Market Share Discounts and Investment Incentives*

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Abstract

The paper investigates pro- and anticompetitive effects of the use of market share discounts (MSD's). While MSD's can be used for exploiting a dominant position and may lead to a welfare reduction, MSD's also can serve as an efficient device for the creation of incentives. Particularly, if a final demand for an upstream manufacturer's good depends on a promotional effort of a retailer, the manufacturer can effectively use MSD's to induce an optimal level of the retailer's effort. Moreover, it is possible that MSD's have a positive impact both on the consumers' surplus and the total industry profits. Thus the use of MSD's should not be treated as an anticompetitive practice a priori, but rather it has to be judged on a case-by-case basis.

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1 Introduction

Vertical restraints, such as loyalty rebates, resale price maintenance, exclusive dealing and exclusive territories are often used in the deal making between manufacturers and retailers. In some cases vertical restraints serve anti-competitive purposes by leading, for instance, to the market exclusion of competitors or to the creation of entry barriers. In other cases these restraints are used to increase efficiency, for example, by eliminating double price marginalization, reaching an optimal level of production or by creating the "right" incentive for vertically related firms. Still in all cases, vertical restraints are of considerable interest to antitrust practitioners.

In this paper I analyze a special type of vertical restraints - market share discounts (MSDs). In particular, I examine the incentives for manufacturers to apply MSD's as well as the impact of MSD's on retailer's investments in the promotion of the manufacturer's product and on welfare. MSD's are discounts that a manufacturer offers to its distributors or retailers if their sales of the manufacturer's brand comprise a sufficiently high percentage of their total sales of a given class of goods. Thus MSD's are a special type of discount which are based on the quantities of goods that the retailer buys from both the manufacturer and its competitors.

The increasing number of cases related to such restraints confirms that manufacturers have begun to use this type of arrangements more intensively in recent years[1]. The case of the Concord Boat Corporation versus the Brunswick Corporation is one of the well-known examples of the use of MSD's[2]. Brunswick manufactured and sold stern drive engines for recreational boats; it had a large share of the market (i.e., 75% in 1983). Beginning in the early 1980s, Brunswick (like its competitors) offered market share discounts. Boat builder customers who agreed to purchase a certain percentage of their engine requirements from Brunswick for a period of time (often a year, sometimes longer) received a discount off the list price for all engines purchased[3]. Some of the boat builders sued Brunswick, alleg-

[1] See "Roundtable on loyalty or fidelity discounts and rebates", DAFFE meeting, May, 2002, Tom et al [2000] and Kobayashi [2005] for review.

[2] See Concord BoatCorp. v. Brunswick Corp., 207 F.3d 1039 (8th Cir. 2000)

[3] Particularly, an agreement to buy 70% of engine requirements from Brunswick might result in a 3% discount, agreement for 65% a 2% discount, and an agreement for 60% a 1% discount.

ing among other claims, that these discount programs excluded competing stern drive engine manufacturers from the market and amounted to monopolization. A court ruled that Brunswick's pricing amounted to de facto exclusive dealing, and foreclosed rival suppliers of marine engines from the market. On appeal, that ruling was reversed on grounds that market conditions were not conducive to foreclosure.

An additional example is the case of Virgin Atlantic Airways Ltd. versus British Airways[4]. British Airways (BA) used incentive programs that provided travel agencies with commissions, and corporate customers with discounts, for meeting specified thresholds for sales of BA tickets (sometimes expressed in terms of market share). Virgin Atlantic claimed that the result was below cost pricing on certain transatlantic routes where Virgin and BA competed, with BA's attendant losses being subsidized by monopoly pricing on other BA routes. Virgin alleged that the below cost pricing slowed its expansion on the competitive routes. Both a district court and a court of appeals concluded that Virgin had failed to demonstrate that pricing was below cost.

In this paper I consider a vertically related two-level industry. At an upstream level a manufacturer and a competitive firnge produce imperfect substitutes. At a downstream level there is only one retailer which trades both goods to final consumers[5]. I consider two types of contract that manufacturer may offer to the retailer: a wholesale price contract and a market share discount contract.

It is supposed that the retailer can make a costly effort investment which results in an increase in the demand for the manufacturer's good. By assumption this effort has no effects on the demand for the competitive sector's firms good. The effort level is non-contractable hence either the wholesale price or MSD's may not be contingent on the retailer's effort level. However the manufacturer may use either the wholesale price or MSD's in order to motivate the retailer to accept the desired level of effort. That allows us to analize the role of MSD's as a tool for the creation of incentives as well as considering the welfare effects of MSD's.

To highlight this role I begin with a consideration of a benchmark case - a particular case of the model when the retailer's effort has no impact on demand. Then I analyze the case when the effort investment results in increasing the demand

[4]Virgin Atlantic Airways Ltd. v. British Airways F.3d 256 (2d Cir. 2001).

[5]The similar setup is adopted in papers of Mills [2004] and Chioveanu and Akgun [2006].

for the manufacturer's good. For both cases and for both types of contract market outcomes and welfare effects are analyzed.

For the benchmark case the following result are shown. Compare to wholesale price settings, if MSD's are applied, both the quantities of manufacturer's good and the manufacturer's profit increase whilest the quantity of the good sold by competitive sector's firm decreases. The total industry profit decreases as does consumers surplus. Thus only the manufacturer gains from the use of MSD's. That allows us to conclude that MSD's have an anticompetitive character in these settings.

The main results obtained for the second case are the following. Firstly, if the wholesale price contract is applied, the manufacturer may be not able to motivate the retailer to undertake the desired level of effort. In this case the market outcome is the same as in the benchmark case of the use of wholesale price. If MSD's are applied then the manufacturer can design the menu of prices in such a way that the retailer makes the desired level of effort: an efficient one from the social point of view. Moreover both the industry profit and the consumer surplus are higher in the case of MSD's when compared with the wholesale price. Another important result is that while the use of MSD's increases the manufacturer's market share, it does not drive competing firms out of the market completely. Thus in terms of social welfare the outcome when MSD's are applied dominates the case of the wholesale price.

Finally, combining the results obtained for both cases, I conclude that judgments on whether MSD's have an anti- or procompetitive effect depends crucially on features of the market environment. While MSD's may serve for a redistribution of profit between the manufacturer and the retailer and may lead to a decrease in social welfare they may also serve as an efficient instrument for the creation investment incentives and may result in an increase in total social welfare. Thus MSD's should not be treated as anticompetitive practices a priori, but rather the treatment of MSD's should be on a case-by-case basis.

Although there is growing number of paper examining different aspects of MSD's, pro- and anti-competitive effects of the market share discounts have not got enough attention in economic literature yet. The rent-shifting effects of MSD's are analyzed by Marx and Shaffer [2004] and Greenlee and Reitman [2004]. Marx and Shaffer [2004] examine the use of market share discounts, slotting allowances and predatory pricing in a three-party sequential contracting environment. In their model two sellers negotiate sequentially with one buyer. Market share discounts and slotting allowances are used to shift rents between the contracting parties, with no short run consequences for social welfare. One result is that this type of rent shifting equilibrium generally results in both sellers remaining in the market. In the long run, the authors suggest that preventing the use of such devices will result in the adoption of strategies that are more likely to result in one of the sellers being excluded. However, the model does not explicitly analyze the welfare effect of such long term effects.

Greenlee and Reitman [2004] analyze the case of two competing firms selling their goods to final consumers by using loyalty rebates or wholesale price contracts[6]. They found that in equilibrium only one firm applies market share discount. Welfare effects of the MSD's use depend on a structure of demand. The welfare analysis in my paper shows also that the welfare effects depend crucially on a model specification.

Majumdar and Shaffer [2007] analyze a case when one manufacturer and a competitive fringe supply goods to a retailer who has private information about the state of demand. They examine conditions under which market-share contracts are profitable, and show that the full-information outcome can be obtained. They show as well that market-share discounts contracts are more profitable than all-units discounts contract.

Chioveanu and Akgun [2006] compare a manufacturer's incentives to apply market share discounts, all-unit discounts and incremental-unit discounts. They show that in situation where there is full information all discounts are equivalent from both manufacturer's and social viewpoints. Under a situation of uncertainty, the attitude toward risk of the retailer can play a crucial role in the form of the loyalty discount applied by the manufacturer.

