Asymmetric Auction Experiments With(out) Commonly Known Beliefs[*]

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Abstract

Are commonly known beliefs essential for bidding behavior in asymmetric auctions? Our experimental results suggest that not informing participants how values are randomly generated does not change behavior much and may even make it appear more rational.

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1 Introduction

Except for highly special classes of games, e.g., the class of dominance solvable games,[1] game theory requires commonly known game rules which comprise all strategically relevant information like the strategy sets, the payoffs[2] as well as the beliefs of all players. If any such aspect is uncertain or ambiguous, one usually transforms such uncertainty into uncertainty resulting from more or less partially observable, fictitious chance moves whose probabilities, reflecting individual uncertainties, are assumed to be commonly known (Harsanyi, 1967/8). Common knowledge of the rules of game is extremely unlikely. One often does not know one's own goals; how can one be sure what motivates others? Nevertheless, most applications assume commonly known rules of game without even mentioning or at least trying to justify this assumption.

In auction theory, the second-price private value auction (Vickrey, 1961) is, for instance, dominance solvable. Studies of other auction types, however, assume commonly known beliefs concerning the values of others for the (unique) indivisible object. The same applies to auction experiments (see Kagel, 1995). In auction experiments one usually determines all valuations randomly and informs all participants about the probabilities governing such random moves.[3] In actual auctions such information about the beliefs of other bidders is very unlikely. Thus, experiments without commonly known beliefs are very much needed.

We report experimental results of two-bidder private value auctions where both bidders know that their valuations are positive and independent but not how they are randomly determined. These results will be compared with those of an auction experiment with similarly determined random values but known probabilities. Clearly, such a comparison allows to assess the effects of commonly known beliefs. Since the usual a priori-symmetry of bidders, implied by the *iid*-assumption[4] in auction theory, could encourage a false consensus-effect (others' values are like mine, see Engelmann and Strobel, 2000) it is essential to study an asymmetric auction.

Güth, Ivanova-Stenzel and Wolfstetter (2001) have already explored experimentally an asymmetric two-bidder auction, solved game theoretically by Plum (1992) and Kalkofen and Plum (1996). In the new experiment we do not provide any information how values are randomly generated, neither for the own nor for the other's

[1] In dominance solvable games all players have a unique (weakly) undominated strategy like, e.g., in prisoners' dilemma games or random price mechanisms (Becker, De Groot, and Marschak, 1963).

[2] If payoffs reflect choice behavior, common knowledge of rationality can be described as commonly known payoffs.

[3] Since beliefs are subjective a participant may, of course, entertain beliefs which differ from the experimentally controlled probabilities. This illustrates that in experiments one can try to control subjective factors but never guarantee them.

[4] In the private value-case the *iid*-assumption means that all private values are independently determined by an identical and independent chance move (distribution).

value.[5]

2 The former and the new experimental design

Plum (1992) solved a two-bidder asymmetric auction model where bidders' private valuations are independently drawn from distinct but commonly known distributions, one of which stochastically dominating the other.[6] We use the special case where bidders' valuations v_i for $i = 1, 2$ are independently drawn from uniform distributions with supports $[\alpha, \beta_1]$ and $[\alpha, \beta_2]$, with $\alpha = 50 < \beta_1 = 150 < \beta_2 = 200$. The equilibrium bid functions for the first-price auction are in this case $b_i^f(v_i) = \alpha + \frac{v_i - \alpha}{1+\sqrt{1+\gamma_i c(v_i-\alpha)^2}}$ for $i = 1, 2$ with $c := \frac{1}{(\beta_1-\alpha)^2} - \frac{1}{(\beta_2-\alpha)^2}$, $\gamma_1 := -1$, $\gamma_2 := 1$. Thus bidder 2 with the more favorable valuation bids pointwise less than bidder 1. Furthermore, the first-price auction is inefficient since it happens with positive probability that the bidder with the lower valuation wins the auction.[7] The second-price auction is incentive-compatible so that $b_i^s(v_i) = v_i$, for $i = 1, 2$ and therefore efficient.[8]

We only repeated the first phase of the former experiment[9] consisting of four cycles with 12 bidding rounds each. In a cycle the first(second) six rounds were first(second)-price auctions with newly generated values and randomly assigned partners in each round. We use the instructions of the old experiment except for the part informing subjects about the value ranges for both bidders and the equal probability of all these integers. Instead it was only mentioned that "values are positive and determined for each bidder independently and by chance". The new experiment uses the same random sequences of values v_1, respectively v_2 and the matching protocol of the former experiment. Participants were, of course, not aware of this. We also made sure that no subject participated in both experiments. In this way we repeated five (of the eight) sessions of the former study.[10]

[5] The experimental instructions simply define the payoff for a given value and do not explain how this value is determined. If a participant asked for the way of determining the value, we would have simply answered "You do not know this". In fact the question was never asked by the subjects.

