamma)}$, where

$$\theta(M, n; \gamma, \alpha) = 2(1-\alpha)(1-\gamma)^3 + (1-\gamma)^2\gamma(3(M-1)+\alpha(5M-3))n-$$
$$-\alpha\gamma^2(1-\gamma)(M-1)(4M-1)n^2 - (1+\alpha)\gamma^3(M-1)^2 M n^3$$

and

$$\varphi(M, n; \gamma) = 2\gamma n^2 + \gamma^2 n^2(4(M-1)n-4) + \gamma^3 n(2+(M-1)n(-4+(2M-1)n).$$

As $\varphi(M, n; \gamma) > 0$ we have that $\frac{dM}{dn} < 0 \Leftrightarrow \theta(M, n; \gamma, \alpha) < 0$ and it may be shown that the last inequality holds for any $M > 1$ and any $n > 1$ if $\alpha + \gamma > 1$.

Now, $\frac{d(Mn)}{dn} = \frac{dM}{dn}n + M = -\frac{d\pi^U/dn}{d\pi^U/dM}n + M > 0 \Leftrightarrow \theta(M, n; \gamma, \alpha)n + \varphi(M, n; \gamma)M > 0$. It may be shown that the last inequality holds for any $M > 1$ and any $n > 1$. *QED.*

The Lemma asserts that the higher the variety produced by each manufacturer the lower the equilibrium number of manufacturers and the higher the total variety. It follows immidiately that $\frac{d(Mn)}{dM} < 0$ and thus the higher the equilibrium number of manufacturers in the equilibrium the lower the total variety.

Proposition 6 *Suppose Assumptions 1',2' and 3 hold. Then the number of manufacturers in the benchmark case is higher than the ones in the case of multiproduct firms while the total variety in the benchmark case is lower than the one in the case of multiproduct firms, that is* $M^* < N^{s*}; M^*n^* > N^{s*}$.

Proof of Proposition. Under assumption 1' and 2' there exist an internal equilibrium with $n^* > 1$. According to Lemma under assumption 3 the equilibrium number of manufacturers decreases in n^* while the total variety increases in n^* and thus $M^* < N^{s*}$ and $M^*n^* > N^{s*}$. *QED.*

Proposition says that, while the number of manufacturers in the benchmark case is higher, the total variety produced is lower and the wholesale price is also higher comparing to the case of multiproduct firms.

The proposition says that the lower is the economies of scope (the higher is α), the lower is the variety offered by each manufacturer. Moreover, although the equilibrium number of manufacturers increase in α, the total total variety decreases in α.

Plugging (23) into (13)-(19) we obtain the following characterization of the symmetric equilibrium when M is endogenous:

$$
\begin{aligned}
q^* &= \overline{q} = \frac{aA\,(1-a)}{(1+R)(a\gamma n^* + (1-\gamma)(1-a))}; && (25)\\
TQ^* &= \frac{RaA(\gamma n^* - (1-\gamma)(1-a))}{(1+R)(a\gamma n^* + (1-\gamma)(1+a))}; && (26)\\
\pi^{D*} &= \frac{a^2A^2n^*(\gamma n^* - (1-\gamma)(1-a))}{(1+R)^2(a\gamma n^* + (1-\gamma)(1-a))^2}; && (27)\\
CS^* &= \frac{R^2a^2A^2n^*(\gamma n^* - (1-\gamma)(1-a))}{2(1+R)^2(a\gamma n^* + (1-\gamma)(1-a))^2}; && (28)\\
W^* &= \frac{R(2+R)a^2A^2n^*(\gamma n^* - (1-\gamma)(1-a))}{2(1+R)^2(a\gamma n^* + (1-\gamma)(1-a))^2}. && (29)
\end{aligned}
$$

Lemma 4 *Suppose Assumption 1',2' and 3 hold for any $\alpha : \underline{\alpha} < \alpha < \overline{\alpha}$. Then the equilibrium values M^* and n^* are such that $\frac{d(M^*-1)n^*}{d\alpha} < 0$, $\frac{d(M^*n^*)}{d\alpha} < 0$, $\frac{dn^*}{d\alpha} < 0$ on $\underline{\alpha} < \alpha < \overline{\alpha}$.*

Proof of Proposition. See the appendix.

Proposition 7 *Suppose Assumption 1',2' and 3 hold for any $\alpha : \underline{\alpha} < \alpha < \overline{\alpha}$. Then the equilibrium values M^* and n^* are such that M^*n^*, TQ^*,π^{D*},CS^*, TW^* decreases in α while w^*, q^*, p^* increases in α.*

Proof of Proposition. See the appendix.