Greenlee et al [2004] analyze a use of bundled market share discounts by multi-product monopolist. They show that it may exclude an equally efficient competitor that produces a single-product, and that the welfare effect is ambiguous.

Ordover and Shaffer [2007] show that when market share discounts are imple-

[6] The term "loyalty rebates" usually used as a synonym for market share discounts.

mented by a dominant firm, who may have easier access to financing compared to a rival, it can sometimes exclude an equally-efficient rival and lower overall welfare.

The theoretical literature on discounts has not generally considered efficiency based reasons for using market share discounts. One exception is Mills's [2004] paper. Mills has examined the competitive effects of a vertically differentiated product manufacturer implementing market share discounts in sales to its distributors. His central idea is that market share discounts are not mainly an exclusionary device, but rather a device for inducing merchandising services that help consumers make well-informed decisions and augment market performance. In order to show this Mills investigates a case where an upstream firm negotiates a separate contract with every retailer to determine the size and the makeup of the firm's joint profits. In this case every downstream firm has an incentive to make an effort to promote the upstream firm's good whenever it is optimal for social welfare. The author then shows that the same result can be implemented by the upstream firm when it uses MSD's. To get this result Mills (implicitly) assumes that the level of promotion effort of the downstream firm is contractable. This assumption is crucial for getting Mill's result. In contrast I assume that the effort level is not contractable and it allows to reveal a role of MSDs as an efficient mechanism for incentives creation.

The rest of the paper proceeds as follows. Section 2 describes the model. Section 3 considers the benchmark case. Section 4 analyses the general case. Section 5 provides the welfare analysis and a numerical example and section 6 gives conclusions.

2 The Model

There is one retailer, R, which sells two substitutable goods to final consumers. The first good is produced by a brand-name upstream manufacturer, M. It is supposed that the brand manufacturer produces with a constant marginal cost, $c \geq 0$. The second good is produced by a competitive fringe. The marginal cost of producion of the second good is zero.

It is supposed that the retailer can make a costly investment effort which will increase the demand for the manufacturer's good. For example, the circumstances could exist whereby consumers are not perfectly informed about the quality of manufacturer's good and the retailer can provide consumers with that information. The level of the effort is discrete, $e = \{0, 1\}$. It is supposed that the effort cannot be made by the brand manufacturer and, moreover, that the level of effort is not contractable. A cost of effort is denoted by $E > 0$.

A representative consumer has utility function of the form:

$$U(q_1, q_2) = A(e)q_1 + q_2 - \frac{1}{2}(q_1^2 + 2bq_1q_2 + q_2^2), \tag{1}$$

where q_1, q_2 are quantities purchased by the consumer, $b \in (0, 1)$,is the degree of goods differentiation and the parameter A depends on the retailer's effort level: $A(1) = A_1 \geq A(0) = 1$. It is supposed that $1 - b - c > 0$. The consumer's surplus is:

$$CS(q_1, q_2, p_1, p_2) = A(e)q_1 + q_2 - \frac{1}{2}(q_1^2 + 2bq_1q_2 + q_2^2) - q_1p_1 - q_2p_2.$$

The utility function (1) generates the demand for the manufacturer's good, $p_1 = A(e) - q_1 - bq_2$, which depends on the retailer's effort level, and the demand for the competitive sector's firms good $p_2 = 1 - q_2 - bq_1$, which does not depend on the level of effort.

I consider two types of contract that the manufacturer may use in dealing with the retailer: a wholesale price contract, which specifies a constant per-unit price, ω, and market-share discounts. In the latter case the manufacturer's contract specifies parameters $\{t_L, t_H, \overline{s}\}$ that form a menu of prices:

$$t_{MSD} = \begin{cases} t_L & \text{if } s \geq \overline{s} \\ t_H & \text{if } s < \overline{s} \end{cases}, \tag{2}$$

where $s = q_1/(q_1 + q_2)$ denote the share of the manufacturer's good in a total sales of the retailer, $\overline{s}$ is the market share threshold that the retailer must meet in order to buy at the reduced price t_L, and the price t_H: $t_H > t_L$ is the manufacturer's price in the case where the retailer does not meet the market share requirement.

Let's t denote either the single price ω or the menu of prices t_{MSD} depending on the type of contract applied.

All producers compete on price and as a result competitive sector's firms set prices equal to the marginal cost and obtain zero profit because of competition *a la* Bertrand among them.

The retailer's profit is:

$$\pi^R = (A(e) - q_1 - bq_2 - t)q_1 + (1 - q_2 - bq_1)q_2 - eE.$$

The profit of the brand manufacturer is:

$$\pi^M = q_1(t - c)$$

where t is either the wholesale price or the menu of prices.

The timing in the model is the following.

At the first stage the manufacturer and competitive sector's firms simultaneously set their prices. The manufacturer sets the menu of prices, t_{MSD} of the form (2) or the wholesale price ω.

At the second stage the retailer takes a decision on the effort level $e = \{0, 1\}$ and levels of quantities q_1 and q_2.

The model analysis is presented in the following way. I begin with investigation of a special case of the model when $A_1 = A_0 = 1$. This case is considered as a benchmark for a comparison with a general case $A_1 > A_0 = 1$. The condition $A_1 = A_0$ implies that the retailer's effort has no effects on the consumers demand and that the manufacturer has no reason to motivate the retailer to undertake the costly effort. For both wholesale price and MSD contracts I analyze market outcomes and examine an effect of the contract type on welfare. That allows us to see whether MSD's have a procompetitive or an anticompetitive effect in the benchmark settings.

Then I examine the role of the type of contract (wholesale price vs. MSD's) in the case when the consumer's demand depends on the retailer's effort level, $A_1 > A_0 = 1$. Again I consider market outcomes for both types of contract.

Profit functions are subscribed by indexes MSD and WP for cases when MSD's and the wholesale price is applied respectively.

3 Benchmark case: No investment effort

3.1 Wholesale price contract

First let us consider the retailer's problem:

$$\max_{q_1,q_2} \pi^R_{WP}(q_1,q_2;\omega) = (1-q_1-bq_2-\omega)q_1 + (1-q_2-bq_1)q_2.$$

The solutions for the first order conditions are: $q_1(\omega) = \frac{1-b-\omega}{2(1-b^2)}, q_2(\omega) = \frac{1-b+b\omega}{2(1-b^2)}$.

The profit of the retailer as a function of the price ω is:

$$\pi^R_{WP}(\omega) = \frac{2-2b(1-\omega)-2\omega+\omega^2}{4(1-b^2)}.$$

Now the problem of the manufacturer can be written as:

$$\max_{\omega} \pi^R_{WP}(\omega) = q_1(\omega)(\omega-c) = \frac{1-b-\omega}{2(1-b^2)}(\omega-c)$$

and it has the solution: $\omega = \frac{1}{2}(1-b+c)$.

The market outcome is characterized by quantities produced by the manufacturer and competitive sector's firms, $\left\{q_1^{WP} = \frac{1-b-c}{4(1-b^2)}, q_2^{WP} = \frac{2-b-b^2+bc}{4(1-b^2)}\right\}$, the price of the manufacturer, $\omega = \frac{1}{2}(1-b+c)$, the final markets prices $\{p_1^{WP} = \frac{1}{4}(3-b+c), p_2^{WP} = \frac{1}{2}\}$ and the profits of the retailer and the manufacturer, $\{\pi^R_{WP} = \frac{5-3b^2-2b(1-c)-(2-c)c}{16(1-b^2)}, \pi^M_{WP} = \frac{(1-b-c)^2}{8(1-b^2)}\}$.

3.2 MSD contract

The retailer's profit maximization problem is:

$$\begin{aligned} \max_{q_1,q_2} \pi^R_{MSD}(q1,q2;t_{MSD}) &= (1-q_1-bq_2-t_{MSD})q_1 + (1-q_2-bq_1)q_2 \\ \text{s.t. } t_{MSD} &= \begin{cases} t_L & \text{if } s \geq \overline{s} \\ t_H & \text{if } s < \overline{s} \end{cases} \end{aligned}$$

Note, if the retailer trades the good of firms from competitive sector then its profit is a solution of the problem:

$$\max_{q_2} \pi^R = (1 - q_2) q_2$$

and it is equal to 1/4. This value plays a role of the retailer's "reservation profit" in a sense that the retailer would be guaranteed at least this profit level in equilibrium.