[6] For a more general analysis in a similar framework see Maskin and Riley (2000).

[7] For example, if $v_2 = 150$, the bidder with the lower valuation wins the auction for all v_1 in the interval between 130.18 and 150.

[8] That bidder 2 will never bid more than his rival's maximum valuation β_1 is also justifiable in case of commonly known beliefs.

[9] In the later phases of Güth et al. (2001) participants could choose the auction type.

[10] As before participants received an endowment of 700 ECU (Experimental Currency Unit) to cover possible losses. In both experiments 100 ECU correspond to DEM 1.50 (EUR 0.77). On average subjects' earnings in the former experiment are with 23.54 DEM slightly lower than in the new one (25.41 DEM).

3 The effects of commonly known beliefs

Tables 1 and 2 compare the average efficiency level $E = \frac{v_{\text{buyer}}}{\max\{v_1,v_2\}}$ (the share of efficient plays in brackets) as well as the average prices for both experiments, *WO* (WithOut) and *W* (With commonly known beliefs) separately for first-, respectively second-price auction and all four cycles. Based on session averages the null hypothesis of equal efficiency E for both experiments cannot be rejected ($p \geq 0.548$, Mann-Whitney-U test, two-sided, for both auction types and all cycles). The same is true for the prices in the first price auction. In the second-price auction the prices in the *WO*-experiment are significantly lower at the 5% level for bidders with experience (cycle 3 and 4).

<Table 1 about here>

<Table 2 about here>

The similarly arranged Table 3 compares the average degrees $D = \frac{v_i - b_i(v_i)}{v_i}$ of under-($D < 0$) or over-($D > 0$) bidding the own value for all values $v_i \in \{50, ...150\}$ and for both, the observed and theoretical bids, where we additionally distinguish between bidder types, weak and strong. In case of the first-price auction D is naturally positive.[11] As theoretically predicted the weak bidder bids more aggressively than the strong bidder, i.e., underbids less. In the second-price auction D should be zero and may therefore turn out negative.

Whereas in the *WO*-experiment underbidding slightly increases in the first-price auction for both bidder types, there is almost no overbidding on average in the second-price auction. But again the null hypothesis of no differences between the two experiments cannot be rejected for all cycles of the first-price auction and for the first three cycles of the second-price auction ($p \geq 0.06$). For the last cycle of the second-price auction the differences (nearly no overbidding of bidder 1, underbidding by bidder 2 in case of *WO* compared to substantial overbidding by both in case of *W*) are significant for both bidders (MWU at the 5% level). The lack of commonly known beliefs crowds out the rather robust overbidding in second-price sealed-bid experiments (see, e.g., Kagel and Levin, 1993, Kagel, Harstad and Levin, 1987, Kagel, 1995).

<Table 3 about here>

<Table 4 about here>

Table 4 lists the average earnings of the bidders. Despite the systematically slightly higher profits in the *WO*-experiment for both bidder types none of the observed differences is significant ($p \geq 0.1$). Altogether we hardly ever observe significant differences between the two experiments suggesting no or rather minor effects of commonly known beliefs.[12] If at all the minor effects render behavior more

[11] In the first-price auction bids $b_i \geq v_i$ are weakly dominated, only bids $b_i < v_i$ can generate a positive profit.

[12] One can argue that due to the repetition and the provided feedback after each round subjects

equilibrium-like with experience.

Our main result is that the common knowledge assumption changes behavior only slightly and hardly ever in significant ways. For the applicability of game theory this seems to be rather comforting: although commonly known beliefs are rather unlikely, the results do not seem to depend much on this assumption. However, this suggests that boundedly rational bidders do not engage in game theoretic reasoning. More specifically, it seems that information about beliefs hardly matters for the way how they decide.

References

Becker, G. M., M. H. De Groot, and J. Marschak, 1963, An experimental study of some stochastic models for wagers, Behavioral Science 8, 41-55.

Engelmann, D. and M. Strobel, 2000, The false consensus effect disappears if representative information and monetary incentives are given, Experimental Economics 3, 241-260.