One might wonder how the competition in the downstream market (measured in terms of both product substitutability and number of retailers), the economies of scope and the market size affect the manufacturers' entry incentives, the retailer's profits, the consumers' surplus and the total welfare. We are in the position to obtain analytical results for the in which each manufacturer produces a high number of products. More specifically, when the parameters $\{\alpha, \gamma, b, A, R\}$ are such that $n^* \to \infty$ then for all α, with $\underline{\alpha} < \alpha < \overline{\alpha}$ we have: $TQ^* = \frac{RA}{1+R}$, $\pi^{D^*} = \frac{A^2}{\gamma(1+R)^2}$, $CS^* = \frac{A^2R^2}{2\gamma(1+R)^2}$, $TW^* = \frac{A^2R(2+R)}{2(1+R)^2\gamma}$, and $M^* = \frac{1}{1-\alpha}$. From these equilibrium expressions, it follows that the stronger are the economies of scope (lower a), the fewer manufacturers enter into the market (lower M^*). Moreover, it follows that the market size A, the scale of production cost b, as well the number of retailers R do not affect the equilibrium number of manufacturers M^*. Instead, the total industry output, the consumers' surplus and the total welfare increase in both A and R and.decrease in γ.

In order to draw conclusions for lower values of n^*, we have to resort again to numerical simulations. This is so because the system ((**??**) and (24)) that describes implicitly the equilibrium values of the wholesale prices and of the number of goods (w^*, n^*) when the number of manufacturers is endogenous does not have a closed form solution. We consider the following set of parameters: $\gamma = 0.6$, $A = 10$, $b = 0.1$, and $\alpha = 0.7$. Table 6 includes some results regarding the role of the number of retailers R. As it can be seen, the number of retailers has a significant impact on total output, consumers' surplus and welfare. In the case of multi-product manufacturers, although it influences each manufacturer's product variety n^*, it almost has no effect on the number of manufacturers M^*. These findings are in line with Proposition 6 and with our discussion of the case in which $n^* \longrightarrow \infty$. Moreover, in the benchmark case an increase in R affects positively the number of manufacturers N^s.

R	n^*	M^*	Mn^*	N^s	TQ^*	TQ^s	π^{D*}	π^{Ds}	CS^*	CS^s	TW^*	TW^s
1	11.98	3.28	39.28	22.73	8.00	7.87	39.08	38.17	19.54	19.08	58.63	57.26
2	14.25	3.28	46.84	26.38	10.74	10.57	17.55	17.17	35.10	34.34	70.19	68.69
3	15.30	3.29	50.33	28.03	12.11	11.92	9.91	9.70	44.58	43.66	74.30	72.76
4	15.90	3.29	52.34	28.97	12.93	12.73	6.35	6.22	50.82	49.79	76.23	74.68
5	16.30	3.29	53.66	29.59	13.48	13.28	4.42	4.33	55.21	54.10	77.29	75.42
6	16.57	3.29	54.59	30.02	13.87	13.67	3.24	3.18	58.45	57.29	77.94	76.39
50	17.97	3.29	59.22	32.17	15.90	15.67	0.061	0.060	76.75	75.28	79.82	78.26
∞	18.18	3.30	59.94	32.49	16.22	16.00	0	0	79.88	78.37	79.88	78.37

Table 6: The impact of R with endogenous M

The following Table provides some results regarding the impact of the economies of scope.

α	n^*	M^*	n^*M^*	TQ^*	π^{D*}	CS^*	TW^*
0.1	3200.27	1.11	3555.19	13.30	6.64	53.12	79.69
0.3	268.15	1.42	382.41	13.23	6.58	52.63	78.94
0.5	57.35	1.98	114.03	13.10	6.48	51.81	77.71
0.7	15.90	3.29	52.34	12.93	6.35	50.82	76.23
0.9	3.25	9.79	31.93	12.77	6.24	49.95	74.92
$\alpha > \overline{\alpha}$	1	28.97	28.97	12.73	6.22	49.79	74.68

Table 7: The impact of α with endogenous M

From Table 7 it follows that α influences both the number of manufacturers and the manufacturer's product variety and its impact on both is significant. In line with the case in which $n^* \longrightarrow \infty$ we see again that the stronger are the economies of scope (lower a), the higher is both the total product variety and the product variety offered by each manufacturer. Moreover, we see that the stronger are the economies of scope, the fewer manufacturers enter into the market (lower M^*). At the same time, a has a "moderate" impact on total output, consumers' surplus and welfare.