Lemma 1 *The equilibrium values of $\{t^L, t^H, \overline{s}\}$ are such that the retailer meets the market share threshold, $s \geq \overline{s}$.*

Proof. See the Appendix ■

Lemma 1 says that in equilibrium the manufacturer's price t_{MSD} is such that the retailer always meets the market share threshold and buys at the price t_L. The intuition is the following. If the retailer does not meet the threshold, that is $s < \overline{s}$, and it buys at the price t_H then the outcome does not change if the manufacturer sets prices $\{t'_L, t'_H, \overline{s}'\}$ such that $t'_L = t_H$, $\overline{s}' = s$ and t'_H is the prohibitively high. Now, if the manufacturer increases the market share threshold slightly $\overline{s}' > s$ then the retailer buys more for the same price and the manufacturer's profit is higher. Thus $s < \overline{s}$ cannot stay in equilibrium.

Hence the exact value of the manufacturer's price t_H does not play a role provided it is high enough. Without loss of generality we can put $t_H = +\infty$.

Corollary 1 *In equilibrium it must be that $s = \overline{s}$.*

Proof. See the Appendix. ■

Note that if under equilibrium the case was produced where $s \neq \overline{s}$ this would imply that the market outcome is the same as in the case of wholesale price and the manufacturer has no possibility of increasing its profit by setting appropriate levels of $\overline{s}$ and t_L, which is contra-intuitive.

As a result of the Corollary 1, the profit of the retailer can be written as:

$$\begin{aligned} \max_{q_1, q_2} \pi^R_{MSD}(q1, q2; t_L) &= (1 - q_1 - bq_2 - t_L) q_1 + (1 - q_2 - bq_1) q_2 \\ \text{s.t. } \frac{q_1}{(q_1 + q_2)} &= \overline{s}. \end{aligned}$$

The first order condition gives the solution:

$$q_1(t_L, \overline{s}) = \frac{\overline{s}(1 - \overline{s}t_L)}{2(1 - 2\overline{s}(1 - b)(1 - \overline{s}))}.$$

Thus the retailer's profit as a function of t_L and $\overline{s}$ is

$$\pi^R_{MSD}(t_L, \overline{s}) = \frac{(1 - \overline{s}t_L)^2}{4(1 - 2\overline{s}(1 - b)(1 - \overline{s}))}.$$

The manufacturer's profit maximization problem now can be written as:

$$\begin{aligned} \max_{t_L, \overline{s}} \pi^M_{MSD} &= q_1(t_L, \overline{s})(t_L - c), \\ \text{s.t. } q_1(t_L, \overline{s}) &= \begin{cases} \frac{\overline{s}(1-\overline{s}t_L)}{2(1-2\overline{s}(1-b)(1-\overline{s}))} & \text{if } \pi^R_{MSD}(t_L, \overline{s}) \geq \frac{1}{4} \\ 0 & \text{if } \pi^R_{MSD}(t_L, \overline{s}) < \frac{1}{4} \end{cases}. \end{aligned}$$

Lemma 2 *In equilibrium the manufacturer extracts the entire retailer's profit above the reservation profit level.*

Proof. See the Appendix. ■

The last proposition means that under equilibrium, the equality $\pi^R_{MSD}(t_L, \overline{s}) = \frac{1}{4}$ holds. This gives a correspondence between a price t_L and market share threshold $\overline{s}$ which must hold in the equilibrium:

$$\overline{s}(t_L) = \frac{2(1 - b - t_L)}{2(1 - b) - t_L^2}. \tag{3}$$

Now the profit maximization problem of the manufacturer becomes:

$$\max_{t_L} \pi^M_{MSD}(t_L) = q_1(\overline{s}(t_L), t_L)(t_L - c) = \frac{1 - b - t_L}{2(1 - t_L)(1 - b) + t_L^2}(t_L - c)$$

and it has the unique solution:

$$t^*_L = \frac{(2 - c)(1 - b) - D_1}{1 - b - c}, \tag{4}$$

where $D_1 = const = \sqrt{(1 - b^2)[2(1 - c)(1 - b) + c^2]}$.

Plugging (4) into (3), we obtain the equilibrium values of $\overline{s}^*$ which together with t_L^* determines other equilibrium values.

The market outcome is characterized by quantities produced by the manufacturer and competitive sector's firms, $\left\{q_1^* = \frac{1-b-c}{2D_1},\, q_2^* = \frac{1-b+bc}{2D_1}\right\}$, the market share threshold, $\overline{s}^* = \frac{1-b-c}{(1-b)(2-c)}$, the manufacturer's price, $t_L^* = \frac{(2-c)(1-b)-D_1}{1-b-c}$, the final market prices $\{p_1^* = \frac{2D_1-(1-b^2)(1-c)}{2D_1},\, p_2^* = \frac{2D_1+b^2-1}{2D_1}\}$ and the profits of the retailer and the manufacturer, $\{\pi_{MSD}^R = 1/4,\, \pi_{MSD}^M = \frac{D_1+b^2-1}{2(1-b^2)}\}$.

Let's note that $b(1-c) < 1$ implies $\overline{s}^* = \frac{1-b-c}{(1-b)(2-c)} < 1$. Hence we can formulate the following proposition.

Proposition 1 *Although the share of the manufacturer is higher in the case of the use of MSD's, the manufacturer never sets the market threshold equals to* 1.

Thus the competitive sector never moved from the market completely and MSD's do not result in an exclusive relation[7]. The intuition here is the following. According to (3) the higher the market share threshold $\overline{s}$ the lower the price t_L maust be in order to provide the retailer's with its reservation profit level. Thus to implement $\overline{s} = 1$ the manufacturer has to set $t_L = 0$, which does not maximize its profit.

3.3 MSD's vs. wholesale price contracts

In the following, I compare the outcomes in the case of MSD's with those in the case of the wholesale price contract.

Proposition 2 *Comparing with the wholesale price contract the use of MSD's leads to the following*

1) an increase in the manufacturer's market share, s,

2) the retailer buys at higher price, that is $t_L > \omega$,

3) an increase in the manufacturer's output, q_1,

4) an increase in the manufacturer's profit,

5) a decrease in the final market price for the manufacturer's good p_1,

[7] This result contributes to a discussion in the antitrust law literature (see for example Tom at al[2000]) on a relation between MSD's and exclusive dealing. See Bernheim and Whinston [1998], Katz's [1989] survey, Marvel [1982], Mathewson and Winter [1987] on exclusive dealing.

6) an increase in the final market price for the good p_2,

7) a decrease in the output of competitive sector's firms q_2,

8) a decrease in the retailer's profit,

9) a decrease in the consumer surplus.

Proof. See the Appendix. ■

Thus the manufacturer, which has some degree of market power, uses MSD's to increase both its output and price and to extract the entire profit of the retail-erabove the reservation level. In the case of MSD's all agents, with the exception of the manufacturer, lose. Hence, under the presented environmental conditions, MSD's can be treated as an anticompetitive tool.

4 Investment Effort.

4.1 Wholesale price contract

The profit maximization problem for the retailer is:

$$\max_{q_1,q_2,e} \pi^R_{WP} = (A(e) - q_1 - bq_2 - \omega)q_1 + (1 - q_2 - bq_1)q_2 - eE$$

where $e \in \{0,1\}$, $A(0) = 1$, $A(1) = A_1$.

The first order conditions in respect to q_2 and q_1 are:

$$\begin{cases} A(e) - 2q_1 - 2bq_2 - \omega = 0 \\ 1 - 2bq_1 - 2q_2 = 0 \end{cases}.$$

It gives a solution: $q_1(\omega, e) = \frac{A(e)-b-\omega}{2(1-b^2)}$, $q_2(\omega, e) = \frac{1-A(e)b+b\omega}{2(1-b^2)}$.

The retailer's profit as a function of the effort level e and the price ω is: $\pi^R_{WE}(\omega, e) = \frac{(A(e)-\omega)(A(e)-b-\omega)}{4(1-b^2)} + \frac{1-A(e)b+b\omega}{4(1-b^2)} - eE$.