Harsanyi, J .C., 1968, Games with incomplete information played by 'Bayesian' players, part 1: The basic model, part II: Bayesian equilibrium points, part III: The basic probability distribution of the game, Management Science 14, 159-182, 320-334, 486-502.

Güth, W., R. Ivanova-Stenzel, E. Wolfstetter, 2001, Bidding Behavior in Asymmetric Auctions: An Experimental Study, Discussion Paper 15/2001, Sonderforschungsbereich 373, Humboldt University at Berlin.

Kagel, J. H., 1995, Auctions: A survey of experimental research, in: J. H. Kagel and A. E. Roth, eds., Handbook of experimental economics (Princeton, New Jersey: Princeton University Press), 501-585.

Kagel, J. H. and D. Levin, 1993, Independent private value auctions: Bidder behavior in first-, second- and third-price auctions with varying numbers of bidders, The Economic Journal 103, 868-79.

Kagel, J. H., Harstad and D. Levin, 1987, Information impact and allocation rules in auctions with affiliated private values: A laboratory study, Econometrica 55, 1275-1304.

Kalkofen, B. and M. Plum (1996): Optimal pricing-rules for private-value auctions with incomplete information, ifo Studien 42, 77 - 100.

in the *WO*-experiment learn over time the full game structure as provided in the *W*-experiment. This might explain the results in cycle 3 and 4, but not in the earlier ones.

Maskin, E. S. and J. G. Riley, 2000, Asymmetric auctions, Review of Economic Studies 67, 413-438.

Plum, M., 1992, Characterization and computation of Nash-equilibria for auctions with incomplete information, International Journal of Game Theory 20, 393-418.

Vickrey, W., 1961, Counterspeculation, auctions, and competitive sealed tenders, Journal of Finance 16, 8-37.

Auction		First-price		Second-price	
Experiment		WO	W	WO	W
cycle	1	.96 (77%)	.97 (80%)	.97 (87%)	.96 (83%)
	2	.97 (83%)	.97 (83%)	.98 (90%)	.97 (87%)
	3	.98 (87%)	.97 (86%)	.98 (91%)	.98 (87%)
	4	.99 (90%)	.98 (87%)	.99 (91%)	.99 (95%)
all		.97 (84%)	.97 (84%)	.98 (90%)	.98 (88%)

Table 1: Average Efficiency E (share of efficient allocations)

Auction		First-price			Second-price		
Experiment		WO	W	$E(p^*)$	WO	W	$E(p^*)$
cycle	1	103.3	107.3	91.1	83.1	83.4	89.3
	2	98.7	103.3	88.6	86.9	91.2	89.8
	3	104.5	106.9	90.5	87.0	92.8	89.8
	4	102.8	105.8	91.1	82.6	89.7	85.5
all		102.3	105.8	90.1	84.9	89.3	88.6

Table 2: Average Realized and Expected Equilibrium Prices

Auction		First-price						Second-price					
Bidder		1			2			1			2		
Experiment		WO[a]	W[a]	D^{*b}	WO[a]	W[a]	D^{*b}	WO[a]	W[a]	D^{*b}	WO[a]	W[a]	D^{*b}
cycle	1	.23	.18	.21	.27	.23	.25	.03	.01	.00	.05	-.02	.00
	2	.19	.17	.20	.27	.22	.23	-.01	-.03	.00	.04	-.03	.00
	3	.18	.16	.21	.23	.20	.25	-.01	-.09	.00	.03	-.06	.00
	4	.17	.16	.21	.21	.21	.25	-.01	-.10	.00	.06	-.05	.00
all		.19	.17	.20	.24	.21	.24	.00	-.05	.00	.05	-.04	.00

Table 3: Average degree D of under-(over-)bidding
[a: observed bids; b: theoretical bids]

Auction		First-price				Second-price			
Bidder		1		2		1		2	
Experiment		WO	W	WO	W	WO	W	WO	W
cycle	1	6.85	5.05	21.75	20.74	13.43	13.88	38.32	36.97
	2	8.75	6.84	20.45	17.68	12.93	9.72	40.02	36.98
	3	6.30	5.08	24.27	22.56	11.60	8.44	35.14	31.86
	4	6.35	6.14	24.51	21.60	14.70	11.88	37.43	33.21
all		7.06	5.78	22.75	20.64	13.17	10.98	37.73	34.76

Table 4: Average Profits