γ	n^*	M^*	M^*n^*	TQ^*	π^{D^*}	CS^*	W^*
0.5	22.44	3.29	73.81	15.49	7.60	60.80	91.20
0.6	15.90	3.29	52.34	12.93	6.35	50.82	76.23
0.7	11.22	3.30	36.97	11.12	5.46	43.75	65.62
0.8	7.57	3.30	25.00	9.76	4.81	38.51	57.76
0.9	4.40	3.30	14.57	8.72	4.31	34.54	51.80

Table 8: The impact of γ with endogenous M

Table 8 shows that product substitutability almost has no effect on the equilibrium number of manufacturers M. However, it has a big impact on each manufacturer's equilibrium product variety n^*. As expected, the closer substitutes the products are (higher γ) and thus the fiercer is the competition the lower are the retailer's profits.

5 Conclusion

In this paper we have developed and analyzed a successive oligopoly model with multi-product manufacturers and endogenous entry in the market's upstream tier.

First, we provide sufficient conditions for existance and uniqunes of the symmetric equilibrium in pure strategies.Second, we have demonstrated that the equilibrium configuration of the upstream tier depends crucially on the economies of scope in the process of new products creation.

When the economy of scope is weak enough the number of manufacturers is high and each manufacturer produces only one product. Manufacturers produce more than one products only if the degree of the economy of scope is sufficiently high. Under this condition, the gain from the creation of new products by existing manufacturers is larger than the gain obtained by a new firm entering the market. We have also demonstrated how a number of other market characteristics, such as the market size, the degree of product substitutability and the number of retailers in the downstream market affect the product variety, the number of manufacturers as well as the firms' profits and welfare.

Throughout we have restricted our attention to the case where the number of downstream retailers is given and trading between the manufacturers and the retailers takes place through linear wholesale price contracts. It would be interesting to attempt to endogenize the structure of the downstream tier too by including a fixed entry cost and to consider different forms of vertical contracts such as two-part tariffs.

Our model provides a useful framework for addressing a variety of questions that arise in vertically related markets as well as for empirical analysis and policy experiments. For instance, it would be interesting to introduce different tax regimes for the upstream and downstream firms and analyze the optimal tax structure.

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ADDITIONAL COMMENTS:

1. IN THE BENCHMARK CASE I WAS WONDERING WHETHER WE COULD EXAMINE ANALYTICALLY THE FOLLOWING - IF YES WE COULD INCLUDE THEM IN THE BENCHMARK CASE SECTION IN PROPOSITIONS:

- In the exogenous M case: How do a, R, M and γ affect w^{*s}, π^{D*s}, π^{U*s}, CS^{*s}, TW^{*s}?

Igor: we have statement about M and π^{U*s}. and we have Proposition 4.

All other things hardly ever can be shown analitically. We don't know actually how behave $Mn^*(M)$ and $(M-1)n^*(M)$. (If M increases then n^* decreases, but what is about Mn^*?)

BTW TW is not monotone in M at all!

- In the endogenous M case: How do a, R and γ affect N^{*s}, w^{*s}, π^{D*s}, π^{U*s}, CS^{*s}, TW^{*s}?

(If we prove them, then we won't need to include the in the tables with the numerical simulations)

2. DO WE NEED THE RESULTS ETC FOR TQ^*? I DON'T THIK THAT THEY ARE SO IMPORTANT - SO PERHAPS WE COULD DROP THEM BECAUSE WE END UP CONFUSING THE READER BY GIVING HIM/HER TOO MUCH INFORMATION.

Igor: TQ has almost the same meaning as the final price - we may want to have something that characterize the final consumption: either total ammount or price.

3. PERHAPS WE COULD INCLUDE THE WELFARE ANALYSIS AND THE NUMERICAL SIMULATIONS ACOMPANIED WITH A DISCUSSION ABOUT POLICY IMPLICATIONS IN A SEPARATE SECTION, e.g. in that

section, one of the results that we have (formally) is $CS^{*} > CS^{*}$ and $TW^{*} > TW^{*s}$ (from Proposition 5).

4. WE SHOULD THINK MORE CAREFULLY ABOUT WHAT TYPE OF RESULTS WE WANT IN THE PAPER AND MAYBE DROP SOME OF THE NUMERICAL SIMULATIONS THAT WE HAVE.

Igor: Yes it may be the case. But for the moment it is better to have more than less.