Retailer's profit functions for different levels of the investment effort, $\pi^R_{WP}(\omega, e)\big|_{e=0}$ and $\pi^R_{WP}(\omega, e)\big|_{e=1}$, are decreasing in ω functions with the following relation on slopes:

$$\left.\frac{\partial \pi^R_{WP}(\omega, e)}{\partial \omega}\right|_{e=1} = \frac{-2A_1 + 2b + 2t}{4(1-b^2)} < \frac{-2 + 2b + 2t}{4(1-b^2)} = \left.\frac{\partial \pi^R_{WP}(\omega, e)}{\partial \omega}\right|_{e=0}. \quad (5)$$

Let's $\widehat{\omega}$ denote the price such that the retailer is indifferent either to make the investment effort $e = 1$ or $e = 0$. The solution of $\pi^R_{WP}(\widehat{\omega}, e)\big|_{e=1} = \pi^R_{WP}(\widehat{\omega}, e)\big|_{e=0}$ is

$$\widehat{\omega} = \frac{1}{2}(A_1 + 1 - 2b) - \frac{2E(1 - b^2)}{A_1 - 1},$$

and the following conditions hold: if $\omega < \widehat{\omega}$ then the retailer's profit is higher if it makes the effort $e = 1$ and if $\omega > \widehat{\omega}$ then the retailer's profit is higher if its level of the effort is $e = 0$. Together with (5) it implies that the lower is the price ω, the higher is the retailer's gain from the investment effort, $\pi^R_{WP}(\omega, 1) - \pi^R_{WP}(\omega, 0)$.

Now let's restrict parameters of the model to rule out trivial cases.

Assumption 1. $\widehat{\omega} > 0$.

Assumption 2. $\widehat{\omega} < \frac{1}{2}(1 - b + c)$.

Assumption 3. $\frac{A_1 - b - \widehat{\omega}}{2(1-b^2)}(\widehat{\omega} - c) < \frac{(1-b-c)^2}{8(1-b^2)}$.

Assumption 4. $b(A_1 + c) < 1$.

Assumption 1 says that if the manufacturer's price is low enough, $\omega \in (0, \widehat{\omega}]$, then the retailer makes the investment effort. It may be rewritten in the form: $\frac{(A_1-1)(A_1+1+2b)}{4(1-b^2)} > E$ and it rules out cases when the cost of effort is "too high" ($E \to +\infty$) or the result of the effort investment is "too small" ($A_1 \approx 1$). If assumption 1 does not hold there is no possibility of implementing the level of effort $e = 1$.

Assumption 2 may be rewritten in the form $\frac{(A_1-1)(A_1-b-c)}{4(1-b^2)} < E$ and it implies that the effort cost is not "too small" or that the effect of the effort investment is not "too high". It is outside of our interest because in this case the retailer makes the effort investment regardless of the type of contract with the manufacturer.

Assumption 3 may be written in the form:

$$E > \frac{(A_1 - 1)[(1 - b - c) + \sqrt{(A_1 - 1)(A + 1 - 2b - 2c)}}{4(1 - b^2)}$$

and it implies that neither the effect or the effort should be "too high" or the cost of effort "too low". In addition it implies that the rate of goods substitution should not be close to 1.

Whilst assumption 1 implies the possibility of implementation of the effort level

$e = 1$ and assumptions 2 implies that the effort $e = 1$ is not implemented with necessity under equilibrium, assumption 3 allows us to concentrate on a case that reveals the role of MSD's as a tool for the creation of investment incentives.

Assumption 4 states that the degree of goods substitution should not be close to 1. Moreover, the higher the level of efficiency of the effort the lower the degree of goods substitution should be.

The profit maximization problem of the manufacturer may be written in the form:

$$\max_{\omega} \pi_{WP}^{M} = q_1(\omega, e)(\omega - c) = \begin{cases} \frac{A_1 - b - \omega}{2(1-b^2)}(\omega - c) & \text{if } \omega \leq \widehat{\omega} \\ \frac{1-b-\omega}{2(1-b^2)}(\omega - c) & \text{if } \omega > \widehat{\omega} \end{cases}.$$

Note that in equilibrium the optimal manufacturer's price does not exceed the level $(A_1 - b + c)$. Thus the manufacturer's profit function has the following properties: it is kinked at point $\widehat{\omega}$ and it increases at both intervals $\omega \in [0, \widehat{\omega}]$ and $\omega \in (\widehat{\omega}, \frac{1}{2}(A_1 - b + c)]$. If $\omega \in [0, \widehat{\omega}]$ then the equilibrium effort level is $e = 1$, while if $\omega \in (\widehat{\omega}, \frac{1}{2}(A_1 - b + c)]$ then $e = 0$.

The immediate result of assumption 2 is that in order to implement the level of effort $e = 1$ the manufacturer sets the price $\omega = \widehat{\omega}$ and gets the profit:

$$\max_{\omega \in [0, \widehat{\omega}]} \pi_{WP}^{M} = \pi_{WP}^{M}\big|_{\omega = \widehat{\omega}} = \frac{A_1 - b - \widehat{\omega}}{2(1 - b^2)}(\widehat{\omega} - c) \tag{6}$$

Now let's consider us the manufacturer's profit for the price $\omega \in (\widehat{\omega}, \frac{1}{2}(A_1 - b + c)]$. While $\omega > \widehat{\omega}$ the retailer does not undertake the investment effort and the manufacturer's profit $\pi_{WP}^{M} = \frac{1-b-\omega}{2(1-b^2)}(\omega - c)$ reaches the maximum at the point $\omega = \frac{1}{2}(1 - b + c)$ with

$$\max_{\omega > \widehat{\omega}} \pi_{WP}^{M}(\omega, e) = \frac{(1 - b - c)^2}{8(1 - b^2)}. \tag{7}$$

According to assumption 3 the manufacturer's profit is higher if it sets the price $\omega = \frac{1}{2}(1 - b + c)$ and level of investment of zerois implemented in equilibrium.

Thus given assumptions 1-3 if the wholesale price contract applied the manufacturer's profit maximization implies a retailer effort of zero. This immediately has the result in that the equilibrium outcome coincides with the benchmark wholesale price outcome.

4.2 MSD contract

The profit maximization problem of the retailer is:

$$\max_{q_1,q_2,e} \pi^R_{MSD} = (A(e) - q_1 - bq_2 - t_{MSD})q_1 + (1 - q_2 - bq_1)q_2 - eE \tag{8}$$

Lemma 3 *Under equilibrium the condition $s = \overline{s}$ holds and the manufacturer's price t_H is prohibitively high.*

Proof. See the Appendix. ■

Hence under equilibrium the retailer chooses q_1, q_2 such that $q_1/(q_1 + q_2) = \overline{s}$ and buys at the price t_L. Plugging $q_2 = q_1 \frac{1-\overline{s}}{\overline{s}}$ into (8) and solving the first order conditions we get the optimal level of q_1:

$$q_1(\overline{s}, t_L, e) = \frac{\overline{s}(1 + \overline{s}(A(e) - 1 - t_L))}{2(1 - 2\overline{s}(1-b)(1-\overline{s}))}.$$

Given $\{t_L, \overline{s}\}$ the retailer makes the effort if and only if

$$\begin{cases} \pi^R_{MSD}(e; t_L, \overline{s})\big|_{e=1} \geq \pi^R_{MSD}(e; t_L, \overline{s})\big|_{e=0} \\ \pi^R_{MSD}(e; t_L, \overline{s})\big|_{e=1} \geq \frac{1}{4} \end{cases}, \tag{9}$$

where the retailer's profit is:

$$\pi^R_{MSD}(e; t_L, \overline{s}) = \frac{(1 - \overline{s} + \overline{s}(A(e) - t_L)^2}{4(1 - 2\overline{s}(1-b)(1-\overline{s}))} - eE.$$

The first inequality in (9) is an incentives constraint and it implies that for the retailer it is profitable to make the effort $e = 1$. The second inequality in (9) is a participation constraint and it implies that the profit of the retailer is greater or equal to its reservation profit.

If the values of $\{t_L, \overline{s}\}$ are such that

$$\begin{cases} \pi^R_{MSD}(e; t_L, \overline{s})\big|_{e=1} \geq \pi^R_{MSD}(e; t_L, \overline{s})\big|_{e=0} \\ \pi^R_{MSD}(e; t_L, \overline{s})\big|_{e=0} \geq \frac{1}{4} \end{cases} \tag{10}$$

then the retailer chooses the effort $e = 0$.

Now the manufacturer's profit maximization problem is:

$$\begin{aligned} \max_{t_L, \overline{s}} \pi^M_{MSD} &= q_1(t_L, \overline{s})(t_L - c), \qquad (11) \\ \text{s.t. } q_1(t_L, \overline{s}) &= \begin{cases} \frac{\overline{s}(1+\overline{s}(A(e)-1-t_L))}{2(1-2\overline{s}(1-b)(1-\overline{s}))} & \text{if the condition (9) holds} \\ \frac{\overline{s}(1-\overline{s}-t_L)}{2(1-2\overline{s}(1-b)(1-\overline{s}))} & \text{if the condition (10) holds} \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

Let's first consider the manufacturers profit in the case where the price t_{MSD} is such that condition (9) holds which implies that the retailer makes the effort $e = 1$.

Lemma 4 *In equilibrium the manufacturer extracts the entire retailer's profit above the reservation level.*

Proof. See the Appendix. ■

Thus the condition $\pi^R_{MSD}(e; t_L, \overline{s})\big|_{e=1} \geq \frac{1}{4}$ binds and this determine the equilibrium correspondence on t_L and $\overline{s}$ of the form:

$$t_L(\overline{s}) = A_1 - 1 + \frac{1 - \sqrt{1 + 4E}\sqrt{1 - 2\overline{s}(1 - b)(1 - \overline{s})}}{\overline{s}} \tag{12}$$

Plugging (12) into the manufacturer's profit function (11) and solving the first order conditions we get the optimal value of $\overline{s}$:

$$\overline{s}^* = \frac{A_1 - b - c}{(1 - b)(1 + A_1 - c)}.$$

Now, the optimal value of t_L is:

$$t_L^* = A_1 - 1 + \frac{(1 - b)(A_1 + 1 - c) - D_2}{A_1 - b - c},$$

where $D_2 = const = \sqrt{(1-b^2)(1+(A_1-c)(A_1-c-2b)}\sqrt{(1+4E)}$.

The profit of the manufacturer is

$$\pi^R_{MSD}(t^*_L, \overline{s}^*, e)\Big|_{e=1} = \frac{(1+4E)(1+(A-c)(A-c-2b)-D_2)}{2D_2}.$$

Thus by setting $\{t^*_L = A_1 - 1 + \frac{(1-b)(A_1+1-c)-D_2}{A_1-b-c},\ t^*_H = \infty,\ \overline{s}^* = \frac{A_1-b-c}{(1-b)(1+A_1-c)}\}$ the manufacturer motivates the retailer to make the effort investment $e = 1$. The profits $\{\pi^M_{MSD}\big|_{e=1} = \frac{(1+4E)(1+(A-c)(A-c-2b)-D_2)}{2D_2},\ \pi^R_{ME}\big|_{e=1} = 1/4\}$, the outputs $\{q_1 = \frac{(A_1-b-c)(1+4E)}{2D_2},\ q_2 = \frac{(1-Ab+bc)(1+4E)}{2D_2}\}$ and prices $\{p_1 = A_1 - \frac{(4E+1)(A_1-c)(1-b^2)}{2D_2}$, $p_2 = 1 - \frac{(4E+1)(1-b^2)}{2D_2}\}$ are realized in this case.

Now let's consider the profit of the manufacturer in the case where it motivates the retailer to choose a zero-level effort. Given the price t_L and the threshold $\overline{s}$ the retailer's profit is $\pi^R_{MSD}(e; t_L, \overline{s})\big|_{e=0} = \frac{(1-\overline{s}t_L)^2}{4(1-2\overline{s}(1-b)(1-\overline{s}))}$. The manufacturer's maximization problem is:

$$\begin{aligned}\max_{t_L,\overline{s}} \pi^M_{MSD} &= \frac{\overline{s}(1-\overline{s}t_L)}{2(1-2\overline{s}(1-b)(1-\overline{s}))}(t_L - c),\\ &\text{s.t. (10) holds}\end{aligned}$$

The participation constraint $\pi^R_{MSD}(e; t_L, \overline{s})\big|_{e=0} = \frac{1}{4}$ gives the correspondence $t_L(\overline{s})$ that guarantees the reservation level of the profit to the retailer:

$$t_L(\overline{s}) = \frac{1-\sqrt{1-2\overline{s}(1-b)(1-\overline{s})}}{\overline{s}}. \tag{13}$$

Plugging (13) into (11) and solving the first order conditions we get the optimal value of $\overline{s} = \frac{1-b-c}{(1-b)(2-c)}$.

The optimal value of t_L is:

$$t_L = \frac{(1-b)(2-c)-D_3}{1-b-c},$$

where $D_3 = const = \sqrt{(1-b^2)(1+(1-c)(1-c-2b)}$.

The profit of the manufacturer in this case is

$$\pi^M_{MSD}(t_L, \overline{s}, e)\Big|_{e=0} = \frac{1+(1-c)(1-c-2b)-D_3}{2D_3}.$$

Thus if

$$\pi^M_{MSD}(t^*_L(e), \overline{s}^*(e), e)\big|_{e=1} \geq \pi^M_{MSD}(t^*_L(e), \overline{s}^*(e), e)\big|_{e=0} \tag{14}$$

the manufacturer sets the price $t^*_L = A_1 - 1 + \frac{(1-b)(A_1+1-c)-D_2}{A_1-b-c}$, market share threshold $\overline{s}^* = \frac{A_1-b-c}{(1-b)(1+A_1-c)}$ and the equilibrium retailer's level of the effort is $e = 1$, otherwise the manufacturer sets the price $t^*_L = \frac{(1-b)(2-c-D_3)}{1-b-c}$, the market share threshold $\overline{s}^* = \frac{1-b-c}{(1-b)(2-c)}$ and the equilibrium level for the retailer's effort is $e = 0$. For the purpose of the paper I am interested in the former case. Let's Ω denote the set of parameters (A_1, b, c, E) for which conditions (14), assumptions 1-3 and the incentives compatibility constraint hold. The following technical lemma states that the set Ω is the non-degenerated set.

Lemma 5 *There is a compact set of the parameters of the model* $(A_1, b, c, E) \in \Omega$ *where inequalities (14), (9) and assumptions 1, 2, 3 are compatible.*

Proof. The numerical example in part 5.1 proves that it contains at least one point. Moreover because all functions used in (14) and assumptions 1-3 are continuous the required conditions hold in the neighborhood of the provided point. ■

The set Ω is characterized by the following properties. For given levels of the marginal cost c and the degree of the rate substitution b the set specifies the cost of effort as a function of the efficiency of the effort $A_1 : 0 < \underline{E}(A_1; b, c) < E \leq \overline{E}(A_1; b, c)$, where bounds $\underline{E}, \overline{E}$ increase in.A_1.For a given level of A_1 the higher level of b corresponds to a smaller interval $[\underline{E}, \overline{E}]$. For instance, if $c = 0, A_1 = 1.5, b = 0.5$ then $\Omega = \{E : E \in [0.166,\ 0.253]\}$, if $c = 0, A_1 = 1.5, b = 0.6$ then $\Omega = \{E : E \in [0.1758, 0.2556]\}$ and if $c = 0, A_1 = 1.4, b = 0.5$ then $\Omega = \{E : E \in [0.12, 0.1892]\}$.

Thus market share discounts allow to the manufacturer to design the menu of prices such that the retailer makes the level of the effort $e = 1$ while if the wholesale price contract is applied, the manufacturer implements the level of the effort $e = 0$.

Hence we can conclude that MSD's can be used by the manufacturer as an efficient device for the creation of investment incentives. Certainly, the manufacturer gains from the use of MSD's. In order to analyze MSD's impacts from the social point of view I conduct a welfare analysis.

5 Welfare analysis

The representative consumer surplus is:

$$U(q_1, q_2) = A(e)q_1 + q_2 - \frac{1}{2}(q_1^2 + 2bq_1q_2 + q_2^2) - q_1p_1 - q_2p_2.$$

Proposition 3 *For the set of parameters Ω the set of statements following is true:*

1. When MSD's are applied, the manufacturer designs the menu of prices such that the retailer's level of the effort is $e = 1$ When the wholesale price is applied the level of effort $e = 0$ is implemented under equilibrium.

2. The total industry profit is higher when MSD's are applied.

3. The total output is higher when MSD's are applied.

4. Both the consumer surplus and the total welfare are higher when MSDs are applied.

Proof. The proposition immediately follows the numerical example in part 5.1 and Lemma 5 ■

The intuition here is the following. The retailer is motivated to make the costly effort only if the quantity of the manufacturer's good that it resells is high enough. That means that the manufacturer's wholesale price should be small enough to achieve this. Thus the manufacturer faces a trade-off: either to set a lower wholesale price to shift the demand upward or to set a higher price and to remain ot the same demand curve. The gain by the manufacturer from an increase in the demand can be smaller than its losses from the price reduction. Thus the wholesale price contract may not be enough to implement the desired level of effort from the retailer. If MSD's are applied then the manufacturer may use the market threshold to enforce the retailer to buy more of the manufacturer's good, up to the level where the costly effort becomes profitable for the retailer. The investment effort shifts the demand for the manufacturer's good and increases both the manufacturer's profit and the consumer's surplus.

5.1 A numerical example

Let's consider a numerical example with the following values for the parameters: $A_1 = 1.5, b = 0.7, c = 0.14, E = 0.2$.

First I consider the case of the wholesale price ω. Given these parameters for the model the retailer is indifferent to making the effort investment or not, if and only if

$$\pi^R_{WP}(\omega, e)\big|_{e=0} = \pi^R_{WP}(\omega, e)\big|_{e=1},$$

that is

$$\frac{(1-\omega)(1-b-\omega)}{4(1-b^2)} + \frac{1-b+b\omega}{4(1-b^2)} = \frac{(A_1-\omega)(A_1-b-\omega)}{4(1-b^2)} + \frac{1-A_1b+b\omega}{4(1-b^2)} - E,$$

with the solution

$$\widehat{\omega} = \frac{(A_1-1)[A_1+1-2b)] - 4E(1-b^2)}{2(A_1-1)} = 0.142.$$

The manufacturer's profit in this case is:

$$\pi^M_{WP}(\widehat{\omega}) = q_1(\widehat{\omega})(\widehat{\omega}-c) = \frac{A_1-b-\widehat{\omega}}{2(1-b^2)}(\widehat{\omega}-c) = 0.0009.$$

For any price above the $\widehat{\omega} = 0.142$ the retailer chooses a level of effort of zero.

The wholesale price ω that maximizes the manufacturer's profit is $\omega = \frac{1}{2}(1 - b + c) = 0.22$ and the profit is

$$\pi^M_{WP} = \frac{1-b-\omega}{2(1-b^2)}(\omega - c) = 0.61 \cdot 0.078 = 0.006.$$

Thus the investment effort $e = 0$ is implemented.

The equilibrium prices and quantities are $(p_1^{WP}, p_2^{WP}) = (0.61, 0.5)$ and $(q_1^{WP}, q_2^{WP}) = (0.0784, 0, 445)$ respectively; the profit of the retailer is $\pi^R_{WP} = 0.2531$; the consumer surplus is $CS^{WP} = 0.1266$. Thus the total surplus is $TS^{WP} = 0.3857$.

If MSD's applied then the manufacturer sets the price $t_L = 0.161$ and the market share threshold $\overline{s} = 0.9322$ in order to implement the effort investment level

$e = 1$.The retailer may choose either scenario indifferently. The first being to make the effort ($e = 1$) and to set the optimal prices $(p_1^{MSD}, p_2^{MSD}) = (0.83, 0.507)$. The quantities in this case are $(q_1^{MSD}, q_2^{MSD}) = (0.6375, 0, 046)$. The second scenario is not to trade the manufacturers good at all and to set $p_2 = 1/2$ and $q_2 = 1/2$. The retailer's profit in both cases is $\pi_{MSD}^R = 1/4$. It is assumed that in this case the retailer makes the investment effort . Then the manufacturer's profit is $\pi_{WSD}^M = 0.0134$, the consumer surplus is $CS^{MSD} = 0.225$. Thus the total surplus is $TS^{MSD} = 0.488$. If the retailer chooses the effort level $e = 0$ its profit is $0.1878 < 0.25$. Thus, given $\{t_L = 0.161, \overline{s} = 0.9322\}$ the equilibrium level of the effort is $e = 1$. To implement the effort level $e = 0$ the manufacturer may set the price $t_L = 0.221$ and $\overline{s} = 0.287$. The manufacturer's profit in this case is $0.0123 < 0.0134$. Thus if MSD's are applied then the equilibrium effort level is $e = 1$.

The results confirmed in the example are the following: comparing with the wholesale price MSD's result in:

1) an increase in the manufacturer's output, q_1, and a decrease in the competitive sector's firms output q_2,

2) an increase in the manufacturer's profit and a decrease in the retailer's profit,

3) the retailer buys at the lower price, that is $t_L < \omega$,

4) an increase in both the final market prices p_1 and p_2,

5) an increase in the total industry's profit, an increase in the consumer surplus and, as a result, an increase in the total welfare.

5.2 Policy implication

As it was shown the manufacturer may increase its profit by applying the MSD's contract instead of the WP contract. When this results in an implementation of socially preferable level of the effort it has a positive impact on both the manufacturer's profit and social surplus. In other cases the manufacturer uses MSD's for rent-shifting purposes that results in a decrease in total welfare. Thus it is important to have tests which allow an antitrust authority to distinguish between these cases.

Suppose the manufacturer, which has a certain degree of market power, changes the type of contract it offers to the retailer from the WP contract to the MSD contract. Because parameters of the utility function are not observable in practices and usually the cost of production is a company's private information it may not be possible to judge the manufacturer's incentives *ex ante,* that is before the new outcome is realized. On the contrary, the antitrust authority may judge the procompetitive or anticompetitive character of the use of MSD's based on the observable characteristic of market outcomes *ex post.* Thus when the quantities of goods sold by the retailer and all prices are known for cases when WP and MSD's are applied, a test based on changes in the manufacturer price may be proposed. If the adoption of the MSD's contract results in the effort level $e = 1$ it implies that the price $t_L^{MSD}\big|_{e=1} < \omega$, while if MSD's serve for the rent shifting between the retailer and the manufacturer the price $t_L^{MSD}\big|_{e=0} > \omega$. Intuition here tell us that the investment effort is profitable for the retailer if it sells a high enough quantity of the good 1 and to reach it the manufacturer uses both the market share threshold s and the price t_L^{MSD}.Thus to implement effort level $e = 1$ the manufacturer sets its price to be low enough, particularly lower than its wholesale price.

6 Conclusion

The paper investigates effects of the use of MSD's. In the first part of the model the case without the possibility of is considered. It is shown that the manufacturer, who has some degree of market power, can use MSD's to extract an additional profit through an increase in its market share and a decrease in the market share of its competitors. In this way, the use of MSD's use can be treated as anticompetitive because it leads to a decrease in both the total industry profit and the consumer surplus.

The second part of the analysis considers the case where the retailer is able to make a costly effort investment that increases the demand for the manufacturer's good. In this case MSD's can be used to motivate the retailer to make an efficient level of investment effort. This happens because the MSD's use guarantees that the quantity of the manufacturer's good sold by the retailer is high enough and this provides the incentives for the retailer to make the effort investment. It is shown that this outcome can not always be reached through the use of a wholesale price contract. The main result is that MSD's can lead to an increase in both the total industry profit and the social surplus. Hence the total welfare in the case of MSD's may be higher compare with the case of the wholesale price.

One possible extension of the model may be in a consideration of the case of many heterogeneous retailers. It may be found that in this case the optimal menu of prices should include as many non-degenerated price as well as market thresholds, as many retailers are at the downstream level. That may allow satisfaction of the incentives compatibility constraints for each of them separately. At the same time as the manufacturer designs the optimal price menu it must take into account that the required market share threshold may be reached by a retailer by means of a just reduction of the share of competitors without making a costly effort investment. Thus, to expand results presented for the case of many retailers, a deep formal analysis of the extended model is required.

Another possible extension of the model is in the comparison of the result of the use of MSD's with the results of other non-linear price schemes. There is particular interest in comparison of MSD's and quantity discounts. Quantity discounts usually are not considered as anticompetitive discounts and their use is not restricted by law. If it is shown that MSD's are more preferable from the social point of view than quantity discounts, it will give more reasons to treat MSD's as an efficiency increasing, procompetitive tool. One possibility of getting this result may be in consideration of a case of stochastic demand when the use of quantity discounts can involve difficulties related to the absolute value of a discount threshold. MSD's may be free of these difficulties in the case where both demands, for the manufacturer's good and for the competitive sector's firms good, have the same shock. I leave these extensions for future investigation.

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A Appendix

Proof of the Lemma 1. I proof the statement by contradiction.

Suppose, in the equilibrium the manufacturer sets $\{t_L^e, t_H^e, s^e\}$ and the retailer does not meet the market share threshold. That is $s = \frac{q_1^e}{q_1^e + q_2^e} < s^e$, where $\{q_1^e, q_2^e\}$ and s are equilibrium quantities and the equilibrium market share of the manufacturer respectively.

Because in the equilibrium the market threshold restriction is not met, the level of the market threshold s^e has no effect on market outcome. In this case the equilibrium price t^e_H coincides with one in the case of wholesale price $t^e_H = \frac{1}{2}(1-b+c)$. As a result, the equilibrium retailer's profit equals one in the case of the wholesale price, $\pi^R_{MSD} = \frac{5-3b^2-2b(1-c)-(2-c)c}{16(1-b^2)}$.

Note that the retailer's profit is higher than its reservation profit. It is because of:

$\frac{5-3b^2-2b(1-c)-(2-c)c}{16(1-b^2)} > \frac{1}{4} \Leftrightarrow 5-3b^2-2b(1-c)-(2-c)c > 4(1-b^2) \Leftrightarrow (b+c)^2 - 2(b+c)+1 > 0 \Leftrightarrow (1-b-c)^2 > 0$,

were the last inequality is obviously true.

Let's construct new menu of prices $t' = \{t'_L, t'_H, s'\}$ in the form:

$$\begin{cases} t'_L = t^e_H \\ t'_H = +\infty \\ s' = s^e + \delta \end{cases},$$

where $\delta > 0$.

Now let's show that the new price t' gives the higher profit to the manufacturer.

Because $t'_H = +\infty$, the retailer has either to meet the market share threshold or to trade the competitive sector's firms good only. In the latter case its profit equals to the reservation profit. In the former case, the retailer faces the same manufacturer's price $t'_L = t^e_H$ but it has to adjust quantities q^e_1, q^e_2 to meet the market share threshold. The optimal adjustment implies a decrease in the quantity q_2 and an increase in the manufacturer's quantity q_1. Because of continuity of the retailer's profit function in $q_1$and q_2, for δ small enough we have that the new retailer's profit is still higher than the reservation profit. Thus, if the new price t' is offered then the retailer chooses new quantity $q'_1 > q^e_1$. Given the manufacturer's price remains the same, $t'_L = t^e_H$, the profit of the manufacturer is higher. Thus $\{t^e_L, t^e_H, s^e\}$ were not the equilibrium values which contradicts to the assumption. ■

Proof of the Corollary 1. I proof the statement by contradiction. Let's $\{t_L, t_H, \overline{s}\}$ and s be the equilibrium manufacturer's menu of prices and the equilibrium manufacturer's market share respectively. By Lemma 1 $s \geq \overline{s}$ and the retailer buys at the price t_L.

Suppose $s > \overline{s}$. Note that small changes in t_L result in small changes in the equilibrium quantities of q_1, q_2 and the condition $s > \overline{s}$ still holds.

If t_L is higher (lower) than the equilibrium manufacturer's wholesale price (which is $\frac{1}{2}(1-b+c)$) then a small decrease (increase) in t_L leads to an increase in the manufacturer's profit π^M_{MSD} with $s > \overline{s}$ still holding. Thus in equilibrium $t_L = \frac{1}{2}(1-b+c)$ and the condition $\pi^R_{MSD} > \frac{1}{4}$ holds. Now, if the manufacturer sets $\overline{s}' = s + \delta$ then the retailer has either to trade the good 2 only or to adjust quantities q_1, q_2 to meet new threshold requirement. In the former case the retailer obtains its reservation profit only while in the latter case its profit decreases only slightly and it still remains higher than the reservation profit. Thus the retailer chooses to by more the manufacturer's good at the same price. The profit of the manufacturer is higher that contradict to assumption that $\{t_L, t_H, \overline{s}\}$ was the equilibrium menu of prices. ■

Proof of the Lemma 2. By Lemma 1 and Corollary 1 we have $s = \overline{s}^e$ and hence $\pi^M_{MSD} = q_1(t^e_L, \overline{s}^e)(t^e_L - c) = \frac{\overline{s}^e(1-\overline{s}^e t^e_L)}{2(1-2\overline{s}^e(1-b)(1-\overline{s}^e))}(t^e_L - c)$.

Let's show that $\frac{\partial \pi^M_{MSD}}{\partial t_L} = \frac{s^2(c-2t_L)+s}{2(1-2s(1-b)(1-s))} \geq 0$.

First, because of $2s(1-b)(1-s) \leq \max\limits_s 2s(1-s)(1-b) = \frac{1-b}{2} < 1 \Longrightarrow 2(1-2s(1-b)(1-s)) > 0$. Hence the denominator is positive.

Second, the nominator is positive because

$$s^2(c-2t_L) + s > \min_s s^2(c-2t_L) + s = [s^2(c-2t_L)+s]\Big|_{s=\frac{1}{2(2t_L-c)}} = 0$$

for any $t_L \geq c$

Thus, for any given level of $\overline{s}^e$, π^M_{MSD} is a non-decreasing in t^e_L function. Thus the manufacturer sets t^e_L to be as high as possible until $\pi^R_{MSD} \geq \frac{1}{4}$.

The retailer's profit π^R_{MSD} is the deceasing in t^e_L function for any $0 < t_L < 1$. Thus the manufacturer sets price such that $\pi^R_{MSD} = \frac{1}{4}$. ■

Proof of the Proposition 2. 1). The manufacturer's market share is $\overline{s}^* = \frac{1-b-c}{(1-b)(2-c)}$ in the case of $MSDs$ and it is $s^{WP} = \frac{q_1}{q_1+q_2} = \frac{1-b-c}{(1-b)(3-c+b)}$ in the case of the WP contract. Because $(3-c+b) > 2 > (2-c)$ we have that $\overline{s}^* > s^{WP}$.

2). Now I show that $t^*_L = \frac{(2-c)(1-b)-D_1}{1-b-c} > \frac{1}{2}(1-b+c) = \omega$,

where $D_1 = \sqrt{(1-b^2)[2(1-c)(1-b)+c^2]}$.

$\frac{(2-c)(1-b)-D_1}{1-b-c} > \frac{1}{2}(1-b+c) \Leftrightarrow$

$2(2-c)(1-b) - (1-b)^2 + c^2 > 2D_1 \Leftrightarrow$

$(1-b^2)+(2(1-b)(1-c)+c^2)>2D_1 \Leftrightarrow$
$\sqrt{(1-b^2)}^2+\sqrt{[2(1-c)(1-b)+c^2]}^2>2D_1 \Leftrightarrow$
$(\sqrt{(1-b^2)}-\sqrt{[2(1-c)(1-b)+c^2]})^2>0.$

Moreover $1-b^2=2(1-c)(1-b)+c^2 \Leftrightarrow 1-b-c=0$ which contradicts to the assumption. Thus $t_L^*>\omega$.

3). $q_1^{MSD}=\frac{1-b-c}{2D_1}>\frac{1-b-c}{4(1-b^2)}=q_1^{WP} \Leftrightarrow D_1<2(1-b^2) \Leftrightarrow$
$2(1-c)(1-b)+c^2<4(1-b^2)$.

By the assumption $c<1-b \Rightarrow$
$2(1-c)(1-b)+c^2<2(1-c)(1-b)+(1-b)^2=$
$=(1-b)[2(1-c)+(1-b)]=(1-b)[3-2c-b]<$
$<3(1-b)<4(1-b^2)$.

4). $q_1^{MSD}>q_1^{WP}$ and $t_L^*>\omega$ give that $\pi_{MSD}^M>\pi_{WP}^M$.

5). Competitive sector's firms outputs in cases of WP and $MSDs$ contracts are $q_2^{WP}=\frac{2-b-b^2+bc}{4(1-b^2)}$ and $q_2^{MSD}=\frac{1-b+bc}{2D_1}$ respectively.

First, let's note that $D_1>1-b^2$ because of
$\sqrt{(1-b^2)[2(1-c)(1-b)+c^2]}\geq \min_c\sqrt{(1-b^2)[2(1-c)(1-b)+c^2]}=$
$=\sqrt{(1-b^2)[2(1-c)(1-b)+c^2]}\,|_{c=1-b}=(1-b^2)$

Thus $q_2^{MSD}=\frac{1-b+bc}{2D_1}<\frac{1-b+bc}{2(1-b^2)}=\frac{2-2b+2bc}{4(1-b^2)}$.

Note that $2-2b+2bc<2-b-b^2+bc \Leftrightarrow$
$b(b+c-1)<0$ which holds by the assumption. Thus $q_2^{MSD}<q_2^{WP}$.

6) and (7). The changes in prices are the immediate result of changes in quantities. Thus both the increase in q_1 and the decrease in q_2 result in the decrease in p_1 and the increase in p_2.

8). $\pi_{WP}^R=\frac{5-3b^2-2b(1-c)-(2-c)c}{16(1-b^2)}>1/4=\pi_{MSD}^R$
because of $\frac{5-3b^2-2b(1-c)-(2-c)c}{16(1-b^2)}>\frac{1}{4} \Leftrightarrow$
$5-3b^2-2b(1-c)-(2-c)c>4(1-b^2) \Leftrightarrow$
$(b+c)^2-2(b+c)+1>0 \Leftrightarrow (1-b-c)^2>0.$

9). Substituting equilibriums values of prices and quantities for both cases of the wholesale price and MSDs we get that the consumers' surpluses are:

$CS^{WP}=\frac{5-4b^2+(b+c)^2+(b+c)}{32(1-b^2)}$ and $CS^{MSD}=\frac{1}{8}$. Thus $CS^{WP}>CS^{MSD} \Leftrightarrow \frac{5-4b^2+(b+c)^2+(b+c)}{32(1-b^2)}>\frac{1}{8} \Leftrightarrow 1+(b+c)^2-2(b+c)>0 \Leftrightarrow (1-b-c)^2>0$ where the last inequality is obviously true. ■

Proof of the Lemma 3. Suppose, in the equilibrium the manufacturer sets $\{t_L^e, t_H^e, \overline{s}^e\}$ and the retailer does not meet the market share threshold, $s \neq \overline{s}^e$. Because in the equilibrium $s \neq \overline{s}^e$, the level of the market threshold $\overline{s}^e$ has no effects on quantities q_1^e, q_2^e.

Suppose that $e^* = 1$. In this case the equilibrium price (either t_H^e if $s < \overline{s}^e$ or t_L^e if $s > \overline{s}^e$) coincides with the price $\widehat{\omega}$. But this contradicts to the assumption 3 which says that the manufacturer's profit is higher if its price is $\omega^* = \frac{1}{2}(1 - b + c)$ and $e = 0$. Thus if $e^* = 1$ it must be that $s = \overline{s}^e$.

Suppose that $e^* = 0$. Then because of $s \neq \overline{s}^e$ the manufacturer's price (either t_H^e if $s < \overline{s}^e$ or t_L^e if $s > \overline{s}^e$) equals to its wholesale price $\omega^* = \frac{1}{2}(1 - b + c)$. As a result, the equilibrium retailer's profit is $\pi_{MSD}^R = \frac{5-3b^2-2b(1-c)-(2-c)c}{16(1-b^2)} > \frac{1}{4}$.

Let consider new menu of prices $t' = \{t_L', t_H', \overline{s}'\}$ in the form:

$$\begin{cases} t_L' = t_H^e \\ t_H' = +\infty \\ \overline{s}' = s^e + \delta \end{cases},$$

where $\delta > 0$.

Now let's show that the new price t' gives the higher profit to the manufacturer.

Because $t_H' = +\infty$, the retailer have either to meet the market share threshold $\overline{s}'$ or to trade the competitive sector's firm good only. To exclude the manufacturer's good from the trade is not profitable because in this case the retailer obtains its reservation profit only. In the former case, the retailer has to adjust quantities q_1^e, q_2^e to meet the market share threshold. The retailer may change its effort level to $e = 1$ also. Regardless changes in the effort level, the optimal adjustment implies an increase in the manufacturer's quantity q_1. Because of the continuity of the retailer's profit function in $q_1$and q_2, for δ small enough the new retailer's profit is still higher than its reservation profit. Thus, if the new menu of prices t' is offered then the retailer chooses new quantity $q_1' > q_1^e$. Given the manufacturer's price remains the same, $t_L' = t_H^e$, the profit of the manufacturer is higher. Thus $\{t_L^e, t_H^e, s^e\}$ were not the equilibrium values which contradicts to assumption. ■

Proof of the Lemma 4. Suppose, in the equilibrium the manufacturer sets $\{t_L^e, t_H^e, \overline{s}^e\}$. By Lemma 3 $s = \overline{s}^e$.

Suppose the equilibrium level of the retailer's effort is $e^* = 0$. Then all arguments of the Lemma 2 applied with small difference in the following way.

First, $\pi^M_{MSD}(t^e_L, \overline{s}^e) = q_1(t^e_L, \overline{s}^e)(t^e_L - c) = \frac{\overline{s}^e(1-\overline{s}^e t^e_L)}{2(1-2\overline{s}^e(1-b)(1-\overline{s}^e))}(t^e_L - c)$,
with $\frac{\partial \pi^M_{MSD}}{\partial t_L} = \frac{s^2(c-2t_L)+s}{2(1-2s(1-b)(1-s))} \geq 0$ for any $t_L < \frac{(1+cs)}{2s}$.

Thus, for any given level of $\overline{s}^e$, π^M_{MSD} is non-decreasing in t^e_L function for $t_L < \frac{(1+cs)}{2s}$ If $\pi^R_{MSD}(t^e_L, \overline{s}^e)\big|_{e^*=0} > 1/4$ then the manufacturer may increase its profit by an increase in t^e_L. The retailer's response on an increase in t^e_L may imply changes in both the effort level and in quantities q_1, q_2. Regardless changes in the retailer's effort level, the profit of the manufacturers increases for any $t_L < \frac{(1+cs)}{2s}$. The profit of the retailer decreases in t_L and it is less then $\frac{1}{4}$ at $t_L = \frac{(1+cs)}{2s}$.Thus, if $e^* = 0$, the optimal manufacturer's price is such that $\pi^R_{MSD}\big|_{e=0} = \frac{1}{4}$.

Now suppose the equilibrium level of the retailer's effort is $e^* = 1$ and the retailer's profit is $\pi^R_{MSD}(t^e_L, \overline{s}^e)\big|_{e=1} > 1/4$. If the manufacturer increases its price to $t^*_L = t^e_L + \delta$ then the retailer's response may imply changes in both the effort level and quantities q_1, q_2. If the retailer changes the effort level, then $\pi^R_{MSD}\big|_{e=0} = \frac{1}{4}$ as it was shown above. Given that $\delta > 0$ is small enough, an adjustment in quantities still provide the retailer with $\pi^R_{MSD}(t^*_L, \overline{s}^e)\big|_{e=1} > 1/4$. The profit of the manufacturer $\pi^M_{MSD}(t_L, \overline{s})\big|_{e=1}$ increases while the retailer's profit $\pi^R_{MSD}\big|_{e=1}$ decreases for $t < \frac{1+(A_1-1)s+cs}{2s}$ with $\pi^R_{MSD}\big|_{e=1} < 1/4$ if $t = \frac{1+(A_1-1)s+cs}{2s}$. Thus, if $e^* = 1$, the manufacturer's optimal price is such that $\pi^R_{MSD}(t_L, \overline{s})\big|_{e=1} = 1/4$. ■